# Rebuttal to *Second review of* "Bispectra of climate cycles show how ice ages are fuelled" by Diederik Liebrand and Anouk T. M. de Bakker

**Mathieu Martinez (Referee)** R2

mathieu.martinez@univ-rennes1.fr

Received: 29 September 2019

In their revised manuscript, Drs. Liebrand and de Bakker made substantial modifications which clarified the method and the interpretations in terms of climate dynamics. Clarification is still possible by defining terms when they are used for the first time in the manuscript. Nonetheless, the manuscript is a great contribution to the understanding of Plio-Pleistocene climatic changes and will be suitable for publication after minor revisions would be done. Below are my detailed comments:

We agree with R2 that some further clarifications could be made. Especially with regard to defining terms when they are introduced. We have made changes to the text (see below).

1) A possible mistake I saw in the revised version, is the definition of the kurtosis. Kurtosis, which, in the absence of outlier, measures how flat or peaked is the top of a distribution of values, ranges between 1 and the infinite. For a normal distribution, the kurtosis value is 3. For a flat-top distribution, the kurtosis is 1. Then, the more peaked is a distribution, the higher is the kurtosis. Subtracting 3 to the kurtosis value means the normal distribution is centred to 0. In that case, what is measured is called the excess kurtosis which ranges from -2 to infinite. See for instance Collis et al. (1998). If the authors decide to keep this formulation, they should replace "kurtosis" by "excess kurtosis" throughout the text and in Figure 2.

   We agree with R2 that Equation 3 computes excess kurtosis, and we have added this information to the text and Fig. 2.

2) **In Fig. 4c and in section 3.2.**, how there can be in the same bispectrum the following triad interactions: $B^{lm}{}_{p}$(40↑, ∞↑, 40↓) (Line 20) and $B^{lm}{}_{p}$(40↓, ∞↓, 40↑) (Line 24)? How can a triad interaction be positive <u>and</u> negative in the same time?

   Good observation of R2. We agree that this is impossible. In fact, this interaction is negative (i.e., blue colours), hence $B^{lm}{}_{p}$(40↑, ∞↑, 40↓) is the correct notation. Where previously, we marked this interaction as positive, we have now corrected the periodicities to the correct positive interaction at $B^{lm}{}_{p}$(44↓, 405↓, 40↑) in the text. Figure 4c remains the same.

3) **Page 7, lines 24:** section 4.3. does not exist. Do the authors refer to section 3.4. instead?

   We agree with R2 that 4.3 does not exist. We meant 4.2 and have updated the text.

4) In the method section, I think that defining terms which are not reorganizing ideas or paragraphs would make the text much straightforward to read. For instance, "conservative energy exchange" is an important concept in the manuscript. This is mentioned for the time in page 4 but only defined

in page 13. In my opinion, the concepts the authors use should be defined when they are mentioned for the time in the manuscript. Below, I list possible rephrasing and additional explanations in the method section. These additional explanations may be more inclusive for readers who are interested in Pleistocene climate changes but feel not confident in higher order statistics. The authors can feel free to account them or modify them if misunderstanding appeared to come from me:

We agree with R2 that new terms/concepts can best be explained when introduced. (see below)

2.2. Quantifying geometries using central moments

The geometry of a distribution of values can be quantified by an infinity of statistical moments. Generally, only the first four moments are used. These four statistical moments are respectively the average, the variance, the skewness (or asymmetry) and the kurtosis. Here, the cycles are assimilated to a distribution of values through time. The skewness is here defined as a dissymmetry of a cycle relatively to a horizontal axis (Fig. 2b). The asymmetry is here the dissymmetry of a cycle through time (Fig. 2c). The kurtosis quantifies the flatness of the extrema of a cycle. Flat-top (flat- bottom) cycles have low-kurtosis values, while sharp-top (sharp-bottom) cycles have a high kurtosis value. Kurtosis ranges from 1 to infinite. A Gaussian curve has a kurtosis value of 3. Thus, the deviation of a curve to the Gaussian shape can be calculate by the "excess kurtosis" which is defined as follows: excess kurtosis = kurtosis – 3. Using third-moment quantities, skewness is determined by Eq. (1) ...

We appreciate this suggestion and we have partly incorporated this text into the Method section.

2.3.1. The bispectrum

The Fourier Transform calculates a spectrum showing the distribution of variance (being related to power and energy, as power is the energy per time unit) with frequency. The Fourier Transforms is however unable to document higher order statistical moments of the signal considered and, for this, higher order spectral analyses must be done. Bispectral analyses describe the distribution of nonsinusoidality with frequency, in both the real and imaginary parts (King, 1996). The skewness of a cycle geometry is related to the real part of the bispectrum while the asymmetry is related to its imaginary part (Fig. 2). The bispectrum shows nonconservative, relative energy exchanges among frequencies of a single time series. Conservative energy exchanges are here defined as exchanges of energy and relative energy exchanges as … (*I think this is important to introduce the terms here otherwise, many readers may be lost depending on their own usage of the terminology*). Energy transfers can occur ...

We appreciate this suggestion and we have partly incorporated this text into the Method section.

2.3.2. Interpreting the bispectrum

**Page 5, Line 30:** "The former are the so-called difference frequencies, while the latter is referred to as a sum frequency": using "former" and "latter" can be confusing because it is not always to what they refer. I usually tend to avoid these terms. I would write instead: "$f1$ and $f2$ are so-called difference frequencies, while $f3$ is referred to as sum frequency".

We agree with R2 and have clarified the text.

I find the subsection “2.3.2 Interpreting the bispectrum” very practical and useful. Nonetheless, in this subsection, the notions of energy gains and losses are mentioned, while they are defined further in subsection 2.3.4. In my opinion. It would be more logical to define the bispectrum, the calculation of geometries from the bispectrum, the energy exchanges, and then show how to interpret the bispectrum. Again, this organisation should ensure that terms and concepts are defined when they are mentioned for the first time in the manuscript, which is important.

On this occasion we disagree with R2. We prefer to let the text follow the order in which we did the computations and in which we present the figures. During the previous review round we already added a sentence to Section 2.3.1. in which we explain the units of energy transfer. For the Method Section this will suffice. In Section 4.2. we further deal with the exact meaning of “energy exchanges” within a palaeoclimatic context.

**Reference:**

Collis, W.B., White, P.R., Hammon, J.K., 1998. Higher-order spectra: the bispectrum and the trispectrum. Mechanical Systems and Signal Processing 12(3), 375-394.

This reference was already part of our reference list.

We would like to take this opportunity to thank M. Martinez for his constructive second review.