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Climate of the Past
Discussions

A Modified Milankovitch theory that reconciles contradictions with the paleoclimate record

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Abstract

Based upon research results over the past five decades, there has been a general acceptance that the ice ages were initiated by astronomical phenomenon. Specifically, marine, ice and terrestrial paleoclimate data have supported elements of the Milankovitch astronomical theory of the ice ages. However, there remain unresolved problems between the empirical findings and theory. The "100 thousand year problem" has been the subject of extensive research since a 100 thousand year cycle that matches the Earth orbit eccentricity period dominates the frequencies found in paleoclimate records. Yet, eccentricity produces an insignificant variation in annual solar energy. Other problems include the "Stage 11 problem", the " missing interglacials problem", how glaciation is sustained over multiple tens of thousands of years and synchronous hemispheric glaciation. I shall show these problems are resolved by modification of the prevailing Milankovitch theory. In particular, two elements of the theory need modification. One is the limitation of eccentricity's role and the other assuming that glaciation results only from cool summer conditions. By applying the Solar Energy Invariance law to define e-seasons, how eccentricity provides conditions for glaciation is demonstated. The results show eccentricity variations provide significant solar energy variations at the top of the earth's atmosphere to produce glaciation that is global. Global glaciation results in colder winter glaciation occurring in one hemisphere simultaneous with cool summer glaciation in the other hemisphere. Analysis with these modifications resolves each of the problems.

1 Introduction

The Milankovitch (1998) theory of astronomical forcing of the ice ages has been the benchmark that paleoclimate proxy records have been compared. Ground breaking research (Hays, et al. 1976) revealed a strong correlation of oxygen isotope proxy cycles to cycle periods matching those of Earth orbit eccentricity, precession and obliquity leading to the general acceptance that astronomical elements are responsible for pacing of the ice ages. However, unresolved contradictions between empirical findings and theory have been revealed. In particular, spectral analysis revealed a dominant 100 thousand year (kyr) cycle in the oxygen isotope records matching the eccentricity period, yet eccentricity has an insignificant impact on annual solar energy leading to studies for an explanation. This is referred to as the "100 kyr problem". The trend in recent studies has been to dismiss eccentricity and to attribute the dominant cycle to multiples of precession or obliquity cycles, consistent with the Milankovitch assertion that eccentricity played a minor role in glaciation. The prevailing version of Milankovitch's theory is limiting both seasonally and geographically by focusing on cool summers in high latitudes of the Northern hemisphere, primarily mid-summer at 65 degrees north latitude (S65N). I shall show that by expanding the theory globally and seasonally that eccentricity does provide necessary solar energy conditions for global glaciation. Other contradictions resolved include the "Stage 11 problem", the " missing interglacials problem", how glaciation is sustained over multiple tens of thousands of years and synchronous hemispheric glaciation.

2 Eccentricity, the Solar Energy Invariance law and e-seasons

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Of the three astronomical elements, eccentricity (e) determines the change in solar energy reaching the top of the Earth atmosphere (TOA) through changing orbit geometry (distance) and dynamics (velocity). The Earth's mean annual solar energy varies according to $(1-e^2)^{-1/2}$ and is virtually invariant at less than 0.3% change over the past million years leading the Milankovitch theory to dismiss eccentricity as the source of the dominant 100 kyr cycle. Many alternatives to eccentricity have been proposed such as Earth orbit inclination variations exposing the Earth to interplanetary dust clouds with a near 100 kyr period (Muller and Mac Donald, 1997), that near 100 kyr cycles originated from every fourth or fifth 22 kyr precession cycle (Ridwell et al. 1999), that obliquity with a 41 kyr cycle could be near 100 kyr after every two or three cycles (Huybers and Wunsch, 2005) or simply that eccentricity's significance was a myth (Maslin and Brierley, 2015). A prevailing theory is that eccentricity provides a weak trigger for producing non-linear feedbacks amplifying the weak signal (Imbrie et al., 1993). Non-linear response sources proposed have included ice sheet dynamics (Imbrie and Imbrie, 1980) and global carbon/greenhouse gas cycles (Shackleton, 2000).

