

1 Supplement code S1

2 Stan code for solubility correction of the wet extraction measurements

```
3 data {
4   int<lower=1> n;      //no. of elements for regression
5   int<lower=1> nwet;  //no. of total wet extraction data
6   vector[n] x;       //x data for regression of equation-3
7   vector[n] xerr;    //x error as 1 stdev
8   vector[n] y;       //y data for regression of equation-3
9   vector[n] yerr;    //y error as 1 stdev
10  vector[nwet] wetx;  //x data for the entire wet extraction data set
11  vector[nwet] wetxe; //wetx error as 1 standard deviation
12  vector[nwet] cosw;  //wet extraction COS measurements
13  vector[nwet] ecosw; //1 stdev uncertainties for COS measurements
14 }
15 parameters {
16   real a;            // slope
17   real b;            //intercept
18   vector<lower=1e-15>[n] truex;
19 }
20 model {
21   x ~ normal(truex, xerr);      // equation-4
22   y ~ normal(b + a * truex, yerr); // equation-5
23 }
24 generated quantities {
25   vector<lower=1e-15>[nwet] twetx; // resample x for the data set
26   vector<lower=1e-15>[nwet] tcosw; // resample COS measurements
27   vector[nwet] corr;           // solubility correction magnitude
28   vector[nwet] corr_cos;       // solubility corrected COS
29   for(i in 1:nwet) {
30     twetx[i] = normal_rng(wetx[i], wetxe[i]);
31     tcosw[i] = normal_rng(cosw[i], ecosw[i]);
32     // calculation of corrected COS based on equation-3
33     corr[i] = b + a * twetx[i];
34     corr_cos[i] = -1 * tcosw[i] * corr[i] / (1 - corr[i]);
35   }
36 }
```


38 **Supplementary code S2**

39 **Errors-in-variables regression of ssNa vs. COS and correction for COS production**

```
40 data {
41     int<lower=1> n; //number of elements in data vectors
42     vector[n] x1; //ssNa interpolated to COS depths
43     vector[n] xerr1; //error for dint ssNa
44     vector[n] x2; //ssNa interpolated to COS ages
45     vector[n] xerr2; //error for aint ssNa
46     vector[n] y; //COS
47     vector[n] yerr; //COS error
48 }
49 parameters {
50     real a1; //depth slope
51     real b1; //depth intercept
52     real a2; //age slope
53     real b2; //age intercept
54     real alpha; //multiplication factor for COS errors
55     vector<lower=1e-15>[n] xt1; //true dintNa
56     vector<lower=1e-15>[n] xt2; //true aintNa
57 }
58 model {
59     x1 ~ normal(xt1, xerr1);
60     x2 ~ normal(xt2, xerr2);
61     y - (a2 * xt2) ~ normal(b1 + a1 * xt1, alpha * yerr);
62     y - (a1 * xt1) ~ normal(b2 + a2 * xt2, alpha * yerr);
63 }
64 generated quantities {
65     vector[n] xt1new;
66     vector[n] xt2new;
67     vector[n] ynew;
68     vector[n] dpcorr;
69     vector[n] apcorr;
70     vector[n] dypred;
71     vector[n] aypred;
72     for(i in 1:n) {
73         xt1new[i] = normal_rng(xt1[i], xerr1[i]);
```

```
74     xt2new[i] = normal_rng(xt2[i], xerr2[i]);
75     ynew[i] = normal_rng(y[i], alpha * yerr[i]);
76     dpcorr[i] = a1 * xt1new[i];
77     apcorr[i] = a2 * xt2new[i];
78     dypred[i] = ynew[i] - dpcorr[i];
79     aypred[i] = ynew[i] - apcorr[i];
80 }
81 }
82
```