

Additional comment to the manuscript cp-2021-119 “Influence of the choice of insolation forcing on the results of a conceptual glacial cycle model” by Gaëlle Leloup and Didier Paillard

Dear Andrey,

Certainly, the “gravity acceleration, Plank constant or Milankovitch frequencies” are not tunable parameters, but since they are dimensional, they may be part of adimensional similarity parameters that are indeed tunable. For example, the parameter “viscosity” may not be tunable itself but it is part of the Reynolds number that may change. Therefore, when we create an inventory of governing parameters, physical constants must be included.

I do not think that the number of parameters is a matter of taste because it defines plausibility of the entire model. I have briefly mentioned in my original comment a hidden parameter “1” but may be it deserves a more close consideration.

I hope you and the authors would agree with me that any model of a physical phenomenon should be derivable from the basic laws of physics, providing some assumptions and bearing the “cost” of such assumptions as tunable model parameters. From this perspective, the presented by the authors model (1) is, indeed, an ice sheet mass balance, that is:

$$\frac{dV}{dt} = AS \quad (R21)$$

Here V is ice volume, S is ice sheet area, and A is accumulation minus ablation. All variables here are dimensional. Simply speaking, the changes in ice volume are caused by net accumulation over its entire area. Since $S=V/H$ (H is ice thickness) we can re-write the mass balance as:

$$\frac{dV}{dt} = \frac{V}{\tau} \quad (R22)$$

where $\tau = H/A$.

The ice thickness H is, generally speaking, the function of ice volume, but since $H \sim V^{1/5}$, setting it to be constant may be (reluctantly) accepted. Setting A to be constant is a strong assumption either. The “cost” of these two assumptions are constant timescales adopted by the authors.

The $\frac{V}{\tau}$ term in (R22) is important for ice-sheet dynamics. During the glaciation stage, for example, it is responsible for a positive feedback, specifically: a growing ice sheet spreads as a viscous media increasing its footprint and thus collecting accumulation from a larger area. Indeed, the replacement of $\frac{V}{\tau}$ by $\frac{1}{\tau}$ in the mass balance equation (R22) would be equivalent to changing ice dynamics from $V \sim e^{\frac{t}{\tau}}$ to $V \sim \frac{t}{\tau}$. It may be acceptable on short timescales ($\frac{t}{\tau} < 1$) but on the timescales used in the study ($\frac{t}{\tau} > 1$) the mutation of $\frac{V}{\tau}$ into $\frac{1}{\tau}$ in the glaciation equation (1) needs to have a physical explanation. Formally, in the presented model, the glaciation equation currently contains not just a “hidden” parameter but a “hidden” function $\frac{1}{V}$. This function needs to be exposed and physically described. Without such justification, model (1) cannot be recognized as a physical model and any results may have a somewhat limited

explanatory value. Whatever physical phenomenon is going to be invoked for $\frac{1}{V}$ validation, the “cost” of it will be at least one more governing parameter.

Even if this new parameter (let us for the specificity call it λ) appears in a ratio $\frac{\lambda}{\tau_g}$ and incomplete similarity in parameters λ, τ_g can be claimed, such that $\frac{\lambda}{\tau_g} = \frac{1}{\tau_{g_new}}$, ($\tau_{g_new} = \frac{\tau_g}{\lambda}$) the bifurcation trajectory due to the evolution of τ_{g_new} can be caused by two physically distinct processes, i.e., by changed ice dynamics (τ_g) and by changing λ – physics, whatever the authors designate it to be.

Mikhail Verbitsky