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Climate of the Past
Discussions

Interactive comment on "Testing Hypotheses About Glacial Dynamics and the Stage 11 Paradox Using a Statistical Model of Paleo-Climate" *by* Robert K. Kaufmann and Felix Pretis

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We thank the reviewer for their careful review of the manuscript and detailed comments. Based on these comments, we should have made clear that the CVAR model does not over-fit the data and how it copes with non-linearities. These issues are described in a point-by-point response.

************************* Reviewer Comment ******************************

If I understand correctly equation (2) and the following text, the model has a *huge* number of parameters: there are several 10x14and 10x15 matrices of model param-

eters, involving hundreds of parameters being fit. If this is indeed the case, I don't know that it is a surprise or an achievement that the model achieves a satisfactory fit. Saltzman (not cited by the authors) developed hisC1"Earth system models" with the philosophy that the goal of glacial models is to produce a good fit to the ice volume record with the smallest number of parameters, although it is not clear if one actually learned about the dynamics of ice ages by such a fit. In any case, he used a nonlinear model and the order of 10-15 parameters, which, if I understand correctly, is much less than is used here. I can see some (vague) similarity between the Saltzman philosophy and the one taken here, in the sense that the authors try to fit the record without suggesting a mechanistic understanding of ice ages. This could still have led to a useful insight if they could indeed show that nonlinearity is not important, although as I discuss below I don't believe they have shown that.

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The current version of the manuscript does not address the reviewer's concern regarding the number of estimated parameters. We will address this in our revised manuscript. The literature has published thousands of papers that estimate CVAR models; many are estimated using far fewer observations than available here (such as 50 observations per time series as commonly found in macroeconomics). As such, the sample of about 400 observations per time series is comparably large.

In statistics, the potential pitfalls of a statistical model over-fitting the data is captured by the degrees of freedom, the number of independent pieces of information to estimate another piece of information. To illustrate, a line [Y(t)= α+βX(t)+μ(t)] will perfectly fit two observations, but the model has zero degrees of freedom and therefore is not statistically meaningful. But fitting that line to one hundred observations would have 98 degrees of freedom and so could represent a statistically meaningful relation.

For the CVAR reported by Kaufmann and Juselius (2013), each dependent variable has 390 observations. The right-hand side specifies 33 variables, which leaves 357 degrees of freedom for each equation. This is a very large number. Empirical statistical results are evaluated against tables that list critical values. The average table used to evaluate t statistics reports values for up to 100 degrees of freedom, 250 degrees of freedom, and then the asymptotic value for an infinite number of degrees of freedom. This implies that a sample of 357 observations falls near the 'top end' of observations used in empirical investigations.

Furthermore, many of the equations contain more than 357 degrees of freedom due to restrictions placed on the estimated coefficients. To identify the system, coefficients on many variables are restricted to zero. For example, the first cointegrating relation in Table 2 of Kaufmann and Juselius, (2013) and Supplementary Table S.1 restricts thirteen variables (other than Temp and CO2) to zero, which increases the degrees of freedom by thirteen. Similar restrictions are imposed on the other cointegrating relations such that of the possible 140 parameters in the 10x14 long-run Π matrix, 106 parameters are restricted to be zero, which indicates that only 24 parameters are actually estimated (and these restrictions are not rejected using likelihood ratio tests). As such, there are more than 357 degrees of freedom in each equation.

Finally, the model is used to generate an out-of-sample forecast. If the CVAR model overfit the in-sample observations, it is highly unlikely (from a statistical perspective) that it would be able to fit the out-of-sample period with about equal accuracy. As such, the accuracy of the out-of-sample period is consistent with the hypothesis that the large number of degrees of freedom minimizes concerns about overfitting.

Thus, over-fitting should not be a major concern, and also does not seem to occur as we show in the example using a van-der-Pol oscillator following the reviewer's suggestion below. We will clarify these issues in the revised version of the manuscript and include references to the work of Saltzman

************************* Reviewer Comment ****************************

First a comment on the data: while ten proxy records are being used (the authors

should plot them), they are likely not very independent, as they tend to mostly vary together with each other, so the number of observations being explained/ fitted is not as large as it might seem superficially, making the number of model parameters effectively even larger.