To demonstrate the significance of eccentricity, I first examine the relationship of solar energy and orbit mechanics and then quantify with insolation curves. Solar intensity varies inversely with the square of Earth-Sun distance and solar energy is solar intensity applied over a period of time. Sir John Herschel derived a theorem (Herschel, 1835; 1902) that "equal amounts of heat are received from the Sun in passing over equal angles round it." I shall refer to this theorem as the Solar Energy Invariance law where the solar energy received by the Earth in one degree of transit is invariant anywhere along the orbit regardless of eccentricity. An interesting example is that the energy over one degree of arc at perihelion (closest distance) equals the energy over one degree of arc at aphelion (furthest distance). The lower solar intensity at aphelion is compensated by a slower velocity and longer arc thus more time of exposure. Conversely, the higher intensity at perihelion is compensated by a higher velocity and shorter arc thus less time of exposure.

Although solar energy per degree of Earth transit is invariant, the solar energy per day does vary with Earth-Sun distance. This allows me to define eccentricity-derived seasons or *e-seasons*. How eccentricity influences daily energy is illustrated in Fig. 1 for a hypothetical 100 kyr cycle where Earth orbit moves from circular to elliptical (e = 0.05) and then back. The figure is a depiction not drawn to scale. The e-seasons are defined by dividing the Earth orbit into 90-degree quadrants. Aphelion and perihelion are at the center of their quadrants. The aphelion and perihelion quadrants will be referred to as the ap and peri seasons respectively. The upper quadrant between peri and ap seasons is the post-peri season. The opposite season is the post-ap season. Since annual energy is virtually constant and applying the Invariance law, each e-season receives one-fourth the annual total throughout the cycle. The figure illustrates the areas swept out by the Earth's radius during its travel through the ap (blue) and peri seasons (orange). As the orbit becomes elliptical (shifting to the left), the aphelion distance becomes longer and the perihelion shorter since the aphelion to perihelion distance (major axis) remains constant. The area swept out in the ap season expands (dark blue) as eccentricity increases while the area for the peri season contracts (dark orange). Despite the area differences, the

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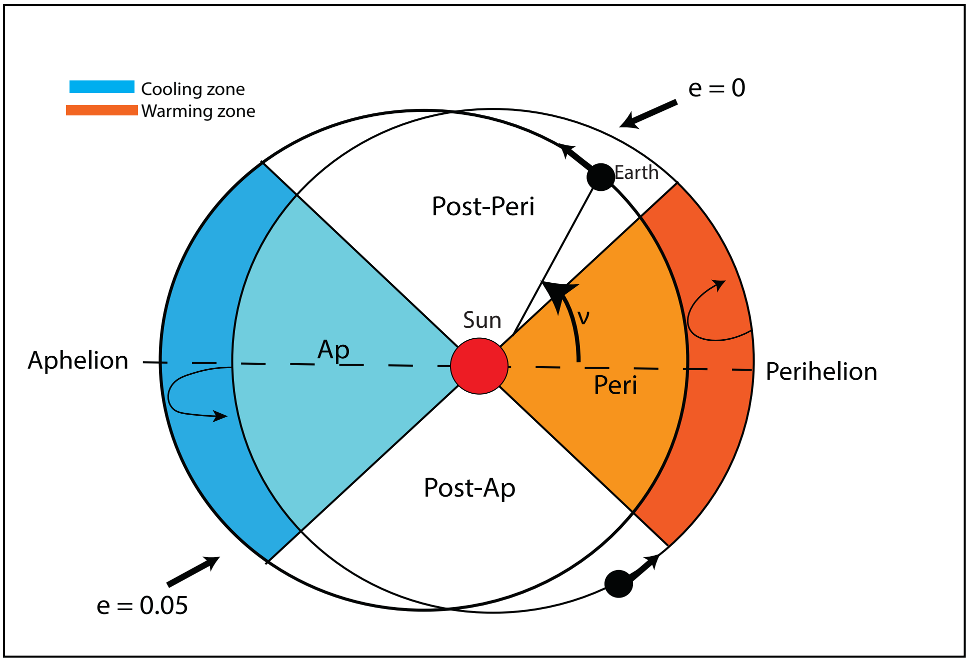

Figure 1. The four e-seasons in a 100 kyr eccentricity cycle. The chart depicts the Earth orbit changing from an eccentricity (e) of 0.0 (circular orbit) to 0.05 (elliptical) and back. The Earth moves counter clockwise in the orbits. The aphelion Earth-Sun distance increases while the perihelion distance decreases as e expands but the sum remains constant. Applying the Solar Energy Invariance law and Kepler's second law, the change will result in ap season cooling (dark blue) and peri season warming in (dark orange). The cooling and warming are global. ν is the true anomaly.

