

In the following responses, we reproduce the reviewers' comments in italics and include our detailed responses in bold text thereafter.

Response to Reviewer 1

The manuscript assesses four different climate field reconstruction (CFR) methods, two of which are based on spectral domain analysis, and the other two are based on ridge regression and canonical correlation analysis, respectively. These methods are evaluated through pseudoproxy experiments (PPE) with five different GCMs (BCC, CCSM, GISS, IPSL, and MPI), using singular value decomposition based hypothesis tests on the mean and covariance functions. The hypothesis test results and the follow up analyses show that the reconstruction performances are affected by (i) how well the overall patterns in a GCM simulation are captured by the leading EOFs, (ii) how temporally stable the leading EOFs are, and (iii) how the sampled locations are representative with respect to the global climate patterns. I think the analysis presented in the manuscript is highly thorough and carefully done, and it sheds light on the factors that can negatively affect the performance of the existing CFR approaches and how CFR methods can be improved in the future. The suggestions below are mostly on improving the presentation and fixing some typos/incorrect statements:

1. *In lines 215- : I think the fact that X is the synthetic truth and Y is the CFR result should be introduced earlier so that readers can understand Sections 3.1.1 and 3.1.2 more easily. In the current format, readers should wait until the end of 3.1.2 to find this out while wondering how X and Y are chosen.*

We appreciate this comment and will modify the text to reflect this fact.

2. *Line 226 “the correlation of climate observations”: I would say “the correlation and the variance of climate observations”.*

We will modify the text in accordance to this comment in the revised manuscript.

3. *Line 232: There are redundant parentheses.*

We will remove the redundant parentheses.

4. *Lines 235: I think it is worth mentioning possible computational issues in conducting singular value decomposition when the data size is large, perhaps mentioning the computational complexity is in $O(\min(mn^2, m^2n))$.*

We agree that mentioning the computational time and complexity of the singular value decomposition is a good idea. This will be added to the revised manuscript.

5. *Line 244: I am wondering if k^2 in the equation for $V(d)$ is a typo. If not, please add a sentence that explains why k^2 is needed.*

The k^2 in the equation for $V_\alpha(d)$ is not a typo. Because the recursive $\hat{\alpha}_k$ is estimated based on the recursive sample comprising observations from 1 to $\lfloor k/2 \rfloor$, the weight k^2 in $V_\alpha(d)$ is to account for the sample size used to estimate α_k . This procedure is capable of incorporating the temporal dependence. Details will be added in the revised manuscript.

6. *Lines 255-257: Perhaps what the authors meant were “In other words, a p-value close to 0 indicates the difference between modeled and reconstructed fields is statistically significant against this null hypothesis, while p-values close to 1 indicates the difference could be explained by random chance.”*

The reviewer is correct and we appreciate that this mistake was pointed out. We will fix this statement in the revised manuscript.

7. *Lines 258-260: I think the authors need more motivation on why they are focusing on the leading five principal components here. My guess is that the leading principal components are mostly large-scale features, which are usually the main interest when studying the changes in spatial patterns (e.g., ENSO). I think the authors want to clarify this point here.*

The reviewer’s conjecture is correct. The leading 5 EOFs consist of more than 80% of the total variability and largely represent the dominant spatial patterns of the random fields. We therefore compare the features of the spatiotemporal fields on these leading five principal components. We will clarify this point in the revised manuscript.