Supplement of

Physically based summer temperature reconstruction from ice layers in ice cores

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Supplementary Tables and Figures

Table S1. Parameters tested in the Monte Carlo simulation.

Parameter	Tested range	Best estimate value	r with RMSE of surface level	r with RMSE of albedo
ρ_n (kg m^{-3})	100 ~ 300	241	–0.834*	–0.171
D_w (mm)	10.0 ~ 50.0	43.5	–0.019	0.002
w_c (%)	4.0 ~ 6.0	4.17	–0.004	0.002
T_r (°C)	3.0 ~ 5.0	3.00	–0.007	–0.005
P_{S_min} (mm w.e.)	1.0 ~ 5.0	4.77	0.042	0.216
α_f (Eq. 3)	0.6 ~ 0.8	0.793	–0.436*	–0.738*
k_{min} (Eq. 4)	3.0 ~ 5.0	4.65	–0.014	–0.021
T_t (Eq. 4)	–0.5 ~ 1.5	0.129	–0.062	–0.088
dk/dT (Eq. 4)	–4.0 ~ –2.0	–2.03	–0.025	–0.116
α_{max} (Eq. 5)	0.8 ~ 0.9	0.897	–0.096	–0.482*
T_{min} (Eq. 5)	–2.0 ~ 0.0	–1.82	–0.004	–0.011
T_{max} (Eq. 5)	2.0 ~ 4.0	3.34	–0.003	–0.000
μ_s (Eq. 6)	25.0 ~ 35.0	32.2	–0.004	–0.014
f_w (Eq. 8)	1.60 ~ 2.60 × 10^{-2}	2.44 × 10^{-2}	–0.254	0.011
η_c (Eq. 8)	14.0 ~ 18.0	17.4	–0.010	–0.005
c_d (Eq. 8)	0.10 ~ 0.90	0.218	–0.006	0.009

Parameters without description in the main text: ρ_n: density of fresh snow, D_w: snow thickness of saturated with water, and T_r: threshold air temperature for rain probability.

* $p < 0.001$

Table S2. Errors for the reconstructed SMTs (σ_T, °C), which are the quadratic sum of those derived from the firn density assumption (σ_d, °C) and the seasonal patterns of the input meteorological variables (σ_s, °C).

Ice core site	Sigma-A	SE-Dome	Aurora	Belukha
σ_T (°C)	0.65	1.57	0.85	0.76
σ_d (°C)	0.07	0.04	0.15	0.13
σ_s (°C)	1.16	1.57	0.84	0.78

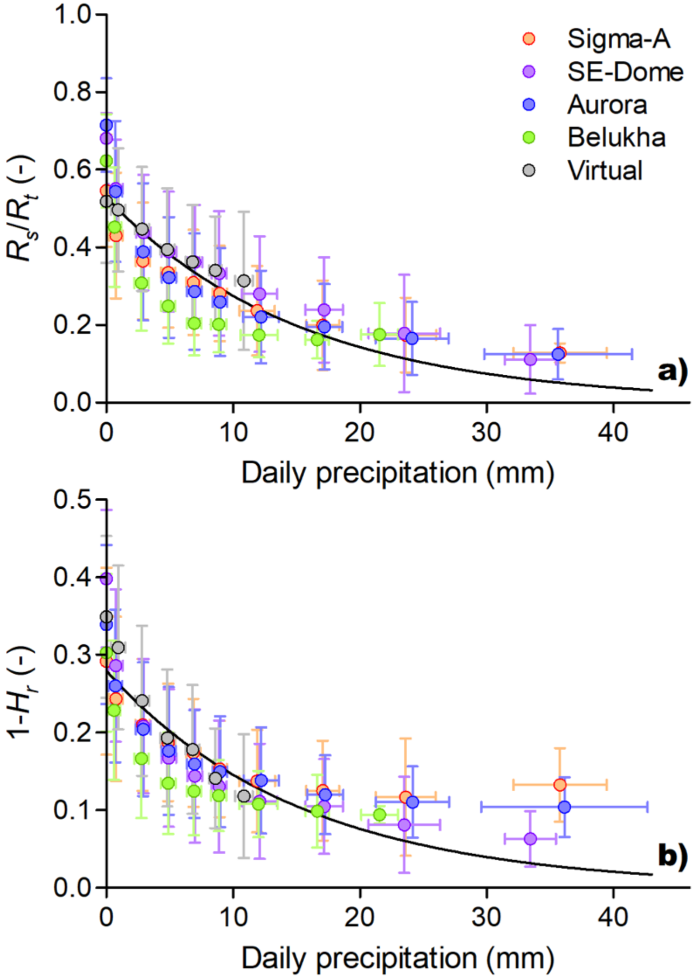

Figure S1. Relationships between daily precipitation and (a) normalised solar radiation (R_s/R_t) and (b) the residual of the relative humidity ($1 - H_r$) for the four ice-core sites and virtual settings for the sensitivity analysis (black circles). The best-fit exponential curves were calculated from all of the data from the four ice-core sites.

