

Rebuttal to *Interactive comment on “Bispectra of climate cycles show how ice ages are fuelled”* by Diederik Liebrand and Anouk T. M. de Bakker

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1. Summary

The authors present an extensive and systematic application of bispectral analysis to the LR04 benthic foraminifera stack. Bispectral analysis allows one to evidence so-called transfers of energy between different frequencies, and may therefore provide support for interpreting nonlinear phenomena known to occur in a system of which we can observe time series. Sections 1 and 2 are devoted to context and methodology, and the main results are given in section 3. Section 4 briefly comments on the suitability of the approach, and section 5 suggests possible climate mechanisms.

As pointed out by the authors, this is not the first time that bispectral analysis is being applied to palaeoclimatic time series. Earlier attempts are due to Teresa Hagelberg in the early 1990s and it is nice to see here an up-to-date application of this technique, illustrated by carefully prepared figures (key Figures are 6, 7, and 9). I have, however, a number of comments which I believe pertain to quite fundamental issues, but which nevertheless may be addressed by the authors.

We find it interesting to learn that R1 considers Figures 6, 7, and 9 as key. We consider Figures 4, 5, and 8 most informative.

2. Major Comments

1. First, the concepts of “energy” and “energy conservation” need to be clarified. In wave theory, the Fourier energy (square of amplitude) is directly interpretable as kinetic energy. The concept of energy conservation therefore has straightforward meaning. In palaeoclimates, the amplitude of a precession beating is not an energy of that form.

This is a very fundamental point that R1 raises. Energy and energy conservation for paleoclimatic case studies are dependent on the record that is being analysed. Here, we solely focus on the LR04 benthic foraminiferal oxygen isotope stack and “energy” is given as a function of the variability in benthic foraminiferal $\delta^{18}\text{O}$, i.e., expressed in $\%{}^3 \text{ kyr}^{-2}$ in the bispectrum, and as $\%{}^3$ when integrating over the bispectrum. However, when other palaeoclimatic time series are considered, the “energy” units will change accordingly.

With respect to the LR04 record: this time series is a globally averaged signal of land-ice volumes and deep-sea temperatures combined. Variability in benthic foraminiferal $\delta^{18}\text{O}$ is largely the result of a nonlinear response of the climate-cryosphere system to the changes in

the distribution of Earth's incoming solar radiation (given in the energy units W m^{-2}), which is often represented by insolation for a particular latitude (e.g. 65°N).

How energy is transferred from Earth's total energy budget (in W s^{-1} , i.e., Joule) into variability in the globally averaged $\delta^{18}\text{O}$ record (in ‰ Vienna Pee Dee Belemnite, VPDB), is depending on "...many climatologic and oceanographic, biologic, sedimentologic and lithologic processes..." (see last line of Section 4.2). It is not the purpose of this study to quantify these Earth-internal processes further. We describe "energy" and "energy conservation" merely qualitatively (i.e., translate asymmetry into a loss and a gain at particular frequencies present in the LR04 stack), without scaling them to the power spectrum of the LR04 stack, to the power spectrum of insolation at e.g. 65°N , or to Earth's total energy budget over a given time. These may be objectives for follow-up studies.

Therefore, why energy transfers should be conservative is not immediately obvious. If I understood correctly, the specific choice of the weight (p. 7, line 2) enforces conservation, but again the physical justification is unclear.

Within the climate system energy losses (i.e., ultimately to space) can occur at numerous stages. These losses will lead to the formation of a particular palaeoclimate record. However, in bispectral analysis and the calculation of the transfer term, these earlier energy losses are not resolved because the bispectrum only considers the available time series, and not what happened beforehand.

When performing bispectral analysis on a specific palaeoclimate record, the conservation of energy during exchanges is assumed (i.e., enforced, obtained, implied) by correcting the energy gains and losses of a particular triad interaction by the values of the frequencies that are involved. This correction is similar/comparable to the Boussinesq scaling used for computing energy exchanges among ocean waves (e.g., (Herbers and Burton, 1997; Herbers et al., 2000)), and allows for qualitative interpretations (i.e., energy gains and losses across frequencies are scaled to one other) (see Section 2.4.1., Fig. 5b, and Fig. 8).

When subsequently making the step to scale these energy exchanges to absolute energy exchanges and make them directly comparable with the time-evolutive gradient of the power spectral density within this specific paleoclimatic record being analysed (i.e., to be able to explain the changes observed within the record through time), the bispectral exchanges have to be corrected for physical processes that play a role in the strength of these exchanges. The dissipation term itself is a completely separate term from the energy conservation that we enforce during triad interactions as documented in the bispectrum (see also Eq. 1 in (Herbers et al., 2000)).

We forego the scaling to absolute transfers here, because of the many unknown/poorly constrained physical, chemical, biological, sedimentological and lithological processes that affect absolute $\delta^{18}\text{O}$ values of the globally integrated LR04 record (see Section 4.2). Further research is needed to advance on this point and obtain estimates of the absolute energies that are exchanged.

To further clarify this point, about the assumed energy conservation in nonlinear triad interactions, we have added text ("if we assume a simple coupling coefficient between frequencies") to Section 2.3.1., added "assumed" to Section 3.4.2. and to Section 6., replaced "using" by "assuming" in Section 4.2.

Similarly, the authors follow the state-of-the art literature and focus on the imaginary part of the bispectrum, but as I understood it the physical rationale for focusing on the imaginary part is in fact grounded in wave theory. Why would we focus on the imaginary part in the present context?

The focus on the imaginary part of the bispectrum is not grounded in wave theory, but in bispectral theory. At equal amplitudes, more energy is transferred among frequencies for time series characterized by asymmetric (imaginary part) than for skewed (real part) wave forms/cycle shapes (approx. an order of magnitude difference). Despite this strong focus on the imaginary part of the bispectrum, we do not rule out a (probably much smaller) contributing role for the real part of the bispectrum in describing (even more) energy transfers. This is a potential topic for future research. (See the first point in the Outlook, i.e., Section 6, and Supp. Fig. 1 and 2).

We agree with R1 that the physical rationale for why nearshore waves are asymmetric is much better understood than why climate cycles are asymmetric. See e.g. the comparison of model to flume/beach data presented in de Bakker et al (2016). In this study, for palaeoclimatic interpretations, we speculate that the asymmetry in the LR04 time series is mainly due to nonlinear (positive) ice feedbacks (albedo, inertia of large ice volumes, land-ice mass-loading threshold) that causes a phase-lag with respect to precession and obliquity, and a phase-coupling with respect to ~110-kyr eccentricity.

We have clarified this point in the text by adding “if time series are dominated by asymmetric wave forms/cycle shapes” to the introduction, and by rephrasing the first bullet point of the Outlook (Section 6).

Perhaps the reader would be reassured to see the bispectral analysis for typical transformation known to be relevant for palaeoclimate dynamics. What happens with bioturbation (which one might intuitively see as a form of non-conservation, or dissipation)? How does bispectral analysis identify demodulation (precession beating being transformed in a response at the period of the beating). What is happening at a period doubling bifurcation? In other words, we need a user’s guide, a reading key of the bispectrum that is well suited to the phenomenology of Pleistocene dynamics. Perhaps these simple examples will also help the reader understand why the focus should be set on the imaginary part of the bispectrum.

The bispectrum is an accepted method in research fields ranging from nearshore waves, neurology, cardiology, to economics, etc. The extension of these (advanced) techniques to palaeoclimatic problems is one of the latest for bispectral applications in this sequence. It is not the purpose of this study to fully educate the reader in bispectral theory, and some background reading/studying may still be required.

We note that the phenomenology of Pleistocene dynamics is highly proxy record dependant (benthic $\delta^{18}\text{O}$ in this case). Hence, there is no single user’s guide that will suit all palaeoclimatic purposes. How the bispectrum is precisely affected by the issues raised above (bioturbation, demodulations, period doubling bifurcations), falls outside the scope of this

study. In general, many of these processes will lead to lower signal-to-noise ratios, and hence, more biased results.

We would like to point R1 (and the interested reader) to the “palaeoclimatic” user’s guide provided by Hagemberg et al. (1991) and King (1996), who show synthetic examples of frequency and phase (de-) coupled time series and their bicoherence spectra.

2. Still in relation with the specific phenomenology of palaeoclimate dynamics, it is important to distinguish ‘cycle’ and ‘frequency’. A saw-tooth signal of 100-ka long is the manifestation of one cycle, that is, a succession of events that form a phenomenon (e.g.: the ice-age cycle). Yet the Fourier decomposition of this signal will feature multiple frequencies (an infinite, countable number of them). Hence, a Fourier peak does not necessarily correspond to what we would like to call a ‘cycle’ or a ‘cyclicity’ in palaeoclimate dynamics. I am a bit worried about the numerous references to a 28-kyr cycle. Wouldn’t it be the main merit of bispectrum analysis to show how frequencies appear in the spectrum and how they are linked to other? In other words, isn’t it precisely the purpose of bispectrum analysis to help one distinguish a frequency from a cycle? (if two frequencies are strongly linked, they are part of a same cycle).