solar energy of each e-season remains the same due to the Invariance law. Kepler's second law states that equal areas are swept in equal times so that ap season cooling results from a longer travel time lowering average daily energy. This is expected as Earth-Sun distance is longer in the ap season. Conversely, the peri season is warming due to a shrinking area, shorter time and higher average daily energy. The post e-season areas remain constant as does their average daily energy. As the orbit becomes increasingly elliptical, the ap season sustains increased cooling and the peri season sustains increased warming until eccentricity equals 0.05, then the process reverses ending back as a circular orbit after 100 kyr. The contrast between the cooling ap season

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and the warming peri season grows with eccentricity increases and diminishes as eccentricity decreases. The cooling and warming is global and not affected by the Earth's spin axis tilt to the orbit plane. Cooling is experienced in both hemispheres simultaneously as is warming. As the Earth cools while moving through the ap season, the traditional season being cooled could be summer, winter, spring or fall. The same situation holds for warming in the peri season. The interplay between e-seasons, traditional seasons and hemispheres will be addressed in Sect. 4.

The great ice sheets of the ice ages resulted from sustained snow growth over thousands of years of continuous glaciation conditions. Croll (1875) postulated that glaciation is produced by long periods of colder winters where snow accumulated as snow replaced rain at higher latitudes. On the contrary, Milankovitch (1998) postulated that glaciation and snow/ice accumulated from sustained cooler summers reducing annual snowmelt. Both Croll and Milankovitch defined winter and summer in terms of half years. Which theory is valid has been a long running debate (Imbrie and Imbrie, 1989). Global cooling in the ap season indicate both the Croll and Milankovitch type glaciations could simultaneously occur in opposite hemispheres. Annual snow growth for building ice sheets would result from more snow created in the colder winter of one hemisphere and less snowmelt in a cool summer of the other hemisphere. Additional snow increases albedo levels would promote further cooling. The warmer peri seasons would provide additional moisture for more snow at high latitudes. That glaciation is global is widely accepted and supported by the paleoclimate record (Imbrie and Imbrie, 1989, Imbrie et al., 1993; Lisiecki and Raymo, 2005; Lang and Wolff, 2011). This empirical support of global glaciation is also empirical support for Croll glaciation since cool summer glaciation in one hemisphere means colder winter glaciation in the other. Bol'shakov (2000, 2011) has consistently advocated the concept of cool summer and colder winter glaciation coexisting in opposite hemispheres. I define deglaciation in this analysis as decreasing eccentricity reversing the glaciation conditions.

It should be noted that both Croll and Milankovitch postulated that more moisture would be created during warmer summers or warmer winters in the peri seasons. The increasing solar energy raises the vaporization rates. This would be true for the early stages glaciation. However, as glaciation matures diminishing returns would result as the shrinking global water reserve due to glaciation offsets the rising vaporization rate resulting in declining moisture volume. Thus, during the latter part of a glaciation phase, ice sheet growth will be expected to decline.

3 Global e-season insolation curves

The eccentricity of the past million years (Laskar, 1993; 2018) shown in Fig. 2a reveal three discernable cycle periods of approximately 65 kyr, 100 kyr and 400 kyr. The eccentricity of the present interglacial, a relative warm period, is 0.0167 (dashed line) and in the midst of a projected 65 kyr cycle (partially shown). The eccentricity ranges from less than 0.01 to 0.06. Daily solar energy is my measure of insolation. The e-season average daily energy is the e-season total energy divided by the e-season length in days (See Appendix A). The e-season insolation curves of Fig. 2b present the average daily

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solar energy as a percentage variation from the circular orbit daily solar energy. The term "insolation" here will refer to the solar energy variation not the magnitude. The daily

