Orbital Insolation Variations, Intrinsic Climate Variability, and Quaternary Glaciations

First of all, we would like to thank all three referees for their very thorough and careful reading of our paper. Following their constructive criticism and valuable feedback, we would like to propose several changes to the manuscript. We are convinced that these changes will substantially improve the quality and clarity of our manuscript and that they will address the referees' objections, questions and suggestions. Whenever we prefer to leave the current version of the manuscript unchanged, where a referee has proposed a change, we have made an effort to justify our view thoroughly. Finally, there is some overlap between the remarks of the referees. We have taken the freedom to answer some comments by more than one referee simultaneously. Whenever a point raised by a certain referee has already been addressed in our reply to another referee, we simply refer to this answer.

In order to improve the readability of our replies we applied a color coding to discriminate our replies from the referees comments. Please understand that we have attached our replies as a pdf document since color coding is not available in this browser based text editor.

Color coding:

Comment by the referee.

Reply from the authors.

Text from the original version of the manuscript.

Suggested improved text.

Referee 3

At lines 102 (section 1) the authors bring in the notion of a Hopf bifurcation with one type of simple system (eqn 5). Then in section 2.2 the description of the subcritical and supercritical Hopf bifurcations are described with another system (eqn 6). I would like to see more diagrams in Section 2.2 (some of us can visualise in our head what is happening when parameters are varied (e.g. through a Hopf bifurcation) but I think it is important to try to improve section 2.1 and 2.2 in a more unified way so at make these sections more accessible to a newer audience that is reading this type of material for the first time. I think a clear illustration with both language and an additional set of figures (possibly using the example systems from equation 5 or 6) would be helpful. For example, one might introduce the sections with language such as, "A Hopf bifurcation occurs when a periodic solution or limit cycle that surrounds an equilibrium point appears or disappears when a (control) parameter is varied. When the stable limit cycle surrounds an unstable equilibrium point, the bifurcation is supercritical. In the case that the limit cycle is unstable and surrounds a stable equilibrium point, the bifurcation is subcritical.” And thenalso illustrated these concepts later on with the simple systems used.

We thank the referee for this very useful and constructive criticism. We addressed this comment in our reply to the Major Comment #1 made by Referee #2.

In general the paragraphs are quite short (e.g Line 54). There is no need to start a new paragraph in a lot of places in the manuscript, please try to make the text flow a bit better.

We will carefully revise the manuscript with respect to this comment and combine paragraphs where it will improve the readability.

L243, 249 monotonic, monotonically

Thank you for pointing this out. Will be corrected accordingly.

In the section on 3.3 on applications D-O events. Figure 8a is a bit confusing , maybe I missed it , but I don’t see how the abscissa and ordinate are defined , it looks like simply x and y, yet they are both scaled to \alpha?

In fact, there are many labels missing in the uploaded version of the manuscript, but they are present in the local version stored on our computers. We apologize for the inconvenience.

However, the caption of Figure 8, still does not sufficiently explain the Figure. We will replace the caption

‘FitzHugh-Nagumo (FHN) model with time scales $\tau_f = 2000$, $\tau_x = 100$, $\tau_y = 60$, and $\alpha = 2$. (a) The cubic term of the fast derivative $P_3(x, y)$ as a function of y for $x = 0$ (solid blue line); dashed lines indicate the same function with $x = \pm 2\alpha/$ sqrt(27). (b) Trajectories of the nonautonomous model, with $\gamma(t) = \sin(t/\tau_f)$, and starting at the times $\{t_0 = -20$ kyr, $t_1 = -16$ kyr, $t_2 = -13$ kyr, $t_3 = -7$ kyr$\}$ in the (x, y)-plane, using different colors for t_0, t_1, t_2 and t_3. (c) The time-dependent forcing $\gamma = \gamma(t)$. (d, e) The same trajectories as in (b), p but plotted in time, as $y = y(t)$ and $x = x(t)$, respectively. ; in panels (c)–(e), the gray shading indicates intervals during which $|\gamma| > 1/3$ and the internal oscillation is suppressed.’