We agree with R1. However, we choose to use “cycle”, “frequency” and “periodicity” more loosely and interchangeably, mainly for textual purposes. We understand that a single frequency (identified in either spectrum or bispectrum) is not necessarily the same as a cycle (identified in a time series), because many cycles are composites of multiple frequencies, most notably skewed, asymmetric, and kurtose cycles.

Despite the (small) differences in the meanings of cycle and frequency, we prefer our less strict semantics to keep the text varied, readable and accessible. However, to also acknowledge the point of R1, we have re-evaluated the usage of “cycle”, “frequency” and “periodicity” throughout the manuscript, and in a few instances changed the wording to make specific references to either time (i.e., cyclic phenomena) or frequency domains clearer. We have also added an explanation to Section 2.3.1., clarifying our intended usage of these words.

3. It is fine in an exploratory paper to focus on one record, here the LR04 stack. However, the possible pitfalls associated with the way the record for this specific application need be better discussed. The chronology of the LR04 stack was established by tuning the record on the output of a simple ice-age model driven by mid-June insolation (the Imbrie and Imbrie 1980 model), with different time constants for the early and late Pleistocene. By design, this approach tends to concentrate power on astronomical bands, with consequences on the bispectrum which are hard to fully anticipate. On the other hand, the process of stacking different records may have unintended effects on the relative weights between the precession and obliquity components (precession being harder to detect, it may be damaged by a stacking process that favours the visible obliquity signal), and, again, consequences on bispectrum hard to anticipate. Precession signals are also relatively more affected than obliquity’s by mixing processes such as bioturbation. Hence, I found a bit hasty and not entirely convincing the author’s conclusion that

stacked records are the best material for their application (p. 20). Splicing high-resolution, carefully chosen records might in fact be an equally attractive choice.

The main rationale to use the LR04 stack in this study is the high signal-to-noise ratio and relatively accurate and precise ages, given the abovementioned tuning assumptions, of which we are fully aware (see caption to Figure 1). We agree with R1 that this does not solely result in benefits, but also in some loss of signal, especially at the higher (precession) frequencies (see also (Huybers and Wunsch, 2004)).

Therefore, we have deleted “Therefore, we argue that for these purposes data stacks are preferred” from Section 6 (last bullet point). In fact, application of bispectra to individual records may prove fruitful for future studies.

4. I must confess being quite critical about section 5. The mechanisms for the explanation of the findings are unnecessarily speculative and slightly misinformed, and seem to me to do more harm than good to the credibility of the paper. A word about the “precession motor”, first. Clearly precession has various possible effects on ice ages dynamics, via the local insolation forcing, possibly the hydrological cycle, why not the carbon or methane cycles. Hence, focusing on monsoon dynamics is unnecessarily reductive. The simulations presented by Werner et al., 2001 suggest that less than 10 % of the precipitation falling on Greenland on in Eastern Canada is of tropical origin. The article is a bit dated but the order of magnitude must be valid. Hence, monsoon might have a direct effect on ice accumulation balance, but the results presented here provide no argument to see it as a dominant one.

The suggested modelling paper by Werner et al, (2001) is mainly concerned with Greenlandic land-ice isotope composition, and does not seem too relevant to our study. During the current interglacial (and those of the past million years or so), Greenland is still largely glaciated. Hence, the moisture source for Greenland is not very relevant in explaining large land-ice volume fluctuations of the past million year (de Boer et al., 2012). The largest land-ice volumes during Middle and Late Pleistocene glacial maxima were located on the North American and Eurasian continents. Hence, moisture sources for these regions during glacial inceptions and maxima may well be largely temperate to (sub-) tropical in origin. Further evidence for the strength of the precession motor comes from lower latitudes (e.g. the sapropels in the Mediterranean), which show the latitudinal migrations of atmospheric (and oceanic) fronts and associated hydroclimate on these shorter, precession time scales (Bosmans et al., 2015).

However, to acknowledge the uncertainty that remains in the understanding of precipitation sources for large land-ice volumes, we have added a reference to Werner et al, (2001), in addition to references to other, more recent modelling studies.

Likewise, the reference to a “resonance of crustal sinking” is, again, unnecessarily sophisticated. Physicists and glaciologists working on ice ages broadly agree that terminations are the manifestation of some ‘non-linear effect’ expressing the instability glacial maxima, and the debate is about the mechanisms of instability (ice-sheet dynamics, ocean and carbon cycle, tectonic CO₂ release). Again, the

contributions or relative importance of these mechanisms cannot be investigated on the basis of a single record, whatever analysis technique is being used.

We agree with R1 that our analysis does not point to crustal sinking as the mechanism. However, we merely state that the bispectral results obtained in this study are in agreement with a nonlinear mechanism, such as crustal sinking and resonance with eccentricity modulated precession (e.g. following (Pisias et al., 1990; Abe-Ouchi et al., 2013)).

Finally, the point 5.2.3 about the “climatic and tectonic boundary conditions” is a bit verbose. A quick glance at the LR04 immediately reveals an evolutionary process, which indeed, is being attributed to tectonic changes with perhaps some evolutionary contribution. The authors are citing many references but the context and the purpose of these references is not always clear, and do not relate to an information that bispectrum analysis would have specifically enlightened.

For contextual purposes, we thought to briefly (one paragraph only) address the long-term climatic evolution during the Pliocene and Pleistocene, mainly to set clear boundaries on what the bispectrum can—and what it cannot—help to understand better. Especially the comparison of the LR04 spectrum to the LR04 bispectrum is of relevance. The time-evolutive spectral analysis show long term frequency evolutions that are absent in the time-evolutive bispectral analysis (compare Fig. 5b to 5c), suggesting shifts in the response of the climate system that are unrelated to the nonlinear processes described by the bispectrum.

In summary, how the bispectrum analysis may contribute to the identification of ice-age dynamics needs to be thought of better. It seems that the main (and really nice) contribution of bispectrum is to act as a powerful test of dynamical system models of Pleistocene climate dynamics.

We agree with R1 that the bispectrum may serve as a powerful test of dynamical system models (GCMs or conceptual) and help the understanding of the Pliocene and Pleistocene climate system. However, we disagree with R1, and do not find it “unnecessarily speculative” and “misinformed” (see R1s point 4) to then also interpret the bispectral results in terms of dynamics (i.e., mechanisms), despite the fact that these interpretations are speculative.

Given the most thorough higher order spectral description of nonlinearities during the Pliocene and Pleistocene to date, which we present, some speculation on the mechanisms is in place. In fact, we would perceive it as a missed opportunity not to at least attempt to (speculatively) link these new observations and descriptions of nonlinearities to mechanisms that have been proposed in the literature. Throughout the Discussion, we have made it very clear that these interpretations are speculative at best.

1. p. 4 l. 5: follow THE convention

Corrected.

2. equation 2: what is the meaning of H^3 ?

This is indeed a mistake. We have changed $As(x) = \frac{\langle H^3(x-\bar{x}) \rangle}{\langle (x-\bar{x})^2 \rangle^{3/2}}$ into $As(x) = \frac{\langle H(x-\bar{x})^3 \rangle}{\langle (x-\bar{x})^2 \rangle^{3/2}}$. This typo was unfortunately not noticed and corrected in in the text of Liebrand et al. (2017). We checked, and the computations that were performed in MATLAB use the latter (correct) formula. H stands for the Hilbert transform.

3. p. 6 l. 4: the reference to Fig. 4a is not straightforward. Perhaps say in more plain language what the reader is supposed to look at on the Figure.

Reading bispectra is not straightforward indeed, which is why we focus the interpretations of the bispectra on the integrations, which transpose the frequency-frequency domain into the time-frequency domain. We did want to show a few clear examples of bispectra, to familiarize the reader with the analysis underpinning the results that are presented later on in the manuscript.

To aid the understanding of Figure 4, we have restructured Section 2.3.2., and moved last paragraph upward. By reordering this section, we now first explain how to read sum frequencies. This should make the reference to Figure 4a also more accessible. All frequencies and periodicities were already given in bispectral notation, which point the reader to the correct “blue area” in the bispectrum.

4. p. 7 l. 1: “Therefore, we make minimum assumptions and use a coupling coefficient that only corrects for a frequency of $W(f_1, f_2) = (f_1 + f_2)$ ”. This seems to be a key passage, of which the implications are not immediately clear to the non-expert. Why does it enforce energy conservation (perhaps this can be explained simply if we consider that the rate of energy loss is counted by cycle), and why having energy conservation allows for “qualitative interpretation”? Again, this links with major comment 1. above, the need to explain in simple term what is “energy”, and how the imaginary part of the triad interaction is an interesting qualitative indicator of energy transfers (comment applies also to p.4 l. 6-11).

See our rebuttal to R1’s Main Point 1 above.

5. p. 8, l. 19: “Nonsinusoidal cycle shapes are generally a good indicator for the successful application of higher order spectral analysis”. Ambiguous sentence. If nonsinusoidal cycles are in the record (quite evidently, late Pleistocene cycles are asymmetric), in what sense does it tell us something about the “successful application” of whatever technique?

Higher order spectra describe nonsinusoidality. However, we agree with R1 that nonsinusoidality on its own, is not sufficient for the successful application of higher order spectra.