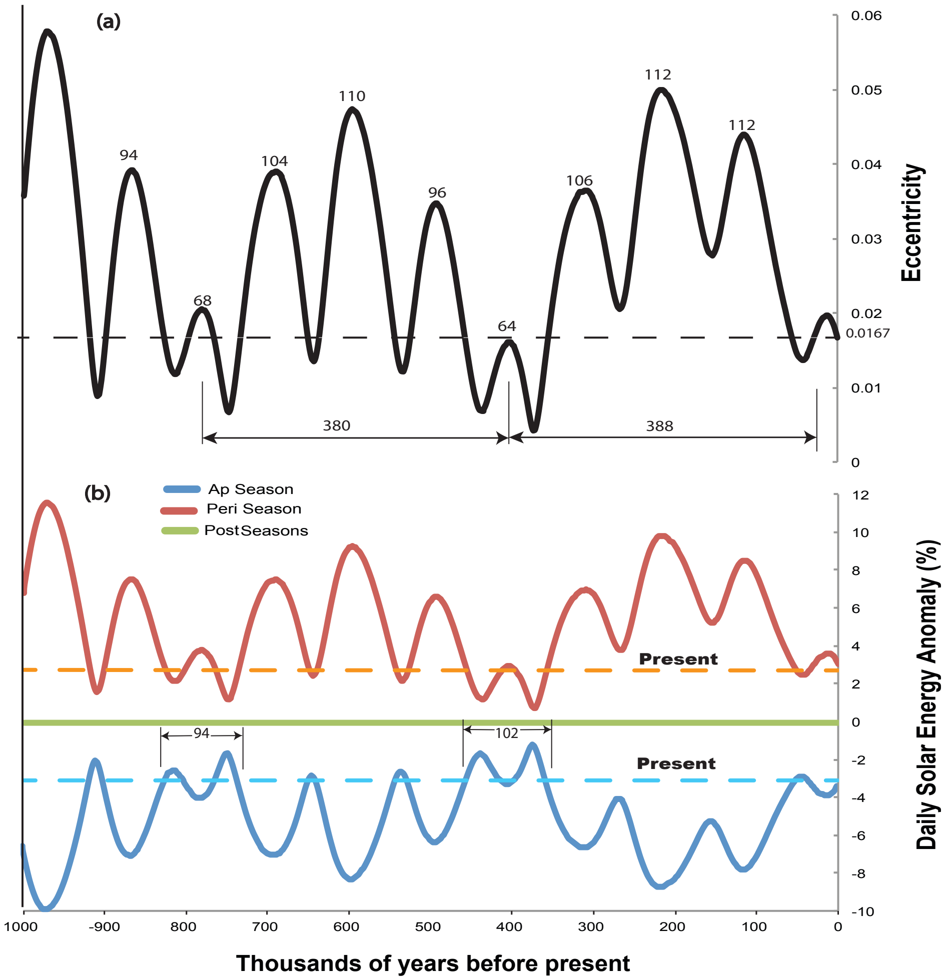

Figure 2. (a) Earth orbit eccentricity for past one million years. The chart exhibits three cycles with periods approximating 65 kyr, 100 kyr and 400 kyr cycles. The dashed line indicates the present eccentricity. (b) Global daily solar energy anomalies for the e-seasons over the past one million years. The anomalies are variations from the reference circular orbit daily solar energy. The post-ap and post-peri seasons have average daily solar energies equivalent to the circular orbit.

energy for post-ap and post-peri seasons are equal to the circular orbit level for the entire one million years so their variations are nil. Since total annual energy is constant,

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increased ap cooling (more negative) is countered by higher peri warming and reduced cooling is accompanied by reduced warming. Conditions for glaciation occur during increasing eccentricity resulting in increasing ap season cooling (more negative) and increasing peri season warming while conditions for deglaciation occur during diminishing eccentricity. The ap season anomaly values range down toward negative 10% and the peri positive variation toward positive 12% indicating a contrast over 20%. Global temperature variations are not directly derived from eccentricity deviations since annual solar energy is constant. Instead temperature increases and decreases are derived indirectly by Earth system responses to solar energy variations including the contraction and expansion of the cryosphere, the corresponding albedo variations and the carbon cycle.

I do not specifically define an interglacial, only that an interglacial would exist when ap season insolation is near or above the present interglacial level and conversely when peri season insolation is near or below the present level. Figure 2b indicates long interglacials occur between the 400 kyr cycles and shorter ones exist between 100 kyr cycles. The 65 kyr cycles are contained within the major interglacials. As eccentricity approach low values, obliquity is influential in producing interglacials. This will be discussed in Sect. 4. The major interglacial at 400 kyr before present (ka) has been widely studied as geological findings indicate conditions may have been similar to present conditions (Howard, 1997; Loutre and Berger, 2003; Rowling et al., 2010; Yin and Berger, 2015) and may provide clues to the Earth's future. The low peri season insolation levels here demonstrate the "400 kyr problem" or "Marine Isotope Stage (MIS) 11 problem" since the prevailing expectation is that a warm period should exhibit high insolation levels. On the contrary, interglacials occur when eccentricity approaches zero as this is a relatively warm state due to a shrinking cryosphere and other reversals of Earth responses. The solar energy and variation for both the warm peri season and the cool ap season are low here. This problem will be addressed again with respect to low summer insolation at the 400 ka interglacial.

