We would like to thank Pierre Francus (Reviewer 1) for his constructive suggestions and comments, which have helped to improve our manuscript entitled “SCUBIDO: a Bayesian modelling approach to reconstruct palaeoclimate from multivariate lake sediment data.” Below, the reviewer’s comments are shown in red, and our responses in black.

I read this article with great interest. It's very easy to read, although the part describing the Bayesian model is more difficult, probably because my understanding of these statistics is very limited. I'll leave it to others to comment on this mathematical part. The proposed approach should be of interest to several researchers active in the study of varves, and I'm looking forward to trying it out myself. Applying this approach to all varved sites should, as the authors write in their conclusion, « (…) produce more reconstructions of an annual resolution to then be incorporated into large data compilations ». It is therefore a significant contribution to paleoclimatology.

We thank you for your interest in our work. Like you, we hope that members of the varve and palaeolimnology community will find this research both interesting and useful.

Although the article is very interesting, I am very surprised that the authors do not show any comparison of their new Bayesian reconstructions with the existing reconstructions for Nautajarvii and Diss Mere. It seems to me that such comparisons would have been far more relevant than comparisons of the Bayesian reconstructions of these two sites with each other, or with reconstructions based on large-scale databases (Temp 12k, LGMR, Holocene-DA, and LMR). If the authors have good reasons for not making this comparison, it's essential to say why.

We thank the reviewer for this comment. Whilst there have been previous palaeoclimate reconstructions published from these lakes, neither lake has produced an annual mean temperature reconstruction per se.

Diss Mere: Martin-Puertas et al. (2023) presented varve thickness data as a proxy of climate variability and compared to a AMOC simulation to explore decadal oscillatory variability in the North Atlantic realm. The thickness of the summer laminae (authigenic calcite precipitation) was also compared to a summer temperature simulation for this region to support the response of the lake to a specific climate parameter. Based on these findings, Boyall et al. (2024) suggested that the Ca-clr record as a proxy for authigenic calcite precipitation, might be also responding to summer temperature as Ca is mostly deposited in the summer months, supported by the findings from a modern lake monitoring investigation at Diss Mere published in Boyall et al (2023). We decided not to include the comparison with the varve data or the Ca-clr profile in the main manuscript as we thought that it may lead to misunderstanding as these proxy records show a clear seasonal bias toward the summer and show a different temperature evolution (i.e. proxies sensitive to summer temperature show a Holocene Thermal Maximum while the SCUBIDO annual mean temperature shows gradual warming thorough the Holocene, see response to Reviewer 2 about the HTM). However, we attach here a comparison between the Diss Mere temperature reconstruction from this study and the Ca-clr record from Boyall et al. (2024) in case you are interested to see the differences between the two records.