We have rephrased this sentence to remove the ambiguity.

6. p. 12: “We only document very minimal direct fuelling of eccentricity-paced climate cycles by precession-paced climate cycles in this zone.” Can we imagine that

this result is influenced by the fact that individual precession cycles are poorly resolved? (bioturbation, undesired effects of stacking).

This may well be the case. The LR04 stack is indeed a globally integrated, land-ice volume dominated record, which may have attenuated precession variability compared to other proxy records, and especially compared to insolation variability at any particular latitude. However, despite the likely bias of this proxy to variability at the lower frequencies, we do see energy transfers from precession to obliquity periodicities (e.g. see Zone 5, OOP), and of obliquity to eccentricity periodicities (e.g. see Zone 2, EEO). Therefore, we argue, that the lack of direct “fuelling” of eccentricity variability by precession (see Zone 6, PEP) is a valid observation,

In the Results Chapter, we prefer to observe and describe without too much interpretation, and have therefore left the text as is. In the Outlook (Section 6) we argue that further higher order spectral analyses, on climate time series that are less land-ice volume dominated, may well be insightful.

7. p. 13: The purpose of the reference to Ahn et al., 2017 is not very clear since it seems that the authors have used the original LR04 stack (hence, Lisiecki and Raymo, 2005).

We have rephrased this sentence by replacing “are” with “may be”.

8. p. 13: Section 3.4.2: another confusing point for the non-expert. Given that weights were chosen such that energy is conserved, how could energy not be conserved? A numerical artefact?

See our rebuttal to R1’s Main Point 1 above.

9. p. 13, l. 17: “A comparison of conservativities indicates that approximately similar amounts of energy are exchanges in interactions involving obliquity, as in those involving eccentricity”. typo: exchanges -> exchanged.

Corrected

The meaning could also be clearer. First, conservativity is a non-standard noun which is not defined in the manuscript (the word appears also in legend of Figure 5).

We have now defined “conservativity” (Section 3.4.2.) and rephrased the figure captions of Fig. 9 and Fig. 10 (N.B. not Fig. 5).

Next, are we speaking of interactions with precession, i.e., are we comparing interactions between precession and obliquity, vs precession and eccentricity?

This sentence (starting with “A comparison of conservativities...”) refers to Figure 10d, in which we compare conservativities of the recombined zones, that contain at least one

precession, obliquity, or eccentricity component (See the first sentence of Section 3.4.3). The answer to R1's question is no.

We have rephrased the text to make this point clearer.

And, again, some more intuitive meaning of "interaction" in the present context (perhaps with a simple example) would be really helpful.

We added a definition of "triad interaction" to Section 2.3.1. However, Figure 10 shows the recombined zonal integrations over the imaginary part of the bispectrum. Frequencies participating in multiple triad interactions are summed and may therefore no longer be visible.

We have added "(triad)" to this particular sentence, to remind the reader of the link to the bispectrum that underpins these computations of energy exchanges.

10. p. 15. l. 2: There may be some confusion between the notion of "reproducibility" (ability to "reproduce" the results based using the data and methodology printed in the manuscript), and "robustness" (insensitivity of results to methodological aspects seemingly unimportant).

Throughout the text we have replaced "reproduce" with "robust/robustness".

11. Figure 1: what are the contours on the continuous wavelet transform plot?

The black contours represent 95% significance. We have added this information to the figure caption.

12. Figure 5: the first bit is cryptic: "Input → "black box" climate → output"

To clarify this figure caption, we have added the relevant panel call-outs ((a), (b), (c)). The meaning of the figure is well-explained in the rest of the caption.

We are aware that strictly speaking bispectra are an "output" analysis, however, our framing here, as a window into the "black box" response, corresponds to the framing of the paper; namely that bispectra 'show how' ice ages are fuelled.

Again, the application of bispectrum analysis is promising and interesting and I would definitely encourage the readers to revise the manuscript. Not much revision may be needed in fact. Focus on the methodology, provide a good 'reading key' so that the naive reader understands better the meaning and implication of the notion of 'energy transfer' in the specific context of palaeoclimate dynamics, and downplay the mechanistic interpretation, which is too speculative and out of scope. Good luck!

Energy and energy transfer do not have a specific context that relates to "palaeoclimate dynamics". These are bispectral properties specific to each proxy time series. See also our rebuttal to R1's Main Point 1 above.

3. References

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We have included this reference in the manuscript.

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We would like to take this opportunity to thank M. Crucifix for his constructive feedback.

Rebuttal to *Interactive comment on “Bispectra of climate cycles show how ice ages are fuelled”* by Diederik Liebrand and Anouk T. M. de Bakker

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1. General comments

Drs. Liebrand and de Bakker provided here new, original statistical analyses of the LR04 $\delta^{18}\text{O}$ stacking to document the nonlinear interactions between the Milankovitch cycles which lead to the generation of new cycles in the palaeoclimatic data and lead to power transfer from the precession band (dominant in the insolation series) to the obliquity and to the 100-kyr eccentricity cycles. In particular, bispectra are used to observe nonlinearities between the insolation forcing and the $\delta^{18}\text{O}$ series, which is an excellent and original idea in palaeoclimatology.

In addition to R2's summary, we would like to emphasize that we apply bispectral analysis only to the LR04 record, and thereby describe nonlinear interactions among climate frequencies as present in this data set (i.e., a transform of the asymmetric cycle geometry), and not between insolation and $\delta^{18}\text{O}$. The comparison to insolation forcing is merely qualitative (see Fig. 6), and is presented for comparison based on a theoretical/physical understanding of astronomical climate forcing.

However, I find section 3 of the manuscript hard to read to someone who is not familiar with the reading the interpretation of bispectra despite I could see the authors made many efforts to make their paper accessible.

To clarify the interpretation of bispectra we provide a thoughtfully constructed reading key in Section 2.3.2. (“Interpreting the bispectrum”). In this section we also refer to Figure 4, to support the interpretation of the bispectrum. This information is crucial for understanding the results (Section 3) and the interpretation of the bispectra.

In section 3.2., I do not understand what is reference to calculate the gains and the losses of energy and I do not understand how the authors can find this information in the bispectra. This step must be clear for any reader to then completely follow the result description in section 3.4.

To read Section 3.2., and understand how gains and losses are computed we refer R2 (and the interested reader) to Section 2.3.2., in which we explain in more plain language how the bispectrum is interpreted, and how the bispectral notation should be read.

In short: an energy gain (EG) is depicted by the warm colours (red), whereas an energy loss (EL) is represented by cold colours (blue). An energy gain at one frequency results in energy

losses at two other frequencies and vice versa: $EG_{f1}+EG_{f2} = EL_{f\beta}$ and $EL_{f1}+EL_{f2} = EG_{f\beta}$. The “energy” reference (i.e., amount of energy that is exchanged in such a nonlinear triad interaction) is given as a function of the variability in benthic foraminiferal $\delta^{18}\text{O}$, expressed in $\text{‰}^3 \text{kyr}^{-2}$ in the bispectrum, and as ‰^3 when integrating over the bispectrum (as is described in Section 3.4.).

We have added information about these energy units to Section 2.3.1. and Section 2.4.

I can see that bispectra document the energy (or power) transfers and the evolutive spectral analyses seem to document these power transfers very clearly. However, I do not understand how the bispectra contribute in understanding the mechanisms of nonlinearities in the $\delta^{18}\text{O}$ already evoked in prior publications and this needs to be clarified.

To date, no similarly detailed (i.e., in a time-evolutive manner) description of nonlinear energy transfers among Pliocene and Pleistocene climate cycles is available. The detailed documentation of these transfers presented in this study ‘show how’ (i.e., through which cycle-cycle interactions) energy is transferred from the insolation frequencies (mainly precession) to those of the ice ages (40, 80, 120, and 95 kyr), and how these transfers evolve through time. In light of these new results, we deem it valuable to tentatively link them to mechanisms that have previously been proposed in the literature. In our opinion, it would be a missed opportunity not to (at least) attempt to draw further conclusions about potential nonlinear mechanisms, given the best available description of nonlinear energy transfers.

A suggestion for a follow-up study would be to test with fully coupled climate-ice sheet models which mechanisms correspond to the specific frequency interactions we observe in the LR04 stack. We have added this suggestion to the Outlook (see the third bullet point in Section 6).

In summary, much clarification is needed to allow a larger community to access this enthralling way to observe cycles in sedimentary series and observe their interactions. I thus suggest this manuscript deserves to be published after revisions will be done.

On this particular point (i.e., “much clarification”), we disagree with R2. Section 2.3. is concerned with the interpretation of bispectra and was designed with great care. It provides an explanation of the bispectrum, and how to read one. This explanation is more detailed than most existing papers on bispectra. Section 2.3. is written with the specific aim of explaining the bispectrum to the nonexpert, and we believe that after reading this section, the remainder of the text is accessible to most palaeoclimatologists/-oceanographers.

The authors can find more specific comments below:

2. Specific comments

Throughout the manuscript, the term "energy transfer" is used. What do the authors refer to when they use this terminology? This must be more clearly stated, unless I missed it in the manuscript.