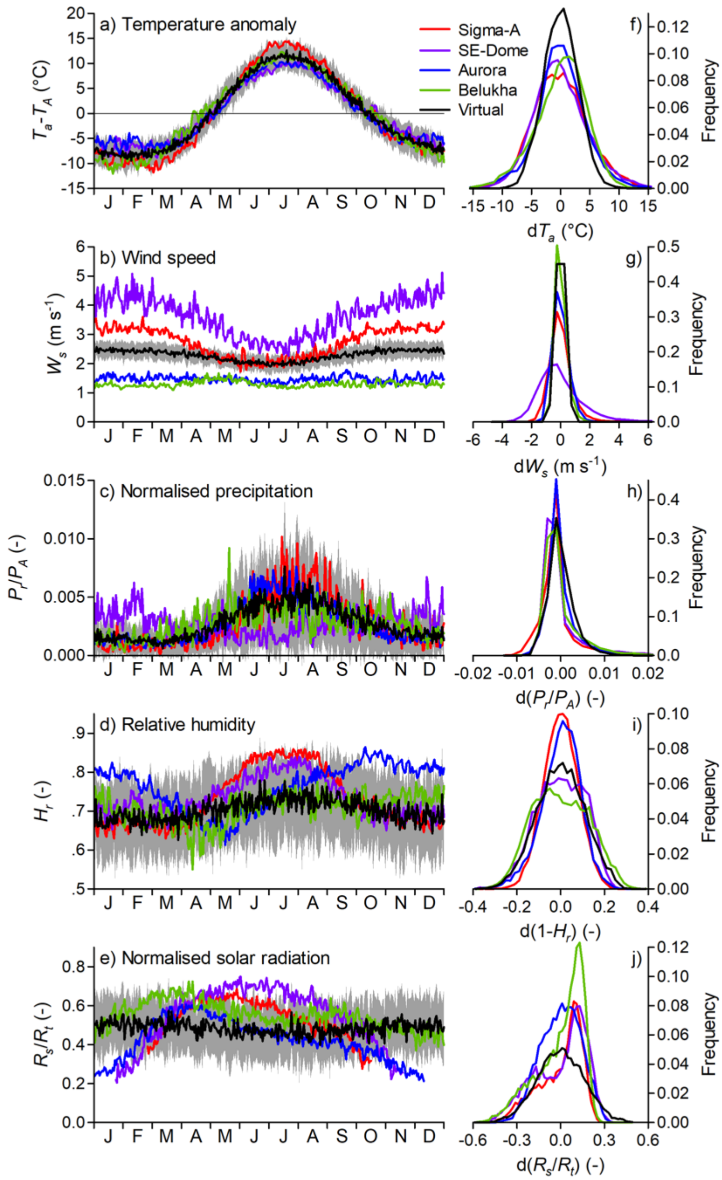

Figure S2. Long-term seasonal patterns of (a) air temperature anomaly ($T_a - T_A$), (b) wind speed (W_s), (c) normalised precipitation (P_r/P_A), (d) relative humidity (H_r), and (e) normalised solar radiation (R_s/R_t). (f–j) Histograms of the daily anomalies in the long-term patterns of the respective variables. Coloured lines denote ERA-Interim data for the four ice-core sites. The black lines with grey shaded regions denote the idealised variables for the sensitivity analysis.

