Answer to RC 3 :

We thank the reviewer for his helpful comments and suggestions that will help us revise and improve the manuscript. We hope the answers and modifications proposed satisfactorily address his remarks.

In the following, the reviewer's comments are in black, our answer in blue and suggested corrections in green.

G. Leloup and D. Paillard

Additional comment to the manuscript cp-2021-119 “Influence of the choice of insolation forcing on the results of a conceptual glacial cycle model” by Gaëlle Leloup and Didier Paillard

Dear Andrey,

Certainly, the “gravity acceleration, Plank constant or Milankovitch frequencies” are not tunable parameters, but since they are dimensional, they may be part of adimensional similarity parameters that are indeed tunable. For example, the parameter “viscosity” may not be tunable itself but it is part of the Reynolds number that may change. Therefore, when we create an inventory of governing parameters, physical constants must be included.

I do not think that the number of parameters is a matter of taste because it defines plausibility of the entire model. I have briefly mentioned in my original comment a hidden parameter “1” but may be it deserves a more close consideration.

I hope you and the authors would agree with me that any model of a physical phenomenon should be derivable from the basic laws of physics, providing some assumptions and bearing the “cost” of such assumptions as tunable model parameters.

We have to disagree with this statement. Different kinds of models, with different kinds of assumptions, allow to study different kinds of questions. Our approach differs from the one of Verbitsky et al (2018). Our model is not physically based, but is a phenomenological model. It certainly does not pretend to be “derived from” or “based on” physical laws, but only pretend to be “consistent” with physics. Our assumptions are therefore not “tunable parameters” but, more simply, modeling choices that are convenient to economically reproduce the phenomenology of ice ages.

Ice sheet mass change is driven by various processes, affecting surface mass balance, ice discharge to the ocean and bottom melt of grounded ice. Here, we do not intend to explicitly represent the numerous physical processes involved in ice sheet volume evolution. The aim of our conceptual model is not to explicitly represent physical processes but rather to help us understand some critical aspects of the climate system.

Our non-physical model can however help us to raise physical questions. To fit the geological record with our model, the deglaciation threshold needs to increase when other parameters are kept constant. This kind of model raises the question of what physical phenomena could be responsible for making deglaciations “harder” to start on the latest part of the Quaternary compared to the earliest part (Paillard, 1998; Tzedakis et al 2017). We demonstrate here that this conclusion does not depend on insolation choices. However, this question cannot be answered with our model.

We propose to clarify the use of conceptual models in the manuscript with addition to the conclusion, l. 296.
"More generally, this kind of glacial-interglacial conceptual model is designed to explain the main features of the Quaternary time period characterized by the waning and waxing of Northern Hemisphere ice sheets under the influence of changing astronomical parameters. In our case, this raises the question of what physical phenomena are responsible for making deglaciations "harder" to start on the latest part of the Quaternary compared to the earliest part. This kind of model is however unlikely to be directly applicable in a more general context, like the Pliocene and earlier periods, or in the context of future climates under the long-term persistence of anthropogenic CO_2 (Archer and Ganopolski, 2005; Talento and Ganopolski, 2021)."

Modifications of the description of the conceptual model are proposed afterwards.