To further clarify what is meant by “energy transfers” in this particular case study on the LR04 record, we have added a sentence to Section 2.3.1. and to Section 2.4.1.

In page 3, line 26, the author mention they used SiZer to resample every 1 kyr. What is the method used by SiZer to resample? Is it a linear resampling? Is it another method of resampling? Can the authors write exactly the method of resampling? Because it can impact the spectrum at high frequencies.

The SiZer method computes the statistically significant zero crossings of the first and second derivatives of an unevenly sampled time series, to compute a ‘family of smooths’ that fit these criteria. We used the raw data of the LR04 stack to compute these smooths and selected the smooth (out of 41 smooths) that preserved the most structure in the data, given the 1 kyr resampling resolution. SiZer smooths are not linear interpolations. We refer to Chaudhuri and Marron (1999) for a more detailed description of the SiZer method.

We added information about our smooth selection criteria to Section 2.1.

With respect to the impact on higher frequencies: The highest frequency considered in this study is 100 Myr^{-1} , equivalent to a periodicity of 10 kyr. A resampling resolution of 1 kyr (1000 Myr^{-1}) yields a Nyquist frequency of 500 Myr^{-1} , which is well above that of the cycle frequencies considered in this study (i.e., 0 to 100 Myr^{-1}). Furthermore, in an earlier stage of this study, we have performed sensitivity tests for different resampling resolutions (not included in the current study or supplements), and found no difference on the astronomical frequencies considered here.

In page 4, line 13, what do the authors call "time averaging operator"?

This is the window length considered. In this study, most often 668 kyr long windows were used (apart from in the supplements where 500 kyr and 1000 kyr long window lengths are considered).

We have added “(i.e., window length)” to the sentence.

Section 3.2. ("Bispectra of Pliocene and Pleistocene climate cycles") is hard to follow in my point of view at least for the following reasons:

The authors mention gain or loss of energy. Gain or loss should be a difference compared to a reference. What is the reference used to calculate these gains and losses?

See our rebuttal to previous comments by R2 (i.e., the third “General Comment” and first “Specific Comment”)

The authors mention positive or negative interactions, e.g. (page 8, lines 24-25): " negative interactions are concentrated at and between triads along the lines from $B_p^{Im}(40\uparrow, \infty\uparrow, 40\downarrow)$ to $B_p^{Im}(40\uparrow, 40\uparrow, 20\downarrow)$ ". I do not know where to observe this in Figure 4. Can the authors either explain this with a theoretical example easy to understand prior to the real data or at least show where to observe this in Figure 4?

In section 2.3.2. we explain the bispectral notation. This information explains how the bispectrum is read and interpreted. In this section we also give a simple example, not theoretical, but based on Figure 4a.

From these two examples, I think much effort have to be made to guide step-by-step a reader who is not familiar in the use and interpretation of bispectra. Otherwise section 3.4., which describes the results of bispectra, will remain inaccessible for many readers. So, I suggest more step by step explanation to make easier the observation and the interpretation of the bispectra.

See previous rebuttal comments to R2. (i.e., Section 2.3 is key in understanding Section 3).

In Figure 5, I do not understand what the authors refer to by writing "Input → "black box" climate → output". What do the authors mean by "black box" here?

We agree with R2 that the title of this figure caption is a bit cryptic. However, this was done on purpose, with the aim to provoke thought about the workings of the Earth System. "Black box" is a commonly used metaphor for a system of which the inner workings are only partially understood. Earth's climate system is such a "black box". Its past behaviour can only be approximated (by proxy records) or understood theoretically/through modelling. Spectra of insolation represent the climate driver (i.e., input) and the spectra of the benthic $\delta^{18}\text{O}$ record constitute the "black box" response (i.e., output).

We have now labelled the corresponding panels in the Figure caption.

Still in Figure 5, I would clearly state what conservative net energy transfer means

We have added text in between brackets for the explanation of panel (b).

In Figures 5 and 6, I would label the frequencies on which energy transfers occur

Both frequencies and periodicities are labelled along the axes. Adding numbers within the figures would, in our opinion, make them more cluttered and less easy to read.

In section 5.2.1. "Based on the bispectral results, we infer that during the Pliocene and Early Pleistocene this predominantly monsoonally-driven precession motor fuels the 40-kyr obliquity-paced ice age cycles, aided by more linear climatic-cryospheric responses resulting from variability in insolation at this periodicity, especially at higher latitudes" » I do not really understand how the authors can deduce that from bispectral analyses. The authors can of course observe transfers of power from the precession to the obliquity band, but how can they link that to the moisture and heat transfer at low latitude? There is a step I do not understand. Comparatively, the interlatitudinal insolation gradient evoked by Bosmans et al. (2015) appears much more intuitive and easier to understand.

The modelling study by Bosmans et al. (2015) is concerned with explaining obliquity signals at low latitudes, and suggests that these signals may originate in the (sub-) tropics. However, we aim to explain the transfer of energy from precession to obliquity cycles in a high latitude land-ice volume dominated climate record. Although obliquity is observed at low latitudes, many of the (sub-) tropical palaeoclimate records remain precession dominated (e.g., monsoonal/loess records, sapropels, caves records, etc.). Therefore, we speculate that insolation changes at the lower latitudes (mainly precession paced) may fuel the transport of heat and moisture to the poles, and the build-up on obliquity time scales (with associated energy transfers from precession to obliquity as documented in the bispectra) of Northern Hemisphere land ice. The study of Bosmans (2015) does not include a dynamic ice sheet and can therefore not capture these hypothesised energy transfers.

I experience the same feeling with section 5.2.2.: how the power transfer observed in the bispectra can help in linking the transfer from the obliquity to the eccentricity with crustal sinking and delayed rebound? I think the authors need to clarify how the bispectra can contribute to the debate

We copy our reply to R1, who also raised this point: Our analysis does not point to crustal sinking as the mechanism. However, we merely state that the bispectral results obtained in this study, are in agreement with a nonlinear mechanism, such as crustal sinking and resonance with eccentricity modulation precession (e.g. following (Pisias et al., 1990; Abe-Ouchi et al., 2013)).

3. Technical corrections

In Figure S3, labels a, b, c in the caption do not correspond with the panels in the figure. Can the author correct that?

We have corrected the figure caption.

4. References

Bosmans, J.H.C, Hilgen, F.J., Tuenter, E., Lourens, L.J., 2015. Obliquity forcing of low-latitude climate. *Clim. Past*, 11, 1335-1346.

We have added this reference to the manuscript.

Rebuttal references:

Abe-Ouchi, A., Saito, F., Kawamura, K., Raymo, M. E., Okuno, J., Takahashi, K., and Blatter, H.: Insolation-driven 100,000-year glacial cycles and hysteresis of ice-sheet volume, *Nature*, 500, 190–194, <https://doi.org/10.1038/nature12374>, 2013.

Bosmans, J. H. C., Hilgen, F. J., Tuenter, E., and Lourens, L. J.: Obliquity forcing of low-latitude climate, *Clim Past*, 11, 1335–1346, <https://doi.org/10.5194/cp-11-1335-2015>, 2015.

Chaudhuri, P., and Marron, J. S.: SiZer for exploration of structures in curves, *J Am Stat Assoc*, 94, 807–823, <https://doi.org/10.2307/2669996>, 1999.

Pisias, N. G., Mix, A. C., and Zahn, R.: Nonlinear response in the global climate system: evidence from benthic oxygen isotopic record in Core Rc13-110, *Paleoceanography*, 5, 147–160, <https://doi.org/10.1029/PA005i002p00147>, 1990.

We would like to take this opportunity to thank M. Martinez for his constructive feedback.

Bispectra of climate cycles show how ice ages are fuelled

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Abstract. The increasingly nonlinear response of the climate-cryosphere system to insolation forcing during the Pliocene and Pleistocene, as recorded in benthic foraminiferal stable oxygen isotope ratios ($\delta^{18}\text{O}$), is marked by a distinct evolution in ice-age cycle frequency, amplitude, phase, and geometry. To date, very few studies have thoroughly investigated the nonsinusoidal shape of these climate cycles, leaving precious information unused to further unravel the complex dynamics of the Earth's system. Here, we present higher-order spectral analyses of the LR04 $\delta^{18}\text{O}$ stack that describe coupling and energy exchanges among astronomically-paced climate cycles. These advanced bispectral computations show how energy is passed from precession-paced to obliquity-paced climate cycles during the Early Pleistocene (~2,500–750 ka), and ultimately to eccentricity-paced climate cycles during the Middle and Late Pleistocene (from ~750 ka onward). They also show how energy is transferred among many periodicities that have no primary astronomical origin. We hypothesize that the change of obliquity-paced climate cycles during the mid-Pleistocene transition (~1,200–600 ka), from being a net sink into a net source of energy, is indicative of the passing of a land-ice mass-loading threshold in the Northern Hemisphere (NH), after which cycles of crustal depression and rebound started to resonate with the ~110-kyr eccentricity modulation of precession. However, precession-paced climate cycles remain persistent energy providers throughout the Late Pliocene and Pleistocene, which is supportive of a dominant and continuous fuelling of the NH ice ages by insolation in the (sub-) tropical zones, and the control it exerts on meridional heat and moisture transport through atmospheric and oceanic circulation.