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One of the reasons that we use the CVAR model to analyse the paleoclimate data is that is solves many of the difficulties associated with traditional regression techniques. These advantages are described in Juselius (2014) who writes, "By exploiting the unit root feature, typical of many economic variables [and other non-stationary time series such as climate data], the CVAR model was shown to solve the problem of (1) time dependent residuals by conditioning on sufficiently many lags and controlling for a changing environment when needed, (2) spurious correlation and regression results, (3) multicollinearity [correlation among the proxy records in our case], (4) normalization, and (5) reduced rank." We will add this information to the revised manuscript. As such, collinearity among the climate variables should not affect the statistical results beyond increasing the estimates of the error variance which is explicitly accounted for in our statistical tests.

************************* Reviewer Comment ****************************

The authors emphasize as their main conclusion that their study rules out previous claims that nonlinearity must be important. This would have been novel and interesting, but I don't find this convincing for two reasons. (1) Their model is somewhat nonlinear. They need to completely linearize it, repeat the analysis, and demonstrate that the results and conclusions are robust.

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We apologize; we should have been clearer regarding the linear nature of the CVAR model. The CVAR model is linear in parameters. That is, the first cointegrating relation in Table 2 of Kaufmann and Juselius (2013) and Supplementary Table S.1 is a linear relation between Temp and CO2 and the third cointegrating relation is a linear relation among Ice, CO2, and eccentricity. These linear long-run cointegrating relations, which lie at the heart of the CVAR, is what we refer to as a 'linear model'. We will clarify this in our revised manuscript.

The apparent non-linear dynamics can stem from two sources. First, the individual variables (specifically, the first difference of each variable) such as temperature adjust towards disequilibrium as a linear function of disequilibrium in the level of the variables in the previous time period. This creates a seeming non-linear change in the level. However, the model is linear in both first differences and levels.

Second, the model is conditioned on orbital geometry, which changes nonlinearly over time. But these nonlinear changes are represented linearly. As such, non-linear changes in orbital geometry have a linear relation with the variables simulated by the model. This linear effect is very different than the nonlinearities and/or threshold effects that are described in the literature that we cite.

************************* Reviewer Comment *****************************

(2) Given the very large number of model parameters, I suspect their approach could fit any low-order *nonlinear* dynamics successfully. To make a satisfactory case, they need to test this hypothesis as follows: build a simple nonlinear model based on 10 weakly coupled nonlinear oscillators (e.g.,Van der pol oscillators, see Crucifix 2013 "Why could ice ages be unpredictable?" Clim.Past, 9, 2253-2267); fit a similar stochastic model to this model output. The null hypothesis could be that such a model output would be possible to fit using the nearly-linear model used by the authors, although the dynamics are clearly strongly nonlinear. If the null hypothesis is not satisfied, the authors would have a stronger case.

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Thank you, the reviewer proposes a very interesting test of our model. Create observations using a nonlinear data generating process, in this case, a van-der-Pol oscillator from Crucifix (2013). Then fit a linear statistical model to the data. If the linear statistical model can fit the data well, this would suggest that a linear model can simulate the nonlinear dynamics used to generate the data. Such a result would weaken our claim that nonlinearities/threshold effects do not play an important role in glacial cycles. Conversely, if the model fails to fit the data well, this would indicate that the linear statistical model cannot the nonlinear dynamics used to generate the data. This result would support our claim because our linear statistical model fits the in- and out-of-sample observations for paleoclimate well.

We test the reviewer's hypothesis by using the van-der-Pol oscillator from Crucifix (2013) to generate two non-linear sets of data. First, we construct a two variable van-der-Pol oscillator in discrete time that is conditioned on a sinusoidal forcing F (similar to Crucifix, 2013), which is perturbed with white noise. The artificial van-der-Pol data is shown in Figure 1. The variable 'y' mimics a suddenly changing time series in the paleo-proxy record (such as temperatures) while 'x' mimics a more gradually changing variable (such as ice).