4 Traditional seasons and hemispheric insolation curves

The Earth's spin axis tilt and wobble distributes the eccentricity-derived solar energy geographically and seasonally. Obliquity (ε) is the angle between the spin axis and a vertical to the Earth's orbit plane, the ecliptic. This angle varies between 22.1 and 24.5 degrees with a period of 41 kyr. The spin vector, defined by its spin direction, lies along the spin axis and points north. Summer solstice occurs when the vector projected onto the ecliptic points directly at the Sun. This defines orbit positions for the autumnal equinox, winter solstice and vernal equinox each separated by 90 degrees respectively. Precession is the axis wobble whose projected vector angle is measured relative to perihelion and has a 22 kyr period. For this analysis, precession refers to its angular rotation only, not to be confused with the term "precession" in many studies referring to the product of eccentricity and the sine of the precession angle. Currently, obliquity is 23.4 degrees and decreasing. The Earth's traditional seasons result from the tilt so that precession has a significant impact on these seasons relative to the e-seasons. For this

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analysis, vernal equinox is defined as the middle of the spring season quartile rather than the start of spring. The other seasons follow suit.

The e-seasons are fixed relative to aphelion and perihelion while the traditional season quadrants rotate relative to the orbit semi-major axis. Figure 3 depicts the clockwise

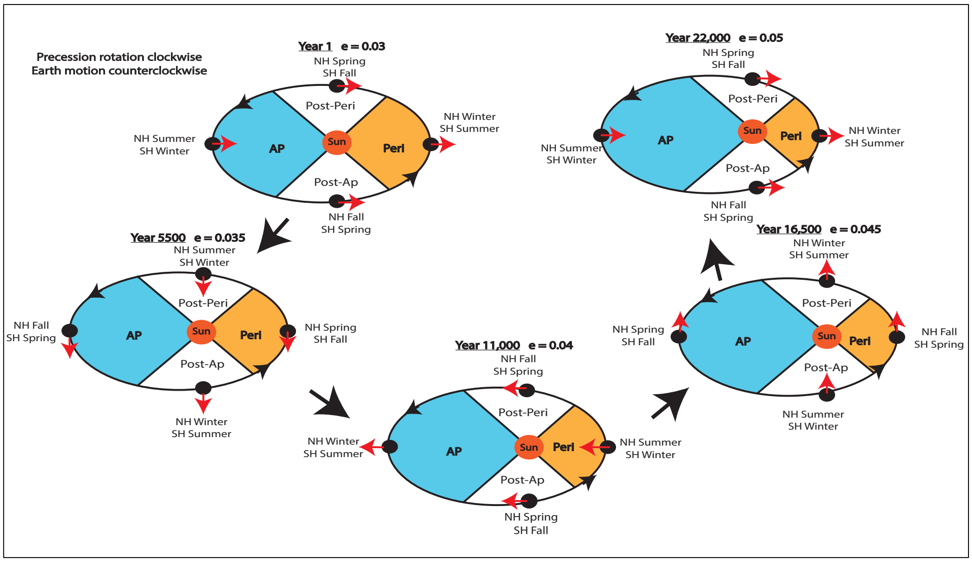

Figure 3. Traditional seasons rotation within the e-seasons over one 22 kyr precession cycle. Earth orbit is shown at five epochs separated by 5.5 kyrs with the Earth traveling in a counter clockwise direction. The Earth's spin vector (red arrow) rotates clockwise over the cycle. The spin vector direction relative to the Sun defines the traditional seasons in each hemisphere and these are labeled at each epoch. The traditional seasons experience cooling or warming as they rotate through the ap (blue)and peri (orange) seasons during the year. The orbit eccentricity varies from 0.03 to 0.05 increasing the cooling and warming in the ap and peri seasons. During year one, cool summer glaciation occurs in the NH and colder winter glaciation in SH. In year 11.0 kyr, the glaciation types are reversed for the hemispheres.

rotation of traditional seasons through the e-seasons in each hemisphere over a precession cycle of 22 kyr. Five epochs are displayed where the traditional season quadrants coincide with e-seasons as eccentricity increases from 0.03 to 0.05. The Earth spin vector (red arrow) projection on the ecliptic determines the season of each hemisphere where pointing at the Sun indicates northern hemisphere (NH) summer solstice. The spin vector

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makes one clockwise rotation during the cycle. Glaciation is taking place throughout the cycle with all traditional seasons participating as cooling in the ap season would provide either conditions for snow where rain would have occurred or less snow melt depending on the season moving through. The warming peri season would result in additional moisture for snow in higher latitudes. Increasing albedo produces further cooling. In year one, cool summer glaciation occurs in the NH while colder winter glaciation occurs in the southern hemisphere (SH) and the reverse occurs in year 11 kyr. In between, cool fall or spring contributes to glaciation. With an increasing eccentricity, the contrast between e-seasons is increasing and sustained snow growth exists throughout the cycle. The glaciation would continue in the next precession cycle as long as eccentricity is increasing. Deglaciation begins when eccentricity decreases.