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1. Introduction

The recurrent ice ages of the Pliocene and Pleistocene, as captured in benthic foraminiferal $\delta^{18}\text{O}$ records, are characterized by long-term trends in glacial-interglacial cycle duration, amplitude, response time, and geometry (Fig. 1 and Fig. 2). These trends mainly reflect the increasingly nonlinear response of the (northern) high latitude cryosphere and global deep-sea temperatures to radiative forcing, i.e. the combined greenhouse effect of incoming solar radiation (insolation) and the partial pressure of atmospheric CO_2 ($P_{\text{CO}_2^{\text{atm}}}$), when global climatic conditions deteriorated (Lisiecki and Raymo, 2005, 2007; Martinez-Boti et al., 2015; Chalk et al., 2017). Spectral analysis, in combination with age control independent from astronomical tuning, formed the decisive evidence in support of Milankovitch's theory of astronomical climate forcing (Hays et al., 1976). These statistical methods, such as the (time-evolutive) fast Fourier transform, or wavelet analysis using a

Morlet-transform, perform well in defining sinusoidal cycle properties, such as frequency, amplitude, and/or (cross-) phase relationships with respect to an astronomical reference curve (Lourens and Hilgen, 1997; Lisiecki, 2010; Meyers and Hinnov, 2010). However, spectral analysis methods are not well suited to describe highly nonsinusoidal signals, such as the skewed and asymmetric cycles that characterize time series of Middle and Late Pleistocene climate (Fig. 2) (Hagelberg et al., 1991; King, 1996; Lisiecki and Raymo, 2007). To mitigate the shortcomings of statistical tests that implicitly assume sinusoidality, higher-order spectral analysis techniques were introduced into the research field of palaeoceanography/-climatology in the 1990s (Hagelberg et al., 1991; Hagelberg et al., 1994; King Hagelberg and Cole, 1995; King, 1996).

Bispectral analysis was conceived in the 1960s within the research field that studies ocean waves (Hasselmann et al., 1963). It is an accepted method of quantifying nonlinear energy transfers among nearshore waves that reach breaking point when the seafloor shallows (Elgar and Guza, 1985; Doering and Bowen, 1995; Herbers et al., 2000; de Bakker et al., 2015). Since the pioneering interdisciplinary studies by palaeoceanographer/-climatologist Teresa (Terri) King Hagelberg and colleagues (notably the physical oceanographer/nearshore ocean wave researcher Steve Elgar) on Pleistocene and Holocene records (Hagelberg et al., 1991; Hagelberg et al., 1994; King Hagelberg and Cole, 1995; King, 1996), relatively few studies have applied higher-order spectral analysis methods to the palaeoclimate archive (e.g., (Muller and MacDonald, 1997b, a; von Dobeck and Schmieder, 1999; Rial and Anaclerio, 2000; Rutherford and D'Hondt, 2000; Huybers and Wunsch, 2004; Huybers and Curry, 2006; Liebrand et al., 2017; Da Silva et al., 2018)). However, two more-recent developments make a re-appreciation for the potential of bispectra for palaeoclimate science timely. First, advancements have been made in the understanding and interpretation of the bispectrum (Herbers et al., 2000; de Bakker et al., 2015). These constitute (i) a shift of focus from bicoherence, which quantifies the strength of the couplings between the frequencies that are present in both the real and the imaginary parts of the (complex-valued) bispectrum, to just the imaginary part of the bispectrum, which can be used to compute nonconservative, relative energy exchanges, if time series are dominated by asymmetric wave forms/cycle shapes, and (ii) integration over the imaginary part of the bispectrum to quantify conservative, net energy transfers, and potentially absolute energy transfers, if net transfers can be scaled to the power spectrum (de Bakker et al., 2015; de Bakker et al., 2016). Second, increasingly noise-free benthic foraminiferal $\delta^{18}\text{O}$ stacks, characterized by precise and accurate age models, have become available to the palaeoclimate community (Lisiecki and Raymo, 2005; Ahn et al., 2017). Such data are a prerequisite for successful application of advanced higher-order spectral analysis methods, because they describe the distribution of nonsinusoidality (i.e., relatively small amounts of variance compared to the sinusoidal cycle properties) over time and frequency (King, 1996).

Both the 40-kyr problem of the Pliocene and Early Pleistocene, and the ~110-kyr problem (a.k.a. the 100-kyr problem) of the Middle and Late Pleistocene, are defined by considerable mismatches in spectral power, at their designated periodicities, between benthic foraminiferal $\delta^{18}\text{O}$ records (dominant) and summer insolation (weaker or absent, respectively) (Raymo and Nisancioglu, 2003; Raymo et al., 2006; Lisiecki, 2010). The mid-Pleistocene transition (MPT) (from ~1200 ka to ~600 ka)

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constitutes the temporal link between the 40-kyr and ~110-kyr “worlds”, when climate cycles in benthic foraminiferal $\delta^{18}\text{O}$ become longer in duration, of higher amplitude, and asymmetric in shape (Hagelberg et al., 1991; King, 1996; Lisiecki and Raymo, 2007). In association with this increasingly nonlinear response of Earth’s climate-cryosphere system to radiative forcing, several climate cycles without a straightforward astronomical origin have been identified, such as those with

5 periodicities of semi-precession, ~28-kyr, ~56-kyr, and ~80-kyr (Rutherford and D’Hondt, 2000; Berger et al., 2006; Lourens et al., 2010). Many, often contrasting, hypotheses have been proposed to address this complex climatic evolution of astronomically forced Pliocene and Pleistocene climate cycles, which range from the merging of the Cordilleran and the Laurentide ice sheets (Bintanja and van de Wal, 2008), a delayed isostatic rebound of the lithosphere-asthenosphere after land-ice mass loading (Abe-Ouchi et al., 2013), to regolith erosion (Clark and Pollard, 1998), and/or $P_{\text{CO}_2^{\text{atm}}}$ thresholds

10 (Chalk et al., 2017), to name just a few (Roe and Allen, 1999; Huybers, 2011). However, more information needs to be extracted from geological archives to distinguish between these mechanisms. We examine the nonlinear response of the Pliocene–Pleistocene climate-cryosphere system to radiative forcing by applying state-of-the-art bispectral analysis techniques (Herbers et al., 2000; de Bakker et al., 2015; de Bakker et al., 2016) to one of the most noise-free palaeoclimate records, namely the LR04 benthic foraminiferal $\delta^{18}\text{O}$ stack (Lisiecki and Raymo, 2005), with the aim to shed new light on (i)

15 the nonlinear origins of climate cycles with and without a direct astronomical connection, (ii) the 40-kyr problem of the Pliocene and Early Pleistocene, (iii) the mid-Pleistocene transition, and (iv) the ~110-kyr problem of the Middle and Late Pleistocene. We show in great detail through which frequency interactions energy is transferred to the 40-kyr, the ~80-kyr and ultimately, ~110-kyr cycles. These new insights can be used to better link climate cycle geometries to nonlinear response mechanisms, and to obtain a better understanding of Earth’s energy balance on astronomical time scales.

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2. Methods

2.1. Benthic foraminiferal $\delta^{18}\text{O}$ record of the Pliocene and Pleistocene

To quantify nonlinear energy transfers among astronomically paced cycles of Earth’s climate-cryosphere system, we use the LR04 compilation of globally distributed records of stable oxygen isotope ratios ($\delta^{18}\text{O}$) measured on the calcite of benthic

25 foraminifera (Lisiecki and Raymo, 2005). This stack spans the Pliocene–Pleistocene time interval, from 5,333 to 0 ka (Lisiecki and Raymo, 2005). We recompiled the original LR04 data set (39,473 data points, <http://lorraine-lisiecki.com/stack.html>) and resampled it at 1 kyr resolution using SiZer, a method that extracts structures in time series by assessing the statistically significant zero-crossing of its derivatives (Chaudhuri and Marron, 1999), because bispectral analysis requires a constant sampling resolution (Fig. 1 and Fig. 2). For the purposes of this study we selected the smooth

30 (out of 41) that preserves most of the structure in the data, given the 1 kyr resampling resolution. This resampling resolution resulted in an oversampling of the original data for ~0.83% of the record ($N = 328$), most of which falls in the earliest Early

Pliocene part of the record, which is excluded from the bispectral analysis for ages >5250 ka. The resultant resampled LR04 record is very similar in structure to the original LR04 stack (Lisiecki and Raymo, 2005), and, because of this similarity, we hereafter refer to the SiZer resampled LR04 record more simply as “LR04 stack”. Prior to bispectral analysis we detrended the LR04 stack using a Gaussian Notch filter (frequency = 0.0 Myr⁻¹, bandwidth = 2.0 Myr⁻¹) to remove periodicities ≥500 kyr (Paillard et al., 1996). We also multiplied the age scale of the LR04 stack with -1, because the direction of time, which determines the sign of asymmetry values and energy transfers in bispectral analysis, increases toward the present. However, for the figures we follow [the](#) convention, and plot ages that increase with geological age.