We repeat this exercise using a ten variable van-der-Pol oscillator. The ten variable oscillator is specified to simulate one variable that changes suddenly and nine variables that accumulate gradually. The ten variable system is shown in Figure 2.

For each of these simulations, we use half of the simulated data (area shaded in grey) to fit an 'in-sample' linear vector autoregression (VAR) model (See Supplemental Material) in which Yt is a vector of two (y and x; or ten in the case of the larger system) variables generated by the van-der-Pol oscillator, F is the sinusoidal forcing, (s) is the number of lags (s) chosen using the Schwarz information criterion (Schwarz, 1978), and Ït_t is a vector of error terms. The VAR corresponds to the CVAR model by Kaufmann and Juselius (2013). We use the statistical model to simulate the endogenous variables over the full sample, which mirrors the approach used in our manuscript.

Results from simulating van-der-Pol Oscillators

Visual inspection of the simulation in Figures 1 and 2 indicate that the linear system does not match the abrupt non-linear pattern. The red lines in Figures 1 and 2 indicate that the model does not account for much of the variation during the in-sample period. Nor does this performance improve during the out-of-sample period. The same holds true for both in- and out-of-sample generated by the ten-variable oscillator (see Figure 2).

This visual impression is confirmed statistically by testing whether the model errors, (the difference between the black and red lines in figures 1) are statistically different from zero. We use the same indicator saturation technique, which is used in the manuscript, to identify periods when the model errors are statistically different from zero for two or more consecutive periods (steps). As indicated by the steps in Figure 3, model errors are statistically different from zero for most of the sample period in the two variable case (Figure 3).

These simulation results show that a linear VAR model is unable to successfully simulate a non-linear van-der-Pol system. Conversely, the linear CVAR climate model is able to simulate glacial dynamics, as described in our manuscript. Together, these results suggest that non-linear dynamics (such as in the van-der-Pol system) may not be play a large role in glacial cycles. We thank the reviewer for this clever test and will include it in our revised manuscript.

************************* Reviewer Comment ******************************

Page 2, line 15-20: That CO2 is not an external variable seems obvious. I don't know that this adds anything new to our understanding.

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We included this note because there is a mismatch between the physical climate system and many of the models used to simulate it. As indicated by the references in

our manuscript, the atmospheric concentration of CO2 is endogenous; it is driven by temperature, sea ice, and many other variables. But many of these mechanisms are not simulated by existing empirical and process-based models of the climate system. Without the ability to simulate these mechanisms, changes in the atmospheric CO2 over time are treated as an exogenous variable. For example, the ice model CLIMBER 2 is conditioned on the radiative forcing of CO2 (Ganopolski and Calov, 2011). We will clarify this in the revised manuscript.

************************* Reviewer Comment *****************************

Page 3, lines 5-10: it seems less plausible that glacial terminations are driven by atmospheric or oceanic dynamics. The trigger needs to be a change to a climate system component that has a long 90 kyr time scale, which reaches some critical threshold, starts changing, and that then leads to changes in the faster components such as sea ice, AMOC, atmospheric circulation, etc. The only such slow component is land ice, which may take 90 kyr to reach some critical size that then affects other components. I realize there is much in the literature about the AMOC triggering terminations etc, but the above argument seems to suggest that these ideas are not likely to be realistic.

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We respectfully disagree with the heart of reviewer's comment "The trigger needs to be a change to a climate system component that has a long 90 kyr time scale, which reaches some critical threshold, starts changing, and that then leads to changes in the faster components such as sea ice, AMOC, atmospheric circulation, etc." One important point of our paper is that there is no need to invoke this nonlinear threshold effect. The linear CVAR model is able to accurately simulate glacial cycles both in and out of sample. As indicated by the 'van-der-Pol oscillator experiment' suggested by the reviewer, it is highly unlikely that the linear CVAR model would be able to simulate glacial cycles in and out of sample if the paleoclimate data were dominated by a non-linear data generating process. We recognize that this may be a different and somewhat controversial way of looking at the data, but we think that the manuscript and the results of the van-der-Pol oscillator experiment suggested by the reviewer represents sound scientific evidence that the community should consider.