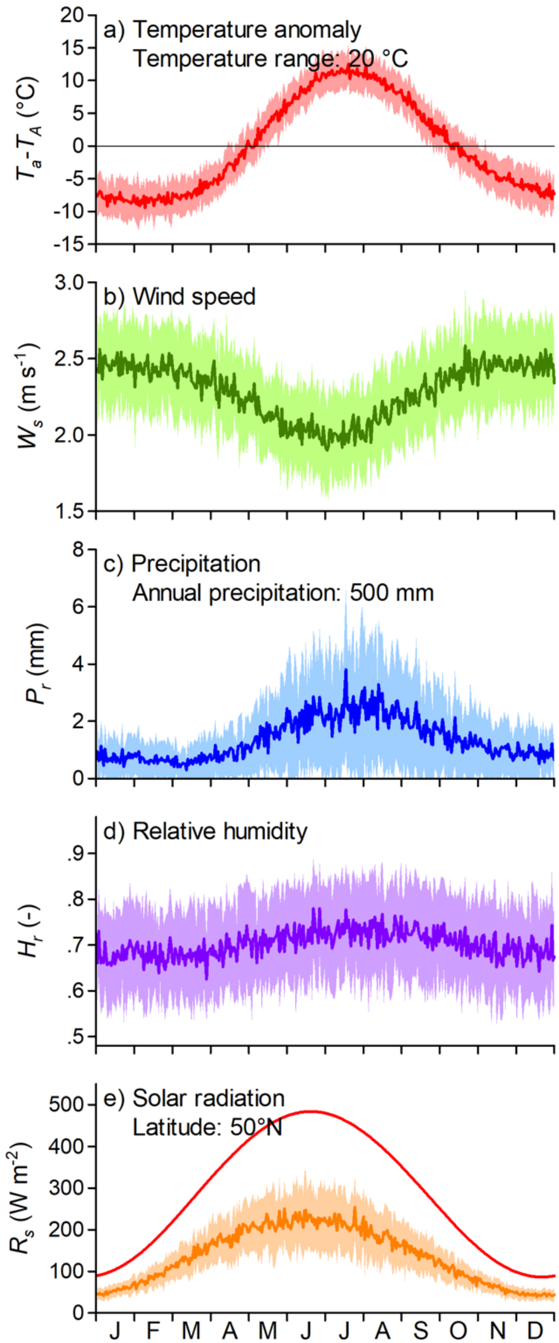

Figure S3. Standard settings of (a) air temperature anomaly ($T_a - T_A$), (b) wind speed (W_s), (c) precipitation (P_r), (d) relative humidity (H_r), and (e) solar radiation (R_s) for the sensitivity analysis. Shaded regions denote the interannual variability. The smoothed red line in (e) denotes solar radiation at the top of the atmosphere.

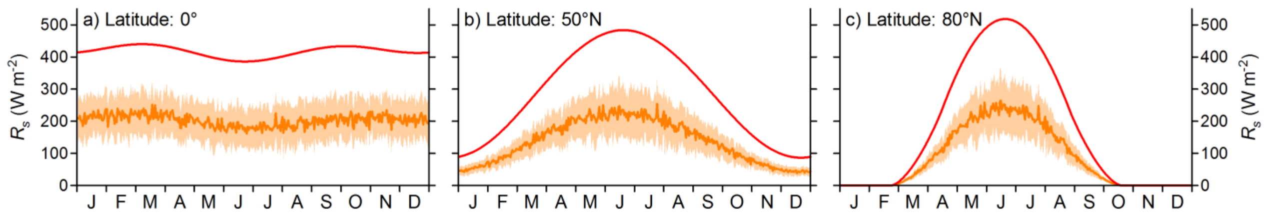

Figure S4. Examples of downward solar radiation (orange lines) affected by changing latitude for the sensitivity analysis. Shaded regions denote the interannual variability. The thick red lines denote the solar radiation at the top of the atmosphere.

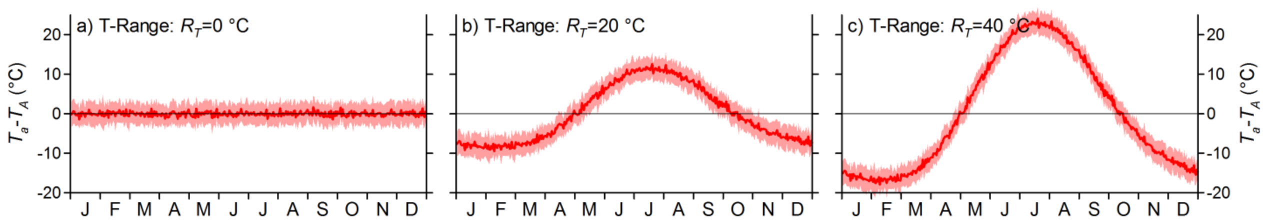

Figure S5. Examples of changing the temperature range (R_T) for the sensitivity analysis. Shaded regions denote the interannual variability.