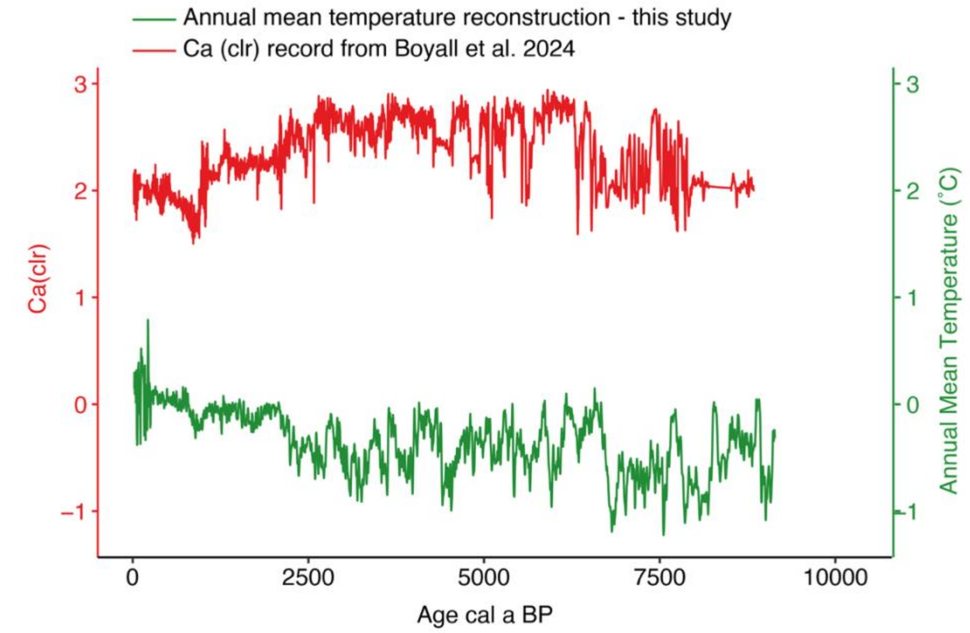

Figure A1-1 Comparison between Diss Mere annual mean temperature reconstruction in green for this manuscript and the Ca-clr record from Diss Mere published in Boyall et al. (2024) in red. Both records are presented at a 10-year moving average.

Nautajärvi: similar to Diss Mere, varves are responding to a combination of different climatic parameters and disentangling them from the varved record is challenging (Ojala et al., 2005). There is a pollen-inferred growing degree days (GDD) reconstruction (Ojala et al., 2008), and whilst this tends to have a good relationship with annual mean temperature, GDD reflects variability in the growing season (summer) and can indicate both long, mild summers, or short and hot summers. The figure below is the Nautajärvi reconstruction from SCUBIDO overlaid with the GDD and whilst the long-term trend is similar, we prefer not to show this comparison within the manuscript given that they are reconstructions of two different climate parameters and thus may not add much to the discission in this current manuscript.

In the revised manuscript we will explain to interested readers why we have compared with the large dataset reconstructions rather than site-specific reconstructions.

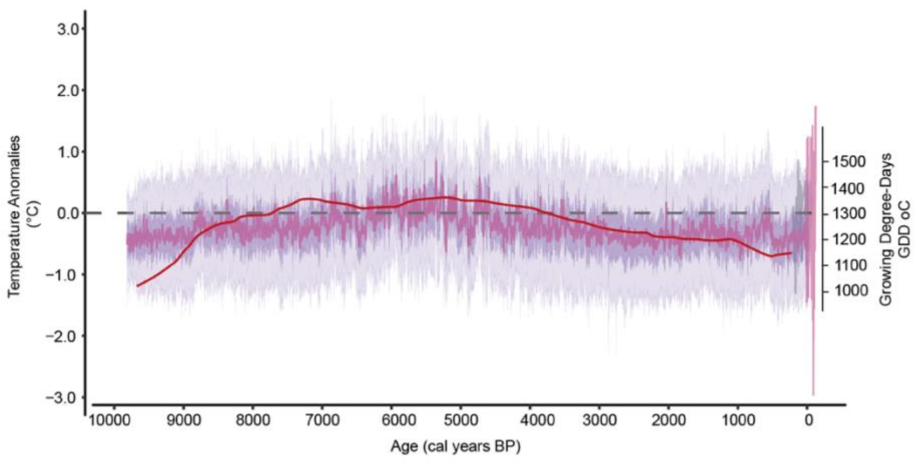

Figure A1-2: Nautajärvi annual mean temperature reconstruction from this study (purple)with growing degrees reconstruction from Ojala et al. (2008) (red line)

Figures 1 and 2 show no relationship between XRF-CS or reconstructed with actual temperature, at least when considering a classical statistical approach. It's something of a surprise to see that a Bayesian approach manages to derive information from these relationships that are not visible. Could you address this in your discussion? It might help convince readers of the validity of your approach.

We agree with all three reviewers that Figure 1 does not demonstrate clear relationships between temperature and the individual XRF elements. However, we would like to emphasise that within this modelling approach we are not using the relatively weak relationship between each element and climate individually and instead are harvesting from the joint response of each of the elements together in a multivariate response regression approach (sometimes known as seemingly unrelated regression; SUR, for example Mbah et al. 2018) which provides us with a more precise posterior estimate of climate. This means that the model is learning from both the direct quadratic relationship between element and climate, and also the group response to climate represented as correlations between the elements. A similar example of this is when pollen is used to reconstruct climate. A single species may have a weak relationship with the climate target variable, but when used in combination with species assemblage it is this joint relationship that can find a good match with a climate variable. This is a very common approach in Bayesian reconstructions of palaeoclimate (e.g. Haslett et al 2006, Parnell et al 2015), though individual relationships are often not shown in papers. We have decided to keep Figure 1 as we believe some readers will be interested in observing the relationships between the individual elements and temperature. However, to ensure that this figure is not misinterpreted, we will include additional sentences to the main text and the figure captions explaining the reasons behind the weak looking relationships.

