Review of the manuscript cp-2021-119 “Influence of the choice of insolation forcing on the results of a conceptual glacial cycle model” by Gaëlle Leloup and Didier Paillard

The motivation for the research is well articulated. Indeed, the insolation amplitudes under different configurations of the orbital forcing may vary significantly enough to potentially influence our understanding of the climate history. To demonstrate the sensitivity of modeling results to the orbital forcing configuration, the authors offer a number of numerical experiments with a specific dynamical model.

Unfortunately, the experimental set-up is not comprehensive enough and therefore the results are not conclusive.

The authors are a bit overoptimistic when they introduce their model as the one having only 5 governing parameters. In fact, in the presented set-up, the model has 13 governing parameters, specifically:

$I_0, V_0, \tau_i, \tau_g, \tau_d, I_{41}, I_{23}, I_{22}, I_{19}, T_{41}, T_{23}, T_{22}, T_{19}$

Here $T_{41}, T_{23}, T_{22}, T_{19}$ are orbital periods (an index shows a numerical value in kyr) and $I_{41}, I_{23}, I_{22}, I_{19}$

are corresponding insolation amplitudes. Only timescales and orbital periods have dimension of time, other parameters are adimensional.

If we select, let say, the obliquity period as a parameter with independent dimension, then, according to π-theorem, the amplitude of the 100-kyr system response V_{100}will be a function of 12 adimensional similarity parameters:

$$V_{100} = \Phi(I_0, V_0, \frac{T_{41}}{\tau_i}, \frac{T_{41}}{\tau_g}, \frac{T_{41}}{\tau_d}, I_{41}, I_{23}, I_{22}, I_{19}, \frac{T_{41}}{T_{23}}, \frac{T_{41}}{T_{22}}, \frac{T_{41}}{T_{19}}) \quad \text{(R1)}$$

It would be physically reasonable to assume $\frac{T_{41}}{T_{23}}, \frac{T_{41}}{T_{22}}, \frac{T_{41}}{T_{19}}$, and I_0 to be constant and, using generalized π-theorem, to migrate to 8 similarity parameters:

$$V_{100} = \Phi(V_0, \frac{T_{41}}{\tau_i}, \frac{T_{41}}{\tau_g}, \frac{T_{41}}{\tau_d}, I_{41}, I_{23}, I_{22}, I_{19}) \quad \text{(R2)}$$

The authors demonstrated that period-doubling bifurcation caused by rising deglaciation threshold V_0 is not very sensitive to the choice of $I_{41}, I_{23}, I_{22}, I_{19}$. Unfortunately, they stopped here, and, by avoiding variations of $\frac{T_{41}}{\tau_i}, \frac{T_{41}}{\tau_g}, \frac{T_{41}}{\tau_d}$ similarity parameters, they left readers with the impression that changing deglaciation threshold V_0 is the only option to reproduce the middle-Pleistocene transition. As Verbitsky et al (2018) and Verbitsky and Crucifix (2020, 2021) have demonstrated, the space of possibilities to produce a period-doubling bifurcation similar to the middle-Pleistocene transition is much wider and (if we continue to speak in terms of the model being reviewed) $\frac{T_{41}}{\tau_i}, \frac{T_{41}}{\tau_g}, \frac{T_{41}}{\tau_d}$ similarity parameters definitely have their roles in the drama. The authors' reasoning to keep τ_i, τ_g, τ_d constant because “these parameters gave correct behaviour in previous studies” is surprising – actually, their past experience with τ_i, τ_g, τ_d speaks about importance of these parameters for the system dynamics and therefore strongly advocates for them to be in the center of the study. Formally, making τ_i, τ_g, τ_d constant is equivalent to claiming complete similarity in parameters $\frac{T_{41}}{\tau_i}, \frac{T_{41}}{\tau_g}, \frac{T_{41}}{\tau_d}$ and excluding them from the solution (R2). Since $\frac{T_{41}}{\tau_i}, \frac{T_{41}}{\tau_g}, \frac{T_{41}}{\tau_d}$ reference values are of the same order of magnitude as V_0, there is absolutely no physical justification for such decision. Indeed, let us imagine that $\frac{T_{41}}{\tau_i}$ similarity parameter is changing in such way that it becomes a dominant similarity parameter relative to $\frac{T_{41}}{\tau_g}, \frac{T_{41}}{\tau_d}$. It means that equations (1d) and (1g) become identical, system (1) becomes linear and independent of V_0, and a period-doubling bifurcation is impossible:

$$V_{100} = \Phi(\frac{T_{41}}{\tau_i}, I_{41}, I_{23}, I_{22}, I_{19}) \quad \text{(R3)}$$

Thus $\frac{T_{41}}{\tau_i}$ similarity parameter can control what the authors call the middle-Pleistocene transition, and the period-doubling bifurcation can be produced by slow change of $\frac{T_{41}}{\tau_i}$ similarity parameter from its relatively high values to its relatively low values under constant threshold V_0. The amplitude of the system response V_{100}will evolve from solution (R3) to solution (R2). Obviously, slow changes of $\frac{T_{41}}{\tau_g}, \frac{T_{41}}{\tau_d}$ may also produce a period-doubling bifurcation. Furthermore, the system (1) has one more "hidden" parameter, i.e., "1" in the glaciation equation. In fact, it is terrestrial ice mass influx that was tacitly set to be constant and equal 1. Recognition of this parameter is important for the exactly same reasons we outlined above for τ_i, τ_g, τ_d, and, indeed, it is yet another potential source of bifurcation.

We do not know what specific bifurcation (or their mix) we observe in the historical records and therefore the following question needs to be answered: How sensitive are ***all*** bifurcations, which system (1) may reveal, to our choice of insolation forcing?

Without answering this question the study is incomplete and inconclusive.

Minor comments:

1. In system (1) the glaciation equation is marked as (d) and the deglaciation equation is marked as (g). All references in the text to (g) and (d) states are therefore incorrect.
2. Line 81 "there is no contribution from the obliquity…" It is incorrect, since the obliquity is definitely present in all forcing spectra
3. Line 93 "The importance of orbital forcing alone seems able to start a glaciation…" "The importance" cannot start anything. The phrase needs to be re-formulated.
4. Line 94 "Therefore, the condition to switch from the deglaciation state to the glaciation state is based on insolation only: it is possible to enter ***deglaciation*** when the insolation becomes low enough". You mean "glaciation" here.
5. Line 245 "frow"
6. Line 273 "To model future natural evolutions of the climate system, one would need to take into account for possible evolutions of the V_0 threshold." English should be revisited.

References

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