2.2. Quantifying geometries using central moments

Skewness and asymmetry are quantified using both the third central moment and bispectral methods (Fig. 2, Supp. Fig. 2, and Supp. Fig. 3) (see Section 2.3.3 for bispectral method) (Elgar, 1987). Kurtosis is quantified using the fourth central moment only, because no trispectra were calculated (Fig. 2). Using third-moment quantities, skewness is determined by Eq. (1):

$$Sk(x) = \frac{\langle(x-\bar{x})^3\rangle}{\langle(x-\bar{x})^2\rangle^{3/2}}, \quad (1)$$

where the overbar indicates the mean value and where $\langle \rangle$ is the time averaging operator ([i.e., window length over which the computation is performed](#)) (Doering and Bowen, 1995), and asymmetry is determined following Eq. (2):

$$As(x) = \frac{\langle H(x-\bar{x})^3 \rangle}{\langle(x-\bar{x})^2\rangle^{3/2}}, \quad (2)$$

where H is the Hilbert transform (Kennedy et al., 2000). Using fourth-moment quantities, kurtosis is defined by Eq. (3):

$$K(x) = \frac{\langle(x-\bar{x})^4\rangle}{\langle(x-\bar{x})^2\rangle^2} - 3. \quad (3)$$

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2.3. Bispectral analysis

2.3.1. The bispectrum

In contrast to spectral analysis, which gives [the distribution of variance of a sinusoidal signal with frequency](#), bispectral analysis describes the distribution of nonsinusoidality with frequency, in both the real and imaginary parts (King, 1996).

These parts are related to skewed and asymmetric cycle geometries, respectively (Fig. 2). The bispectrum shows nonconservative, relative energy exchanges among [frequencies of a single time series](#). Energy transfers can only occur, and be analysed by the bispectrum, if both the frequencies (f) and phases (Φ , φ , in radians) of these [nonsinusoidal](#) cycles are

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coupled: i.e., $f_1 + f_2 = f_3$ and $\varphi_1 + \varphi_2 = \varphi_3$, respectively (Hagelberg et al., 1991). Thus, for any possible combination of three frequencies the bispectrum assesses whether there is a coupling, and if so, whether energy exchanges occur. The transfer of energy in these so-called “triad interactions” is nonlinear, because the changes in cycle amplitudes (A) do not sum, and have a currently unknown, and probably variable, coupling through time (i.e., $A_1 + A_2 \neq A_3$, see Section 4.3). We note that the meaning of “cycle”, “frequency” and/or “periodicity” if different when referring to either the time or frequency domain. For example, a “single” cycle in a time series is often composed of multiple frequency components, that can be deconvolved by (bi)spectral analysis. However, for textual purposes, we use these words interchangeably, regardless of their reference to a specific domain.

10 The bispectrum is defined by Eq. (4):

$$B_{f_1, f_2} = E[C_{f_1} C_{f_2} C_{f_1+f_2}^*], \quad (4)$$

where $E[\dots]$ is the ensemble average of the triple product of complex Fourier coefficients C at the difference frequencies f_1 , f_2 , and their sum f_3 (i.e., $= f_1 + f_2$), and the asterisk indicates complex conjugation (Hasselmann et al., 1963). In this study, we focus on the imaginary part of the bispectrum (hereafter often referred to as “bispectrum”, unless indicated otherwise),

15 following studies on ocean waves (e.g., (Herbers et al., 2000; de Bakker et al., 2015)), because most energy transfers are associated with asymmetric cycle shapes (i.e., wave forms) and the climate cycles of the Middle and Late Pleistocene have high amplitudes (Fig. 1) and are highly asymmetric (Fig. 2). Furthermore, conservative energy exchanges between the coupled frequencies that are located in the imaginary part of the bispectrum can be computed if we assume a simple coupling coefficient between frequencies. Prior to bispectral analysis we detrend the data, and subsequently apply a tapering function
20 using a Hann (a.k.a. Hanning) window that also mitigates spectral leakage (Hagelberg et al., 1991), and then multiply the data with an energy correction factor to adjust for the change in amplitude that results from the windowing. Energy transfers in the bispectrum that are computed on the LR04 stack, are expressed in $\%^3 \text{ kyr}^{-2}$.

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2.3.2. Interpreting the bispectrum

25 The x - and y -axes in the bispectrum, which correspond to f_1 and f_2 respectively, are mirror images of each other and share a symmetry axis at $x = y$ (Fig. 3) (Hasselmann et al., 1963). Therefore, only the positive, one-eighth part of the bispectrum is depicted, which is subdivided into 15 zones (see section 2.2.5, Table A1). For triad interactions (recall, interactions among three frequencies), only two outcomes exist: either two frequencies (by definition: f_1 and f_2) gain energy (though not necessarily in equal measures), and one frequency (i.e., f_3) loses energy, or f_1 and f_2 lose energy (ditto), and f_3 gains energy.
30 The former are the so-called difference frequencies, while the latter is referred to as a sum frequency. Frequencies often participate in multiple interactions, and depending on the interaction, they act as either a difference frequency or a sum

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frequency. A particular frequency can thus receive energy through one interaction, and simultaneously lose energy in another interaction.

To help the interpretation of the bispectrum, we depict the main astronomical frequencies in the bispectrum with coloured lines (Fig. 3). Vertical and horizontal lines correspond to the difference frequencies (f_1 and f_2 respectively), and diagonal lines correspond to the sum frequency (f_3). Junctions between lines highlight the locations in the bispectrum where interactions occur between three periodicities (i.e., triad interactions), of which at least one is an astronomically-paced climate cycle, which can also interact with itself (Fig. 3, see also Table A2). No frequency axis is associated with sum frequency f_3 , but its value can be read off at the crossing points with the x - and y -axes of the bispectrum, or by summing the x and y coordinates of the difference frequency axes at any point along the diagonals (Fig. 3).

We follow the convention, and define a triad interaction as negative or positive if f_3 either loses (blue colours) or gains (red colours) energy, respectively (Fig. 4). In written form, energy gains at a particular frequency are marked by an upward pointing arrow (\uparrow), and losses by a downward pointing arrow (\downarrow). Bispectral notation of triad interactions are given as $B(f_1, f_2, f_3)$ in the frequency domain (Myr^{-1}), and as $B_p(p_1, p_2, p_3)$ in the periodicity domain (kyr). For palaeoclimatological purposes we will mainly focus on the periodicities, which corresponding frequencies can easily be looked up in Table A2. We refer to the relevant part of the bispectrum with “*Re*” (i.e., real) and “*Im*” (i.e., imaginary), which are given in superscript (B^{Re} and B^{Im} , respectively). Thus, if we consider a negative triad interaction, located in the imaginary part of the bispectrum, between p_1 , a 40-kyr obliquity paced cycle (i.e., $f_1 = 25.0 \text{ Myr}^{-1}$), and p_2 , a ~95-kyr eccentricity paced cycle (i.e., $f_2 = 10.5 \text{ Myr}^{-1}$), that results in a loss of energy at p_3 , the ~28-kyr periodicity (i.e., $f_3 = 35.5 \text{ Myr}^{-1}$), this is given as: $B_p^{Im}(40\uparrow, 95\uparrow, 28\downarrow)$ (i.e., $B_f^{Im}(25.0\uparrow, 10.5\uparrow, 35.5\downarrow)$) (Fig. 4a).

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We note that within this study, we almost exclusively focus on the imaginary part of the bispectrum. This approach contrasts with previous palaeoclimate investigations that used the bicoherence spectrum (see e.g. (Hagelberg et al., 1991; von Dobeneck and Schmieder, 1999; Wara et al., 2000; Huybers and Curry, 2006; Da Silva et al., 2018)). In short, bicoherence is a measure of the coherency between the real and imaginary parts of the bispectrum (i.e., where both parts have strong interactions), and is marked by positive values only (see e.g. (de Bakker et al., 2014)). Conversely, the real and imaginary parts of the bispectrum transform skewed and asymmetric cycle geometries into their composite frequency components, and are characterized by both positive and negative interactions.

2.3.3. Quantifying geometries using the bispectrum

Adopting the bispectral method to extract cycle geometries (Fig. 2, Supp. Fig. 2, and Supp. Fig. 3) (Elgar, 1987), skewness and asymmetry are computed from the biphase. The biphase reflects the ratio between the real and imaginary parts of the bispectrum, which (recall) correspond to skewed and asymmetric cycle shapes, respectively. The biphase is defined by Eq.

(5):

$$Sk(x) + iAs(x) = \left[12 \sum_n \sum_l B(f_n, f_l) + 6 \sum_{p=1}^{\frac{N}{2}} B(f_p, f_p) \right] / E[x^2]^{3/2}, \quad (5)$$

where n and l range from 1 to the Nyquist frequency N , with $n > l$ and $n + l \leq N$ (Elgar, 1987), and f_p refers to the primary frequency (a.k.a. natural or resonance frequency, or first harmonic). We note that positive skewness (asymmetry) values correspond to a dominance of negative interactions in the real (imaginary) part of the bispectrum, and vice versa, negative values for these geometries are indicative of an overall dominance of positive interactions.