In response to the concerns about Figure 2, we agree that in the previous version of the manuscript the relationship between true and reconstructed climate was not illustrated well. Since this initial submission we have re-assessed the calibration period and have now started the calibration period at 1700 CE rather than 1669 CE as there were several gaps in the Diss Mere XRF data between 1659 CE and also documented uncertainties in the HadCET dataset. We have also identified in our script used for validation that there was a small coding bug which meant that we were misaligning our true instrumental temperature and reconstructed temperature by one year (this was due to us rounding up the decimal places in the ages). This has since been adjusted and a new Figure 2 will be presented in the manuscript which displays a good relationship between true and reconstructed temperature. But we also attach it here for you to view now (Figure A1-3), as well as the Nautajärvi plot (Figure A1-4) which will be going into the Supplementary Information.

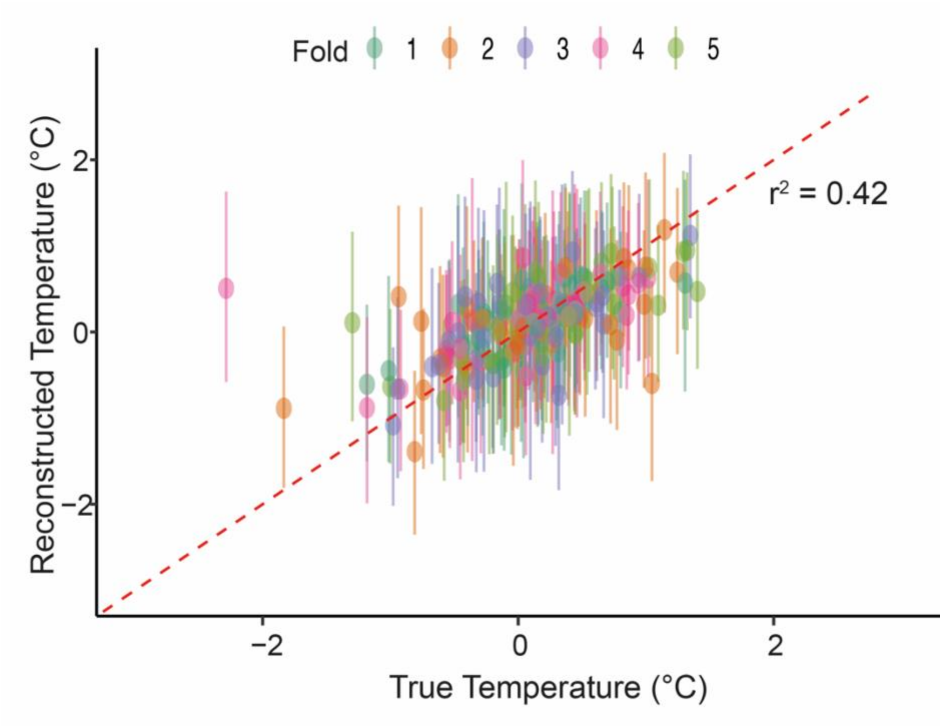

Figure A2-3: The results from the out-of-sample cross validation results for Diss Mere with true temperature used in the calibration period against reconstructed temperature from the SCUBIDO model. Colours represent the different folds and lines represent the 95% confidence interval. This will be replaced as Figure 2 in the updated version of the manuscript.