Insolation due to eccentricity, precession and obliquity is derived at the hemispheric level for summer and winter seasons over the past 500 kyr. The measure of insolation is the average daily solar energy for the hemispheres as a percentage variation over the 500 kyr mean (See Appendix B). Figure 4a illustrates the insolation curves of the ap, peri, NH summer and NH winter. Fall and spring curves are not shown but would exist between the summer and winter curves. The NH summer curve (red) has a profile similar to the S65N curve (Berger and Loutre, 1992; Paillard, 2001: Laskar, 2018) in both phasing and relative amplitude. The precession cycles are responsible for the oscillation of insolation for summer and winter bounded by the ap and peri insolation. Sustained cooling for hemispheric glaciation during a 100 kyr cycle is attained by the precession derived alternation of cool summer and colder winter glaciations. This provides a resolution of how glaciation is sustained over tens of kyr. This alternation of glaciation types also explains the missing or "skipped" interglacials in the multiple precession or obliquity theories (Ridwell et al., 1999; Huybers and Wunsch, 2005) of the dominant 100 kyr cycles. That is, the S65N insolation only addresses summer so that high summer insolation levels project an interglacial while the paleoclimate record may indicate a glacial period. However, a winter insolation curve would project low insolation and a colder winter glaciation consistent with the paleoclimate record.

Obliquity distributes the variation of the summer and winter curves above and below the e-season envelopes. Obliquity is prominent when eccentricity approaches zero. For example, the ap season peak near 150 ka does not reach the current interglacial reference line suggesting eccentricity does not provide conditions for an interglacial. However, obliquity at this point in its cycle has moved insolation above the dashed reference line providing interglacial level conditions. At the major interglacials, the obliquity influence is seen by the insolation oscillating inside and outside the envelopes. Statistical studies (Huybers, 2011; Feng and Bailer-Jones, 2015) have shown a strong correlation of obliquity with peak temperature proxy data suggesting obliquity is the causal forcing of interglacials. My analysis indicates that this occurs only when eccentricity is low.

Figure 4b provides insolation curves for the NH summer and SH winter. The SH and NH insolation profiles have the same phase and similar relative amplitude indicating

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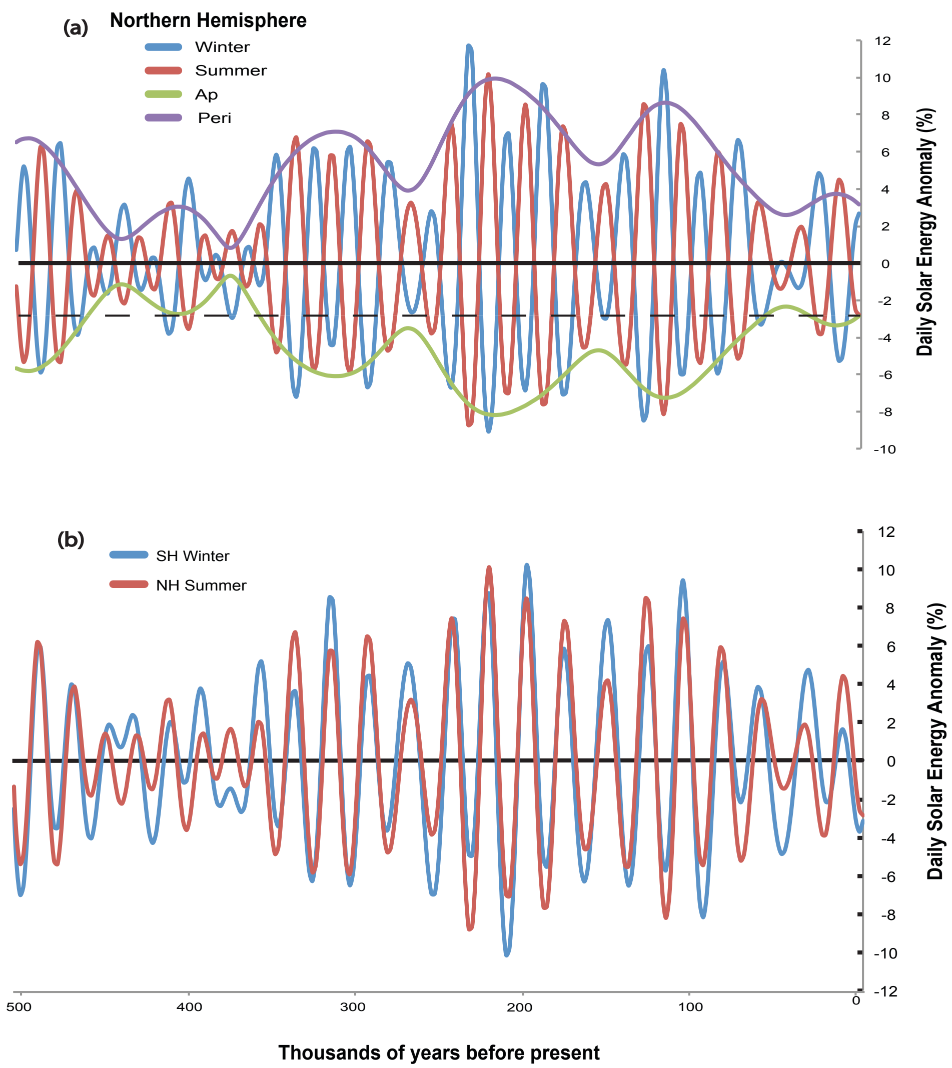