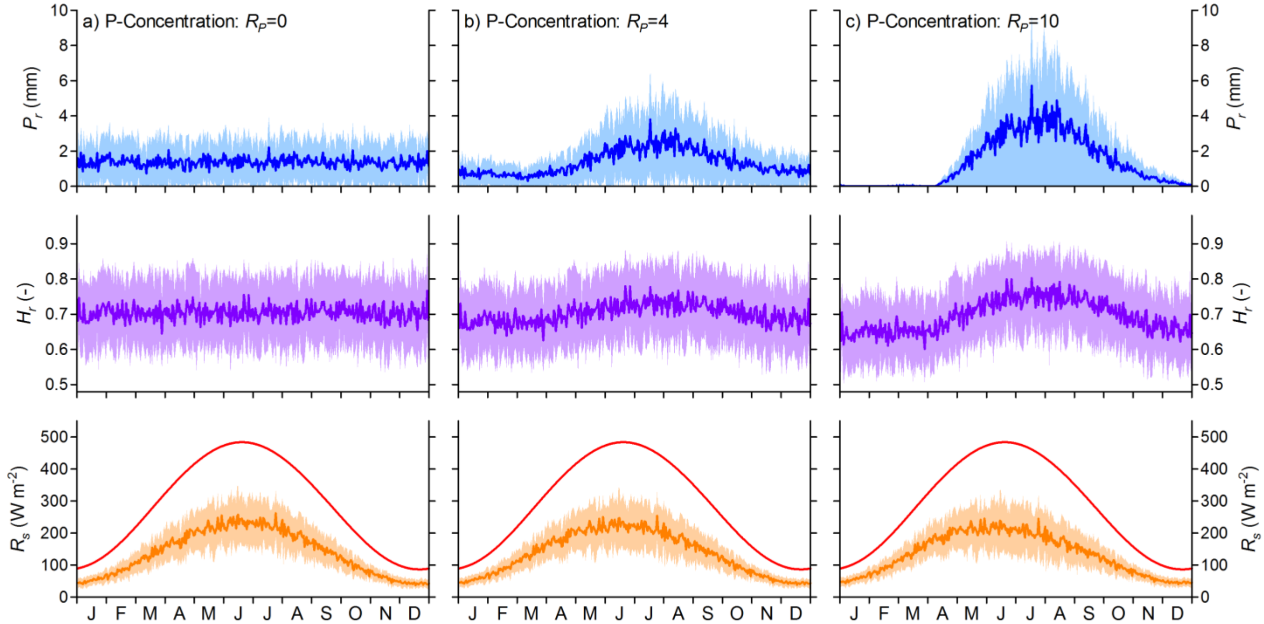

Figure S6. Examples of changing the precipitation concentration (R_P) for the sensitivity analysis. Shaded regions denote the interannual variability. The smoothed red line in the solar radiation (R_s) plots denotes solar radiation at the top of the atmosphere.