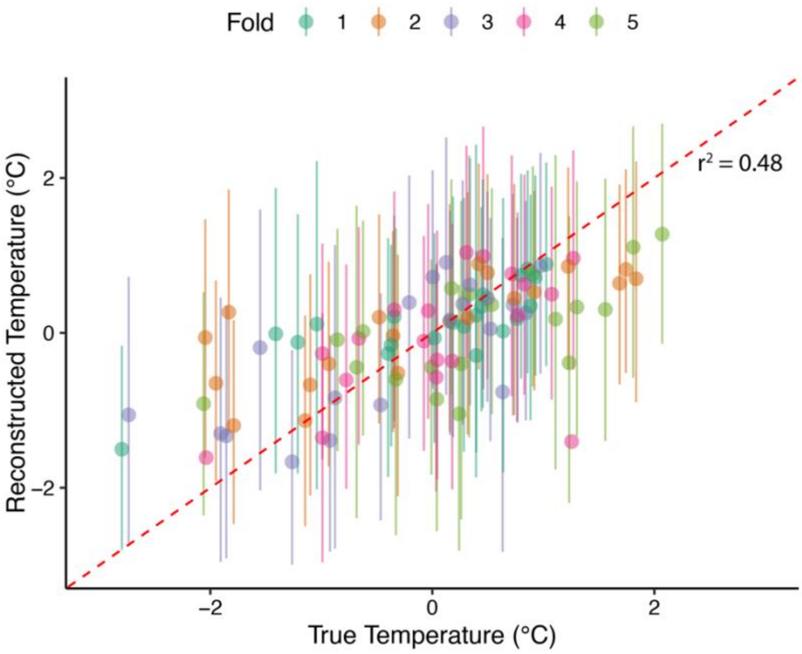

Figure A1-4: The results from the out-of-sample cross validation results for Diss Mere with true temperature used in the calibration period against reconstructed temperature from the SCUBIDO model. Colours represent the different folds and lines represent the 95% confidence interval. This will be replaced as Figure 2 in the updated version of the manuscript.

If the upper part of Diss Mere contains no varved record, it would be desirable to better describe the chronology of the part that served as Modern calibration dataset between 1659 CE and 1932 CE. I suggest adding a supplement, but it seems important to me to have a figure so that the reader can appreciate the quality of the age model in this time interval.

The full chronology and age model for the non-varved sediments is described and published in Boyall et al. (2024). This is based on a combination of tephra layers that link the non-varved and the varve chronology, radiocarbon dates and the 1963 CE 137Cs peak. The average age uncertainty for the non-varved section of Diss Mere is ± 65 years and thus is higher than the varve chronology. During the calibration period (1700 – 1932 CE), the age uncertainty is smaller at the top of the sequence with a maximum uncertainty of ± 22 years between 1932 CE and 1800 CE, however this increases gradually to ± 110 years for the following century (Supplementary Figure 1). The sedimentation rate within this period is very high (0.15 cm/year) and includes up to 20 data points per year.

We agree with the reviewer that the information about the calibration period should be included in the supplementary information and thus we will add this, and a age-model for this period. This will go as Supplementary Figure 2 but is Figure A1-5 in this response.