Figure 4. (a) Daily solar energy anomaly for NH summer (red) and winter (blue) over the past 500 kyrs. The anomaly is the daily solar energy variation from the season average over the 500 kyr period. The ap (purple) and peri (green) season insolation are shown for comparison. The dashed line indicates present level. (b) Daily solar energy anomalies for NH summer and SH winter. The chart illustrates the phase synchronization of the NH summer and SH winter insolation.

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synchronous glaciation between hemispheres. When cool summers produces glaciation in the NH, SH colder winter glaciation occurs simultaneously. As indicated earlier, empirical evidence supports global glaciation. An issue with the Milankovitch summer insolation curves is that glaciation is asynchronous between hemispheres. A proposed hypothesis to produce sychonization in support of the Milankovitch theory has been that the ice sheets in the NH are dominant and have a global influence, thus after a lag period, the SH would follow the climate variations of the NH (Raymo, 1997: Clark et al., 1999; Raymo and Huybers, 2008). Here, this issue is resolved with global glaciation.

5 Conclusions

It has been one hundred years since Milankovitch first published the fundamentals of his theory (Milankovitch, 1920). That his theory is still the benchmark in examining paleoclimate records is a testament to the respect for his work. However, the theory needs broadening with repect to (1) using annual solar energy as the sole criteria for judging eccentricity forcing power (2) excluding winter insolation and colder winter glaciation and (3) limiting the geography to high latitudes. Broadening the theory beyond these three limitations has provided a resolution to a number of contradictions between theory and the paleoclimate record. My primary conclusion is that glaciation is a global phenomenon and the dominant 100 kyr cycle found in spectral analyses of the paleoclimate records for the past million years represents eccentricity. Increasing eccentricity provides increasing contrasts between cooling and warming e-seasons during the year facilitating snow growth and glaciation. This increased global cooling allows for both the Croll colder winter and Milankovitch cooler summer glaciations to occur at the same time in opposite hemispheres and alternate between hemispheres due to precession. Global temperature reduction takes place from the sustained snow growth and creation of ice sheets as well as other feedbacks. When eccentricity decreases, deglaciation takes place and ice sheets regress leading to higher temperatures and interglacial conditions. The interglacials between the 100 kyr cycles are relatively short while major period interglacials occur between the 400 kyr cycles. Obliquity becomes significant at lower eccentricity levels.

There are two theoretical issues needing further examination. First, there is the lag time of roughly 25 kyr between the ap season peaks here and oxygen isotope proxy peaks found in the paleoclimate record (Raymo, 1997; Lisiecki and Raymo, 2005; Berger et al., 2016). Secondly, how the concepts here would apply to the mid-Pleistocene transition problem. This problem relates to the dominance of the 41 kyr cycles in paleoclimate records beyond one million years (Paillard, 1998; Raymo and Nisancioglu, 2003; Clark et al., 2006; Nyman and Ditlesen, 2009; Willeit et al., 2019) and what caused the dominance to switch to the 100 kyr cycle in the million years that followed.