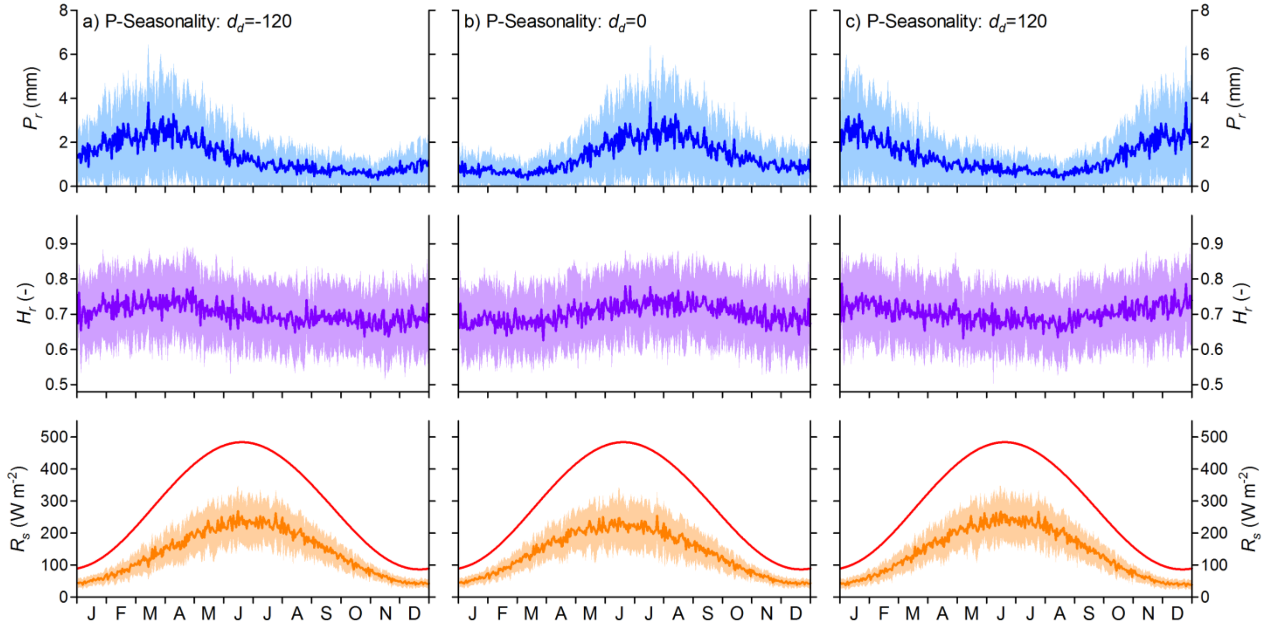

Figure S7. Examples of changing the precipitation seasonality (d_d) for the sensitivity analysis. Shaded regions denote the interannual variability. The smoothed red line in the solar radiation (R_s) plots denotes solar radiation at the top of the atmosphere.

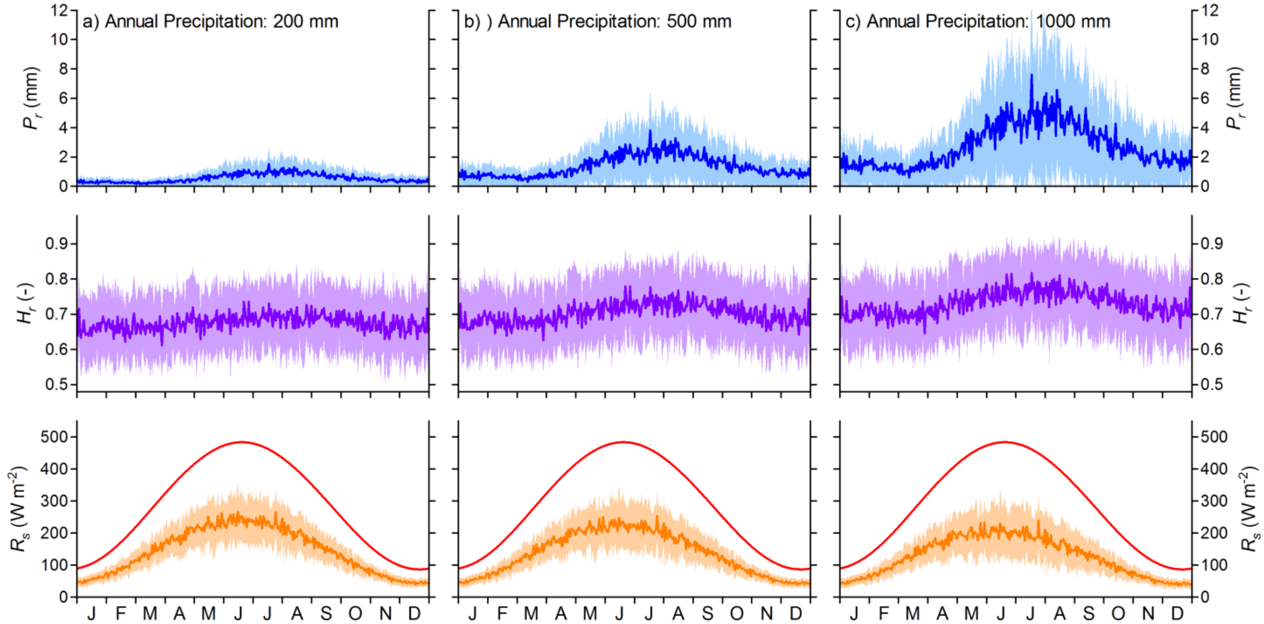

Figure S8. Examples of changing the annual precipitation amount for the sensitivity analysis. Shaded regions denote the interannual variability. The smoothed red line in the solar radiation (R_s) plots denotes solar radiation at the top of the atmosphere.