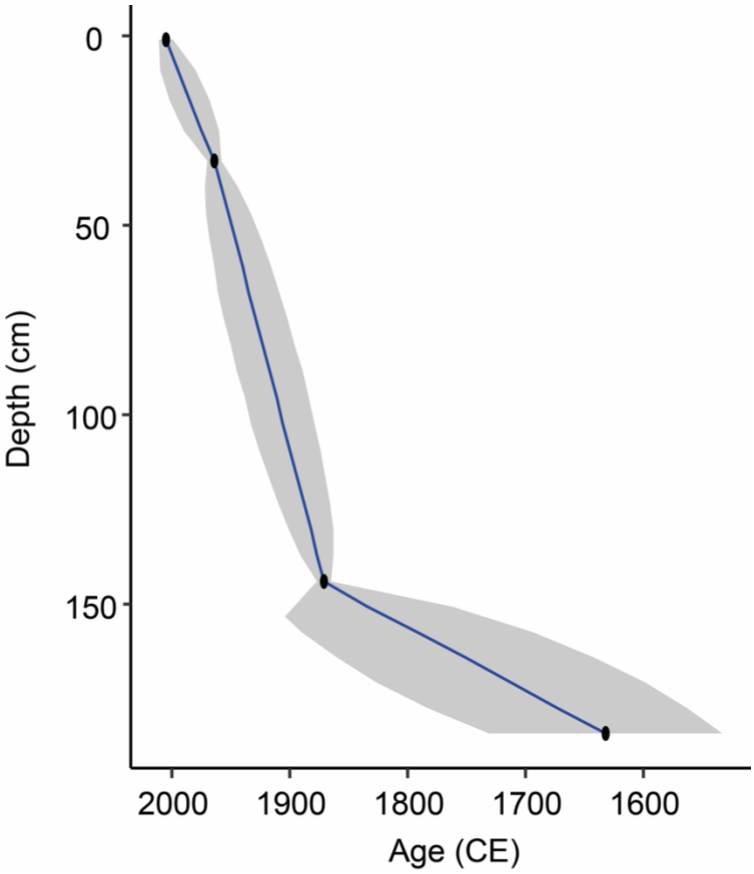

Figure A1-5: Age mode for the calibration period (1700-1932 CE) for Diss Mere. This age model is part of the published age-model from Boyall et al. (2024).

Finally, it would be good to define all the variables in the equations presented. Some are indeed not defined, such as MVN, Mi, ωi,... Perhaps in a list of abbreviations and variables in an appendix?

This was a great idea, and we thank the reviewer for this. We have created this table and will add this as Supplementary Table 1.

I have further minor comments which are listed below.

L23: add « Micro » before X-Ray.

We will correct this in the revised manuscript.

L73: what is the starting temporal resolution Erb? Please specify in the text

We will correct this in the revised manuscript.

L136: replace "different" by "two"

We will correct this in the revised manuscript.

L148-149: CLR-transformation requires that elements having too many "0" values are dismissed. Have you done that?

Yes, only the elements which did not contain a large number of null values were included. We will clarify in the text when we introduce the Diss Mere data.

L307-309: along the same lines, how these elements have been selected? Maybe should you also specify the dwell time for each acquisition step.

We will clarify this in the text that they were chosen based on having a <15% standard error. We will also add in the dwell time.

L319: I suggest replacing "As a result of these findings" by "Therefore"

We will correct this in the revised manuscript.

L333: what temperature are you referring to here, because two lines further on, you say you're using the average annual temperature.

We will clarify that we mean annual mean temperature.

L338-339 : « which differentiate SCUBIDO from other recent reconstructions based on varved sediments (Zander et al., 2024)". This information is available later (L375-377) and more detailed there. I would delete this part of the sentence.

We will correct this in the revised manuscript.

L350: How have you resampled to annual means? Using the varve boundaries? Or using the equation of the age model?

We will clarify in the text that we have just used linear interpolation. We could not use the varve boundaries as this was in the uppermost non-varved section.

L367-370: Please try to use less jargon, or explain what a "Burn-in period", and the "chains" are.

We will correct this in the revised manuscript.

L392: a reference to the literature would be useful here.
We will correct this in the revised manuscript.

L414: please use "µXRF-CS" everywhere or nowhere.
We will correct this in the revised manuscript.

L431: it seems that the last 2000 years are less variable in Diss Mere because this corresponds to the non-varved section.
We also believe that this could be the case, though this happens slightly before the end of the varves and thus did not include this interpretation in the previous version. However, we will add this into the sentence as a potential cause for the loss of variability.

L437: Do you want to say "lithology" instead of "stratigraphy".
We will correct this in the revised manuscript.

Table 1 is missing some information, such as the how the elements have been selected, the dwell time, the resolution, the anode composition of the X-ray tube.
We had initially not included this as the information is present in Lincoln et al. (2025). But we will now add some of this information into Table 1 and will refer readers to Lincoln et al. (2025) for more detail.

Figure 4 "please specify in the caption that Nautajarvi is the pink curve and Diss Mere the green one.
We will correct this in the revised manuscript.

L554: "handling" instead of "handing"?
We will correct this in the revised manuscript.

Please add a section "Data availability" as required by the editorial policy. Reference to the codes (Gitub) and, if you can, the XRF-CS data. Maybe also indicate where the Temp 12k, LGMR, Holocene-DA, and LMR datasets have been downloaded from.
We will correct this in the revised manuscript.

Additional references not included in the manuscript bibliography but mentioned in the response:

Mbah, C., Peremans, K., Van Aelst, S., & Benoit, D. F. (2018). Robust Bayesian seemingly unrelated regression model. Computational Statistics, 34(3), 1135–1157. https://doi.org/10.1007/s00180-018-0823-x