Appendix A Equation for computing daily solar energy for e-seasons

The measure of e-season insolation for this analysis is the average daily solar energy. For each e-season, the solar energy received is one-fourth the annual global total (E_t). Since

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the mean annual solar energy is essentially constant (varies according to $(1- e^2)^{-1/2}$) and applying the Solar Energy Invariance law, each e-season will receive a constant amount of energy regardless of orbit geometry. This analysis was performed with anomalies relative to the daily solar energy of a circular Earth orbit so that the E_t magnitude was not required. The e-season average daily energy is the ratio of the e-season total energy divided by the e-season duration in days. The true anomaly, ν, is the angle between the Earth's position and perihelion as shown in Figure 1. The equation to compute the season duration is the following time equation for the Earth travelling an arc from true anomaly positions $\nu_a\ to\ \nu_b$:

$$t_{ab} = \frac{T}{2\pi}\left[\left(v_b - v_a\right) - 2e\left(\sin v_b - \sin v_a\right)\right] \qquad \text{(A1)}$$

T is the orbit period of 365.25 days. Eccentricity is the symbol e. Based upon Kepler's third law that orbit period squared is proportional to the semi-major axis (half the aphelion-perihelion distance) cubed, T is constant for all Earth orbits since its semi-major axis remains constant. This time equation is the Milankovitch (1998) derivation valid for small e. Milankovitch derived this equation based upon an earth-centered geometry. For computing e-season insolation curves, the true anomalies for the beginning and ending of each e-season is based upon fixed positions relative to perihelion. The true anomaly for an e-season's arc in radians is as follows: ap season is ¾ π to 1¼ π, post-ap season is 1¼ π to 1¾ π, peri season is 1¾ π to ¼π and post- peri season is ¼ π to ¾ π. At the maximum e of 0.058 during the one million years, the Earth takes 81.8 days to traverse the peri season and 100.8 days for the ap season. The traverse time for both the post-perihelion and post-aphelion quadrants is 91.3 days. The traverse time for any 90-degree quadrant of a circular (e = 0) orbit is 91.3 days. Thus, the post e-seasons average daily energies are equivalent to the circular orbit. The values for astronomical elements, eccentricity, precession and obliquity, were obtained from the La2004 database at http://vo.imcce.fr/insola/earth/online/earth/online/. [24] The sampling step was 2000 years.

Appendix B. Equations for computing hemispheric daily solar energy

The objective of this analysis is to extend the Milankovitch theory and resolve contradictions revealed due to the limitations of the theory. As such, this objective can be achieved by the derivation of insolation at the hemispheric level. The measure of insolation is average daily solar energy for the NH and SH. Unlike the constant solar energy of each e-seasons, the hemispheric traditional season energies are impacted by obliquity and precession. The solar energy for a hemisphere's season is based upon equations developed by Milankovitch (1998). Again, the global annual solar energy is essentially constant. Specifically,

$$N = \frac{E_t}{4\pi}\left[\left(\lambda'' - \lambda'\right) - \sin\varepsilon\left(\cos\lambda'' - \cos\lambda'\right)\right] \qquad \text{(B1)}$$

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where N_s = Northern hemisphere season solar energy
E_t = Earth's total annual solar energy
λ = Angle of Earth's position to vernal equinox (longitude)
ε = Earth's tilt angle (obliquity)

And for the southern hemisphere

$$S = \frac{E_t}{4\pi}\left[\left(\lambda'' - \lambda'\right) + \sin\varepsilon\left(\cos\lambda'' - \cos\lambda'\right)\right] \qquad \text{(B2)}$$

Equations (B1) and (B2) provide the solar energy the hemisphere receives when the Earth travels from longitude λ' *to* λ''. Note that the sum of these two equations is a mathematical expression of the Solar Energy Invariance law, that is, the total global energy is just a linear function of the arc travelled. For summer, the longitude is ¼ π to ¾ π, fall is ¾ π to 1¼ π, winter is 1¼π to 1¾ π, and the spring season is 1¾ π to ¼π. The average daily solar energy is obtained for each hemisphere season by dividing the seasonal energy by the number days to travel the season arc. That is, dividing either equation (B1) or (B2) by (A1). In computing time of transit for traditional seasons, the season beginnings and endings are moving relative to perihelion. Precession moves the vernal equinox location relative to perihelion. The angle between the vernal equinox and perihelion is defined as the longitude of perihelion, ω. For the computation of season days in (A1), true anomaly (ν) is equal to the sum of ω and λ. Since the angle λ is a constant, dependent on season, the time is a function of the moving ω and ε. An adjustment to ω may be needed depending on the whether the astronomical element data set used Earth or Sun center calculations. Adjustments were not necessary for the La2004 database used here.

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