by

‘FitzHugh-Nagumo (FHN) model with time scales $\tau_f = 2000$, $\tau_x = 100$, $\tau_y = 60$, and $\alpha = 2$. (a) The cubic term of the fast derivative $P_3(x, y)$ as a function of y for $x = 0$ (solid blue line); the red lines point to the maximal (2α/ sqrt(27)) and minimal (2α/ sqrt(27)) values of P_3(x=0, y) – these are the maximal values by which P_3 can be shifted up or down, while maintaining all three roots. Correspondingly, dashed lines indicate the shifted function with $x = \pm 2\alpha/$ sqrt(27). The violet lines labeled y_min and y_max mark the right and left boundaries for the roots y_l and y_r, respectively. y_l and y_r can never be located in between the two violet lines. (b) Trajectories of the nonautonomous model, with $\gamma(t) = \sin(t/\tau_f)$, and starting at the times $\{t_0 = -20$ kyr, $t_1 = -16$ kyr, $t_2 = -13$ kyr, $t_3 = -7$ kyr$\}$ in the (x, y)-plane, using different colors for t_0, t_1, t_2 and t_3. (c) The time-dependent forcing $\gamma = \gamma(t)$. (d, e) The same trajectories as in (b), p but plotted in time, as $y = y(t)$ and $x = x(t)$,

respectively. ; in panels (c)–(e), the gray shading indicates intervals during which |γ| > 1/3 and the internal oscillation is suppressed.'

For example at line 371, the description of the gamma and the fixed points that arise. I don't see any description on how the y = \gamma nullcline intersecting the cubic polynomial P_3(x,y) (manifold) is what determines the unstable or stable fixed points of the system. There are a lot of \alpha symbols illustrated on Figure 8a but there is no clear description in my opinion.

Figure 8a shows P_3(x,y), it does not show the nullcline of y with respect to x. However, in a revised version of the manuscript we will add a panel to Figure 8, that shows the nullclines of x and y in the x-y plane with different values for \gamma.

In FIgure 8c the authors show the non-autonomous forcing for \alpha(t) and then on line 378 they introduce the non-autonomous \gamma and it is not clear what physical implications that \alpha provides although \gamma is related to CO2 eventually, and how the two non-autonomous forcing are related.

We agree that we did not provide sufficient physical explanation on what we are doing with the FHN model. This will be improved in a revised version of the manuscript. Please see also our response to the comment on line 407 made by Referee #1.

The authors have not referenced or discussed how this work relates to or improves upon the work of Roberts and Saha (2016) which also illustrate non-autonomous dynamics on a FHN type model. In particular Roberts and Saha draw particular attention to how they modulate the slow manifold through time dependent changes and attempt to relate it to physical mechanisms (e.g. insolation forcing). They also introduce the time dependent sinusoidal forcing on the linear nullcline in the slow component of the slow-fast system. I'm not sure which processes are more important in attempting to explain the last glacial cycle millennial scale variability; either through an amplitude modulation of the slow manifold based on obliquity paced variations as Roberts and Saha have done or the time dependant variation of the linear nullcline using CO2 as the authors have done here.

We thank the referee for bringing to our attention the Roberts and Saha (2016) paper. This work should definitely be mentioned in Section 3.3 of our manuscript. Together with additional explanation on the choice of forcing, we will point to their study and elaborate on how our example relates to this work in a revised manuscript.

The legend for t0,t1,t2,t3 in Figure 8 looks incomplete.

Sorry, this must have happened during the update process.

I like figure 8b , I would almost like to see the 4 curves illustrated separately as the red dominates. I'm not sure if there is an easy way to do this.

We will try to improve the visibility of the different trajectories by changing the linewidths and the line styles.

I like the section on the MPT , but the additional value seems to come from the incremental understanding achieved from relating it to more recent concepts from NDS and RDS. I don't particularly think the title is completely appropriate , but I don’t have a good alternate suggestion. The authors mention orbital insolation , but there are also internal mechanisms, plus a lot of discussion on NDS and RDS , but I’m not sure you can formulate this into an adequate short title.

Thank you for the suggestion. After an internal discussion we concluded that we would like to keep the section title as is.