2.4. Power spectral density and net energy transfers

2.4.1. Integration of the spectrum and bispectrum

To quantify power spectral density, we apply a standard integration of the power spectrum. To obtain conservative, net energy transfers per frequency (defined by the nonlinear source term S_{nl}), and hence, to extract the total gain or loss in energy per climate cycle over a specific time interval (i.e., window), we integrate over the imaginary part of the bispectrum and multiply it with a coupling coefficient. For palaeoclimate time series, no coupling coefficient has been determined previously. Therefore, we make minimum assumptions and use a coupling coefficient that only corrects for frequency $W_{(f_1, f_2)} = (f_1 + f_2)$. A comparable correction for frequency is also part of the Boussinesq scaling that is used for ocean waves (e.g., (Herbers and Burton, 1997; Herbers et al., 2000)). The correction for frequency $W_{(f_1, f_2)}$ insures energy conservation during triad interactions, because more energy is exchanged among the higher frequencies. Furthermore, this correction allows for qualitative interpretations. Furthermore, if a more appropriate coupling coefficient for palaeoclimatic purposes can be found, it could also scale the net energy transfers of the bispectrum (already corrected using S_{nl} during integration) to absolute energy transfers that are equal to power spectral density (see Section 4.3). We express the integral of S_{nl} in terms of an integration, or summation, over the positive quadrant of the bispectrum alone, or equivalently in sum and difference interactions following Eq. (6):

$$S_{nl, f} = S_{nl, f^+} + S_{nl, f^-}, \quad (6)$$

where the sum contributions are expressed by Eq. (7):

$$S_{nl, f^+} = \sum_{f \in F^+} W_{f', f-f'} I \{B_{f', f-f'}\}, \quad (7)$$

Deleted: To help the interpretation of the bispectrum, we depict the main astronomical frequencies in the bispectrum with coloured lines (Fig. 3). Vertical and horizontal lines correspond to the difference frequencies (f_1 and f_2 respectively), and diagonal lines correspond to the sum frequency (f_3). Junctions between lines highlight the locations in the bispectrum where interactions occur between three periodicities (i.e., triad interactions), of which at least one is an astronomically paced climate cycle, which can also interact with itself (Fig. 3, see also Table A2). No frequency axis is associated with sum frequency f_3 , but its value can be read off at the crossing points with the x - and y -axes of the bispectrum, or by summing the x and y coordinates of the difference frequency axes at any point along the diagonals (Fig. 3).

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and the difference contributions are expressed by Eq (8):

$$S_{nl} = -2 \sum_{f \in F^-} W_{f+f',-f'} I \{B_{f,f'}\}. \quad (8)$$

The sum contributions are obtained by integrating diagonally over the bispectrum, and the difference contributions are obtained by integrating along vertical and horizontal lines, perpendicular to the x and y -axes, respectively (Fig. 3). We track the evolution of net energy transfers through time by integrating over bispectra that are determined for consecutive and partially overlapping windows of the time series (i.e., a moving window) (Fig. 5, Fig. 6, and Supp. Fig. S3). Integration can be performed including all frequencies (i.e., over the entire imaginary part of the bispectrum), or over specific zones only, in case subsets of interactions need to be further examined (Fig. 3, Table A1) (de Bakker et al., 2015; de Bakker et al., 2016).

After integration over the bispectrum, either totally or zonally, energy transfers computed on the LR04 stack are expressed in %³. Blue colours indicate a loss of energy at a specific frequency, red colours a gain.

2.4.2. Spectral and bispectral zones

To obtain more insight in the energy that is exchanged among eccentricity, obliquity and precession cycles we integrate the spectra and bispectra over separate zones (Fig. 3, Table A1). For bispectra, such a zonation approach was first applied in research concerned with nearshore ocean waves to distinguish between infragravity and sea-swell frequencies (de Bakker et al., 2015), and is adapted here to investigate the most important climate cycle bandwidths of the Pliocene and Pleistocene. The boundaries between the climate cycle zones are (arbitrarily) defined at frequencies of 17.8, 35.6 and 80.0 Myr⁻¹ (i.e., periodicities of ~56.2, ~28.1 and 12.5 kyr). In total, 15 bispectral zones are defined (Fig. 3, Table A1), each of which represents a unique combination of three main groups of astronomically-paced climate cycles and suborbital periodicities (i.e., supraorbital frequencies) that can interact and exchange energy with one other. We note that many periodicities without primary astronomical origin are included these “eccentricity”, “obliquity”, and “precession” zones. We focus here on the results of the eight most important zones, namely those involving astronomically-paced climate cycles (Fig. 7, Fig. 8, Fig. 9, Supp. Fig. S4 and Supp. Fig. S5). To better highlight the roles of all nonlinear interactions involving eccentricity, obliquity, or precession, we recombine (i.e., sum) the individual zones (Fig. 10).

3. Results

3.1. Geometries of Pliocene and Pleistocene climate cycles

The geometry computations confirm the qualitative visual inspection of the LR04, globally averaged, benthic foraminiferal $\delta^{18}\text{O}$ stack, and show that strong nonsinusoidal cycle shapes are present in the data, especially during the Middle and Late

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Pleistocene part of the record (Fig. 2) (Lisiecki and Raymo, 2007). We focus the geometry interpretations on the time interval with the greater amplitude variability in the $\delta^{18}\text{O}$ record, which start to increase (gradually) from $\sim 3,000$ ka onward. Between $\sim 3,000$ and ~ 350 ka, the skewness of climate cycles varies between -0.5 and 1.0 . Peak values of ~ 1.0 are reached between $\sim 2,500$ and $\sim 1,500$ ka, after which skewness rapidly decreases at $\sim 1,500$ ka to negative values of -0.5 that slowly increase to 0.0 during the MPT (Fig. 2). Glacial-interglacial cycle asymmetry varies between about -0.5 and 1.5 . The most prominent steps in asymmetry occur at $\sim 2,500$ and $\sim 1,000$ ka (the latter during the MPT), when values increase from about -0.5 to 0.5 and from approximately -0.2 to 1.2 , respectively. These results compare well to those previously obtained on (stacks of) benthic foraminiferal $\delta^{18}\text{O}$ records (Hagelberg et al., 1991; King, 1996; Lisiecki and Raymo, 2007). Kurtosis computations identifies leptokurtic (thin-peaked) cycle shapes that vary between 0.5 and 3.0 from $\sim 3,000$ ka onward. Unlike skewness and asymmetry, which are marked by multi-Myr decreasing and increasing trends, respectively, no long-term trend can be discerned in Pleistocene cycle kurtosis (Fig. 2).

3.2. Bispectra of Pliocene and Pleistocene climate cycles

Nonsinusoidal cycle shapes may indicate the potential for obtaining more information about nonlinear interactions among frequencies using higher order spectral analysis (Hagelberg et al., 1991; King, 1996). We present three examples of Pliocene and Pleistocene bispectra (i.e., their imaginary parts), based on the LR04 stack, which are characterized by clear triad interactions (Fig. 4, see also Section 2.2.2.). The first example is a bispectrum across the Late Pliocene to Early Pleistocene interval (i.e., the “40-kyr world”), which depicts negative and positive interactions, where both difference frequencies f_i and f_j , and sum frequency f_s are equal to 25 Myr^{-1} (i.e., 40 kyr), respectively (Fig. 4c). More specifically, negative interactions are concentrated at and between triads along the lines from $B_p^{Im}(40\uparrow, \infty\uparrow, 40\downarrow)$ to $B_p^{Im}(40\uparrow, 40\uparrow, 20\downarrow)$, and from $B_p^{Im}(40\uparrow, 40\uparrow, 20\downarrow)$ to $B_p^{Im}(36\uparrow, 40\uparrow, 19\downarrow)$, indicating a gain at the 40-kyr periodicity. Most notable is the strong negative interaction at $\sim B_p^{Im}(40\uparrow, 93\uparrow, 28\downarrow)$ to $\sim B_p^{Im}(40\uparrow, 83\uparrow, 27\downarrow)$. Other bispectral coordinates marked by negative interactions are those between $B_p^{Im}(95\uparrow, 95\uparrow, 48\downarrow)$ and $\sim B_p^{Im}(70\uparrow, 150\uparrow, 48\downarrow)$. Positive interactions are concentrated on the line between the bispectral coordinates $B_p^{Im}(80\downarrow, 80\downarrow, 40\uparrow)$ and $B_p^{Im}(40\downarrow, \infty\downarrow, 40\uparrow)$, which also indicate a gain at the 40 kyr periodicity by transfers from numerous other frequencies. Overall, the Late Pliocene to Early Pleistocene bispectrum depicts strong energy gains at the obliquity paced 40-kyr periodicity through many triad interactions, some of which are with other astronomically-paced climate cycles.