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Page 8, lines 10-15: it seems to me that the model's ability to simulate glacial cycles during the out-of-sample period does not mean the model is correct. It just seems to suggest that the dynamics of the glacial cycle are the same throughout the past 800kyr. It seems still possible that the model simply fits the record due to its very large number of parameters.

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We agree scientists can never be sure that a model is correct and that the in- and out-of-sample primarily suggests stability of the processes. However, we hope that the simulation exercise using the van-der-Pol oscillator (see reply above) strengthens our argument.

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Because of the issues mentioned above, it seems that me that the manuscript as it stands now does not make a strong case for the suggested conclusions. I recommend a major revision, that it my opinion needs to require new analysis and results rather than just a rewrite and further explanations. I hope the authors find these comments are helpful.

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Please also note the supplement to this comment:
https://cp.copernicus.org/preprints/cp-2020-58/cp-2020-58-AC4-supplement.pdf

Interactive comment on Clim. Past Discuss., https://doi.org/10.5194/cp-2020-58, 2020.

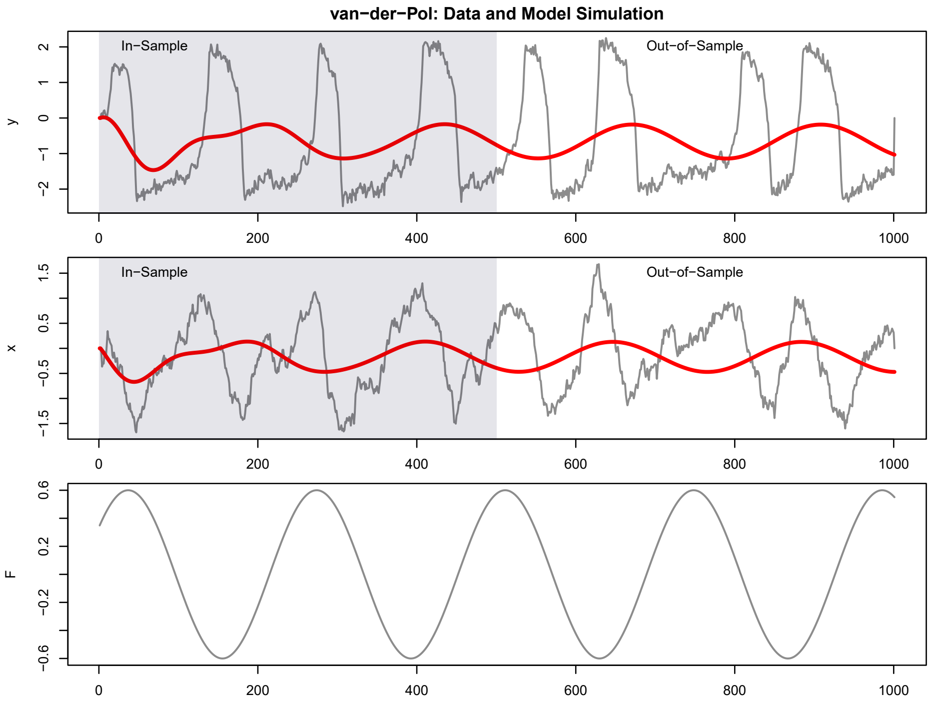

Fig. 1. Simulating artificial data generated by a two-variable van-der-Pol Oscillator (Crucifix 2013). Grey shows artificial time series y and x driven by the exogenous sinusoidal forcing F. Red shows the sim

