Supplementary Information

Supplementary Information 1: mathematical details on the marginal data posterior fitting.

Marginal data posteriors

In order to fit the marginal data posteriors (MDPs) we use the θ which was calculated in the calibration stage and introduce the fossil XRF data (XRF^f). We use k MDPs of each time slice as a reference to the sum of each multivariate normal distribution for each of the 11 chemical elements (j)

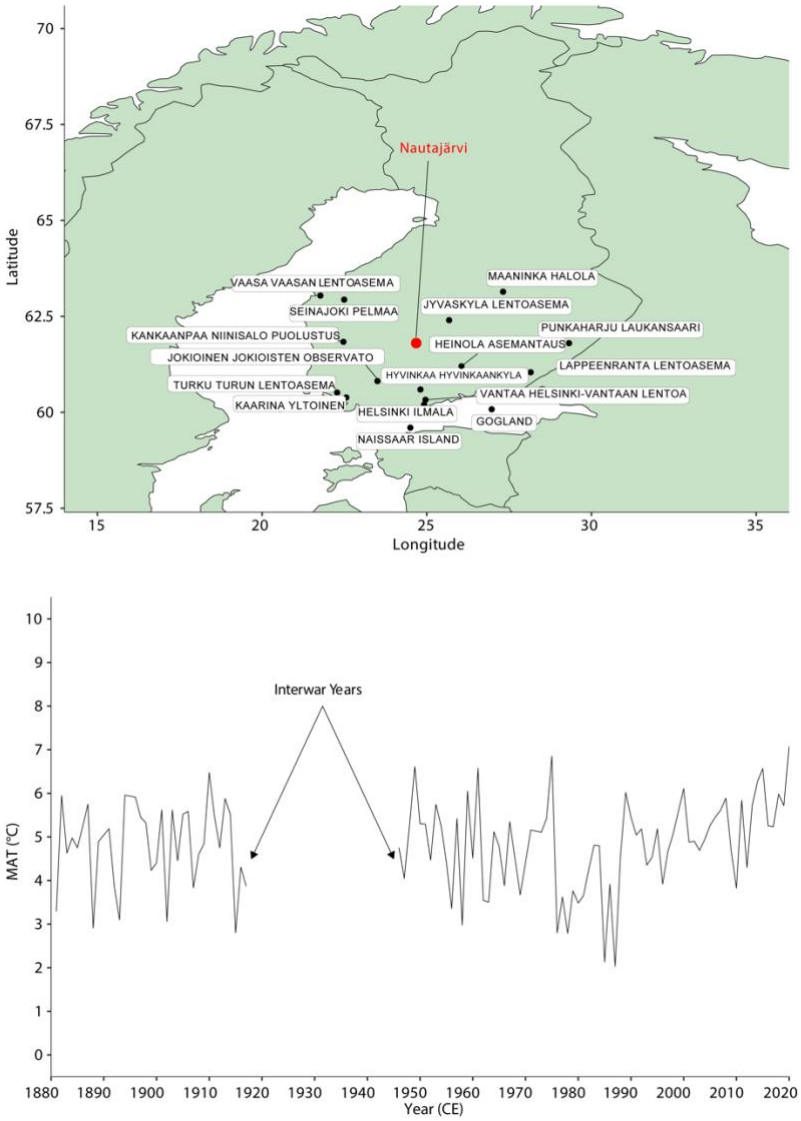
$$MDP_{ik} | \, C_i \sim MVN(\mu_{MDP_i}, prec_{MDP_i})$$

This stage also requires the use of a climate grid which is 50 possible climate values which are placed on a 50 x 50 regular grid. For example, if we were estimating temperature anomalies, the grid may look like a 50-part sequence from -3 °C to +3 °C, or precipitation may look like -100 mm to +100 mm. By having the climate grid helps the model choose sensible climate scenarios.

The output of the final MDP calculation is a MDP for each layer of the fossil XRF data which can be used within the final reconstruction of climate through time.