From this perspective, the presented by the authors model (1) is, indeed, an ice sheet mass balance, that is:
$dV/dt = AS$ (R21)
Here V is ice volume, S is ice sheet area, and A is accumulation minus ablation. All variables here are dimensional. Simply speaking, the changes in ice volume are caused by net accumulation over its entire area.
Since S=V/H (H is ice thickness) we can re-write the mass balance as:
$dV/dt = V/\tau$ (R22)
where $\tau = H/A$.
The ice thickness H is, generally speaking, the function of ice volume, but since $H{\sim}V^{1/5}$, setting it to be constant may be (reluctantly) accepted. Setting A to be constant is a strong assumption either. The "cost" of these two assumptions are constant timescales adopted by the authors.
The V/τ term in (R22) is important for ice-sheet dynamics. During the glaciation stage, for example, it is responsible for a positive feedback, specifically: a growing ice sheet spreads as a viscous media increasing its footprint and thus collecting accumulation from a larger area. Indeed, the replacement of V/τ by $1/\tau$ in the mass balance equation (R22) would be equivalent to changing ice dynamics from $V{\sim}e^{t/\tau}$ to $V{\sim}\ t/\tau$. It may be acceptable on short timescales (t/τ < 1) but on the timescales used in the study (t/τ > 1) the mutation of V/τ into $1/\tau$ in the glaciation equation (1) needs to have a physical explanation. Formally, in the presented model, the glaciation equation currently contains not just a "hidden" parameter but a "hidden" function $1/V$. This function needs to be exposed and physically described. Without such justification, model (1) cannot be recognized as a physical model and any results may have a somewhat limited explanatory value. Whatever physical phenomenon is going to be invoked for $1/V$ validation, the "cost" of it will be at least one more governing parameter.

Indeed, we agree with the reviewer that our model is not a physical model, We therefore do not provide any explicit physical explanation to "justify" the precise formulation of our equations.
The terms V/τ_d and $1/\tau_g$ represent trends linked to the current state of the system : slow glaciations and quick deglaciations.

The term $1/\tau_g$ allows to account for processes that do not depend on the ice sheet area and may represent a function of many possible physical phenomena (ice sheet basal temperature for instance is certainly a key “long-term” physical variable, but many other candidates are likely to be involved - isostasy, carbon cycle, to name a few…)

We suggest some additions (starting l.70) to the manuscript to make this point clearer :

“For the glacial-interglacial cycles, it is not a new idea that the climate system can be represented by relaxation oscillations between multiple equilibria, like a glaciation and a deglaciation state (Paillard, 1998; Parrenin and Paillard, 2003, 2012).
The model used in our study is an adapted and simplified version of the conceptual model of (Parrenin and Paillard, 2003). The aim of conceptual models is not to explicitly model and represent physical processes but rather to help us understand critical aspects of the climate system. Here, we do not intend to explicitly represent the numerous physical processes involved in ice sheet volume evolution, affecting surface mass balance, ice discharge to the ocean and bottom melt of grounded ice. Instead, we represent the climate system by two distinct states of evolution : the "glaciation state" (g) and "deglaciation state" (d).
We make the assumption that the evolution of the ice sheet volume in these two states can be simply described by two terms. The first one, common to the glaciation and deglaciation states, is a linear relation to the summer insolation : when the insolation is above average, the ice sheet melts, whereas when the insolation is low enough, the ice sheet grows. The second term, specific to the system state, represents an evolution trend linked to the system state : a slow glaciation in (g) state and a rapid deglaciation in (d) state.

The evolution of the ice volume in these two states in our model is described by :

(g) $dv/dt = -I / \tau_i + 1/ \tau_g$
(d) $dv/dt = -I / \tau_i - V/ \tau_d$

where *v* represents the normalized ice volume. τ_i, τ_d τ_g, are time constants.
I is the normalized summer insolation forcing at 65° N, a typical latitude for Northern Hemisphere ice sheets.”

Even if this new parameter (let us for the specificity call it λ) appears in a ratio $\lambda/\tau g$ and incomplete similarity in parameters λ, τg can be claimed, such that $\lambda/\tau g = 1/\tau g\_new$, ($\tau g\_new = \tau g/\lambda$) the bifurcation trajectory due to the evolution of τg_new can be caused by two physically distinct processes, i.e., by changed ice dynamics (τg) and by changing λ – physics, whatever the authors designate it to be.

The “1” is not a hidden parameter, as we could rewrite the equation (1g) :
$dV/ dt = -I / \tau_i + \alpha$ with $\alpha = 1/\tau_g$.
The $\alpha = 1/\tau_g$ parameter could be linked to various physical processes, that we do not intend to represent explicitly with our model.
A complete parameter enumeration of our conceptual model (which is not physically based) is therefore a list of 5 mathematical parameters (I_0, V_0, τ_i, τ_g, τ_d) while the true physics behind may likely involve many more physical ones.