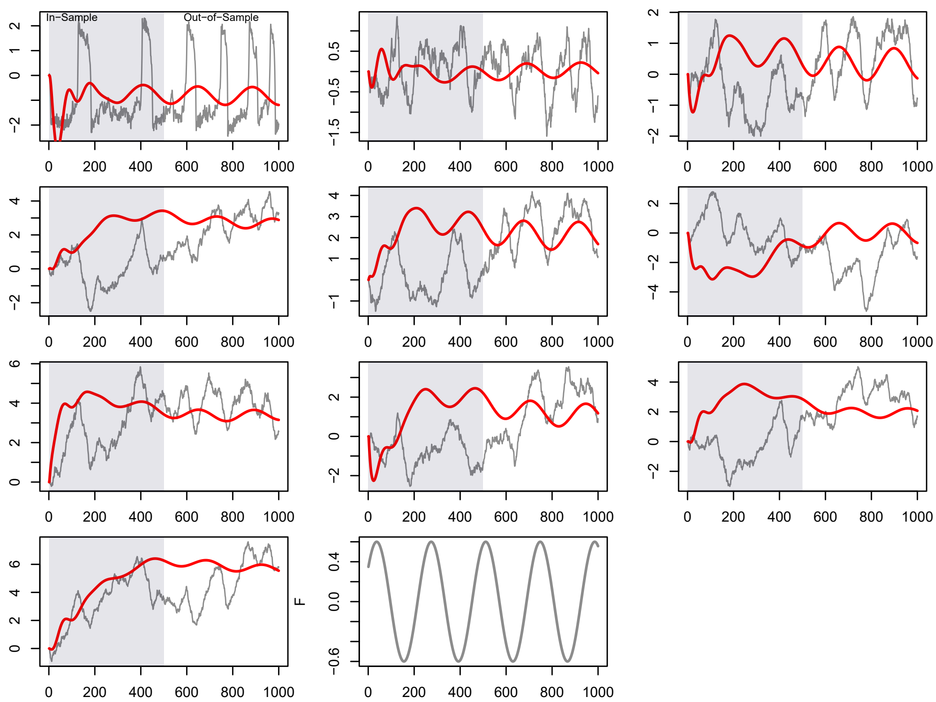

Fig. 2. Simulating artificial data generated by a ten-variable van-der-Pol Oscillator (Crucifix, 2013). Grey shows artificial time series y and x driven by the exogenous sinusoidal forcing F. Red shows the si

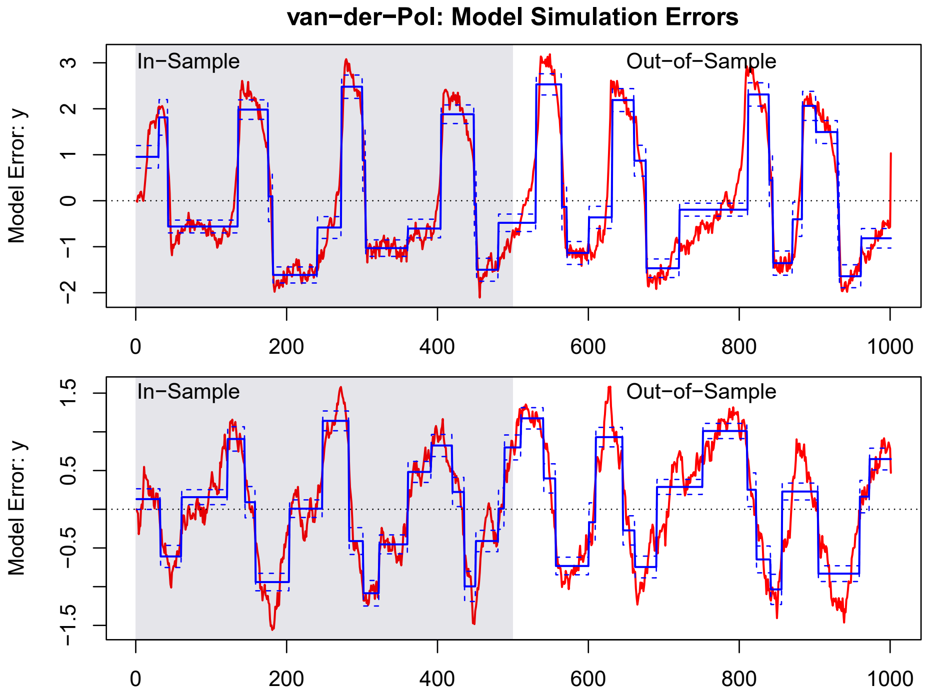

Fig. 3. Simulation errors when modelling the non-linear van-der-Pol oscillator using the linear VAR model. Red shows the simulation errors of y and x, blue shows the time-varying mean of the simulation errors