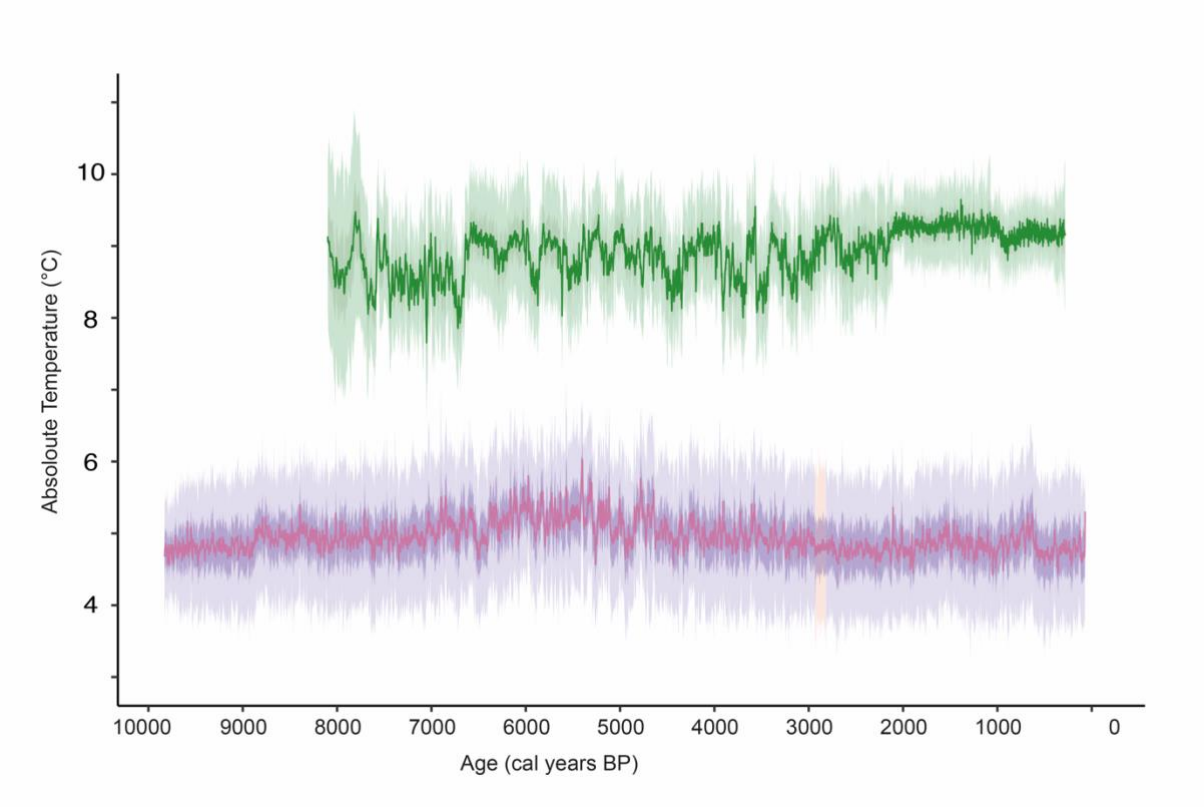
In the walk-through example (Section 4.0 main text), the climate grid chosen for Diss Mere was -3 °C and +3 °C having a range of 6 °C in total which is sufficient to cover the variability in temperatures reconstructed in the Holocene for other studies (Osman et al., 2021).

Supplementary Information 2: locations of meteorological weather stations for Nautajärvi calibration



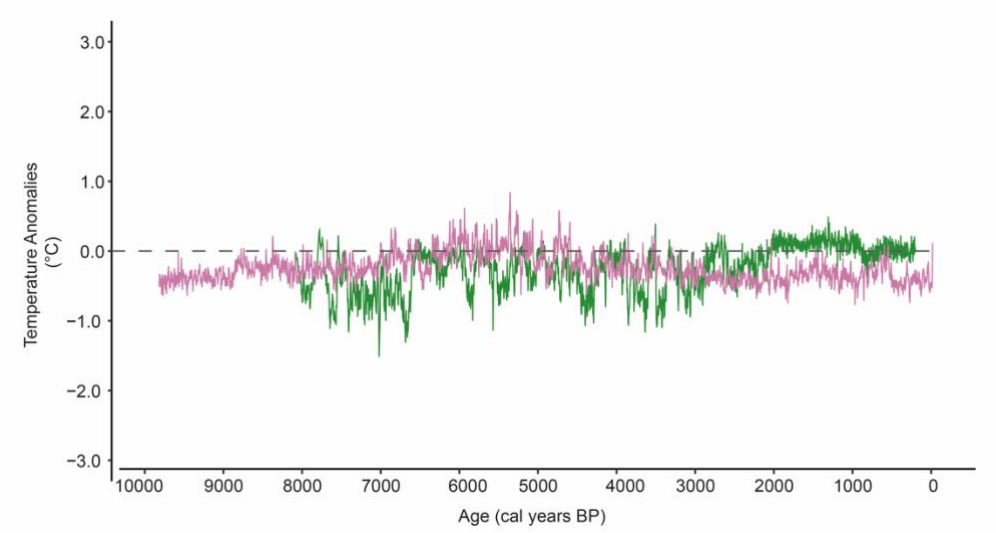
Supplementary Figure 1: A figure showing the locations of the weather stations used to build a composite record of annual mean temperatures for Nautajärvi and a timeseries showing the record with an interwar period of no data.

Supplementary Information 3: Absolute temperatures of the Diss Mere and Nautajärvi reconstruction



Supplementary Figure 2: A figure showing the Diss Mere (green) and Nautajärvi (purple) reconstructions in absolute temperatures

Supplementary Information 4: Diss Mere and Nautajärvi temperature reconstructions



Supplementary Figure 3: Diss Mere (green) and Nautajärvi (purple) reconstructions without uncertainty envelopes. Reference period 1991-2020