The second example is a bispectrum across the mid-Pleistocene transition (i.e., the “ ~ 80 -kyr world”), and shows several negative interactions where either p_1 or p_2 is equal to ~ 80 kyr (Fig. 4b). Particularly strong negative interactions are located at $\sim B_p^{Im}(80\uparrow, 200\uparrow, 57\downarrow)$, $B_p^{Im}(80\uparrow, 80\uparrow, 40\downarrow)$, and at $B_p^{Im}(40\uparrow, 80\uparrow, 27\downarrow)$. Also $B_p^{Im}(40\uparrow, 58\uparrow, 24\downarrow)$ is strongly

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negative. We document one major positive interaction at $B_p^{lm}(69, 95, 40)$, which connects to $B_p^{lm}(59\downarrow, 125\downarrow, 40\uparrow)$. Due to the more loosely distribution of triad interactions in this bispectrum, compared to the one of the Late Pliocene to Early Pleistocene, it is more difficult to interpret dominant energy gains and losses directly from the bispectrum of the MPT. However, we observe an overall gain of energy at the 70- to 80-kyr periodicities, and a dual role for the 40-kyr periodicity, which operates as both energy receiver and supplier simultaneously. We note that the ~80-kyr periodicity is not an obvious triad of the primary astronomical cycles (Table A2), apart from the junction at $B_p^{lm}(95\uparrow, 405\uparrow, 77\downarrow)$, where no interactions are documented in this particular bispectrum. The 80-kyr periodicity thus starts as a “lower” harmonic or “subharmonic” (i.e., where lower and sub refer to the frequency domain) of the 40-kyr periodicity, which in turn is a first harmonic of obliquity forcing, as well as a “combination tone” of precession cycles (Table A2) (Pestiaux et al., 1988). This subharmonic origin for the 80-kyr periodicity is supported by the strong negative interaction at $B_p^{lm}(80\uparrow, 80\uparrow, 40\downarrow)$.

The third example is a bispectrum across the Middle to Late Pleistocene (i.e., the “~110-kyr world”), which shows clear and punctuated triad interactions concentrated where p_2 equals 95 kyr (Fig. 4a). The main interactions are located at the following triads, or their connecting lines: at $\sim B_p^{lm}(95\uparrow, 405\uparrow, 77\downarrow)$, weakly at $B_p^{lm}(95\uparrow, 125\uparrow, 54\downarrow)$, from $B_p^{lm}(95\uparrow, 95\uparrow, 48\downarrow)$ via $B_p^{lm}(69\uparrow, 95\uparrow, 40\downarrow)$ to $B_p^{lm}(80\uparrow, 80\uparrow, 40\downarrow)$, broadly around $B_p^{lm}(40\uparrow, 95\uparrow, 28\downarrow)$, at $B_p^{lm}(40\uparrow, 51\uparrow, 22\downarrow)$, from $B_p^{lm}(31\uparrow, 95\uparrow, 24\downarrow)$ to $B_p^{lm}(29\uparrow, 95\uparrow, 22\downarrow)$, and weakly (but still visible) at the triple junction $B_p^{lm}(24\uparrow, 95\uparrow, 19\downarrow)$. Interactions at these triads (Table A2) are all negative, indicating that energy is mainly transferred to the 95 kyr (and 125 kyr) cycle from the three precession periodicities, the ~28-kyr periodicity and the 40-kyr obliquity periodicity. A very weak positive interaction is located at $B_p^{lm}(125\downarrow, 405\downarrow, 95\uparrow)$. Strong energy gains at the ~110-kyr eccentricity modulation of precession thus characterize the bispectrum of the Middle to Late Pleistocene interval.

3.3. Qualitative comparison of summer-half insolation to the climate-cryosphere proxy record

Figure 5 shows three time-evolutive spectral and bispectral analyses for the Pliocene-Pleistocene time interval. It compares the total power spectral density of summer-half insolation at 65°N (Fig. 5a) (Laskar et al., 2004) to the total energy transfers and total power spectral density of the LR04 benthic foraminiferal $\delta^{18}\text{O}$ stack (Fig. 5b and Fig. 5c, respectively). This insolation curve constitutes a classical shorthand for the complex external forcing of the ice ages (not shown, but see Fig. 1e and Fig. 2e for individual records of precession, obliquity and eccentricity) (Köppen and Wegener, 1924; Milankovitch, 1941). Figure 6 highlights five, equally spaced (500 kyr apart), time-slices of the three panels of Figure 5. Figure 7 depicts three, zonally averaged, frequency-slices, of the three panels of Figure 5, namely: “eccentricity”, “obliquity”, and “precession” zones (see Section 2.4.2.), as well as the total of all frequencies combined.

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3.3.1. Power spectral density of insolation

The time-evolutive power spectra of insolation, are marked by high spectral power at the precession and obliquity periodicities, and the near-absence of spectral power at the eccentricity periodicities (Fig. 5a, Fig. 6, and Fig. 7a) (Hays et al., 1976; Berger, 1977). Furthermore, it is characterized by variability in spectral power governed by the long term (eccentricity) modulations of obliquity and precession. These amplitude modulations have durations of ~180 kyr (not detected here, due to window length) and 1.2, 2.4 and 4.8 Myr for obliquity, and 405 kyr and 2.4 Myr for eccentricity modulated precession (Fig. 5a and Fig. 7a). Concurrent with the MPT we document the sharpest decrease in total spectral power of the entire Pliocene and Pleistocene, which is largely due to a decrease in precession power (Fig. 7a). This is indicative of a reduced perturbation of the Earth's system that results from a smaller difference in insolation forcing between the hemispheres.

3.3.2. Energy transfers among climate cycles

Figures 5b and S3 depict the conservative net energy transfers among (astronomically forced) climate cycles as present in the LR04 stack. The net energy transfers show how energy is redistributed over the power spectrum among astronomically-paced and nonastronomically-paced climate cycles. They reveal strong increases in both energy gains (in red) and losses (in blue) toward the present, and from ~1,000 ka onward several consecutive shifts in energy transfers from lower to higher periodicities occur (Fig. 5b). Energy gains are highly concentrated at the 40-kyr periodicity from ~2,500 ka onward and are marked by a modest 1.2 Myr beat in amplitude (Fig. 5b and Fig. 7b). A second location of strong energy gains is the ~80-kyr periodicity from ~1,000 ka onward (Fig. 6b), which at ~700 ka shifts (Fig. 5b) to a dominant gain for the 95-kyr component (Fig. 6a). Further energy gains are located at a ~200-kyr periodicity from ~750 ka onward, a 50-kyr to 60-kyr periodicity between ~1500 and 500 ka, and (very weak) at the 24-kyr periodicity from ~650 ka onward (Fig. 5b). Energy losses are also observed in the total integration over the bispectra (Fig. 5b, Fig. 6 and Fig. 7b). However, these losses are generally of smaller amplitude per cycle and are distributed across a greater number of periodicities, in comparison to the highly concentrated energy gains (Fig. 5b and Fig. 6). Despite this more dispersed pattern of energy losses, we highlight the more conspicuous decreases: the 40-kyr periodicity from ~550 ka onward, the 30- to 36-kyr periodicities between 1,500 and 1,000 ka, the 30- to 26-kyr periodicities between ~2,500 and ~2,000 ka, the ~28-kyr periodicity from ~800 ka onward, the 24-kyr periodicity between ~1,000 and ~800 ka, the 22-kyr periodicity from ~700 onward, the 19-kyr periodicity from 2,000 ka onward, and the 15-kyr periodicity intermittently from ~1,000 ka onward (Fig. 5b).

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3.3.3. Power spectral density of climate cycles

Time-evolutionary power spectral analysis of the LR04 record is characterized by high amplitude variability at the 40-kyr obliquity-paced cycle, from approximately 4,000 ka onward (Fig. 5c, Fig. 6, and Fig. 7c). In addition, distinct ~80 and ~110 kyr cycles of rapidly increasing amplitude are documented from ~1,200 ka onward, marking the MPT (Fig. 6b) and post-MPT (Fig. 6a) intervals, respectively (Lisiecki and Raymo, 2005, 2007). A relatively weak ~200-kyr cycle is present from ~750 ka onward (Fig. 5c). No strong precession response is present (Raymo and Nisancioglu, 2003; Raymo et al., 2006). The 40-kyr cycle in $\delta^{18}\text{O}$ partially mimics the 1.2 Myr amplitude modulation of the obliquity cycle, especially from ~3,000 ka onward (Fig. 5c) (Lourens and Hilgen, 1997; Lisiecki and Raymo, 2007). We draw attention to a subtle “arc” in spectral power, which starts with a strong 405-kyr cycle, at ~3,500 ka, that becomes gradually shorter in duration until it resonates with the 95-kyr eccentricity modulation of precession, at ~2,650 ka, and then culminates in several weak 70 to 80 kyr cycles at ~2,000 ka (Fig. 5c, see also Fig. 1b) (Lisiecki, 2010; Meyers and Hinnov, 2010; Viaggi, 2018).

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3.4. Zooming in on energy transfers

To better understand the different roles of specific (bandwidths of) climate cycles in nonlinear interactions, we determine conservative net energy transfers per bispectral zone (Fig. 8, Fig. S4, and Fig. S5). These zones reflect the “layers” that make up total energy exchanges as depicted in Figure 5b. We assess if triad interactions are conservative of energy for each of the bispectral zones and for the entire bispectrum (Fig. 9). Lastly, we recombine the bispectral zones again for frequency bandwidths involving precession-, obliquity- or eccentricity-paced climate cycles to highlight their influences in nonlinear interactions (Fig. 10).

3.4.1. Zonal energy transfers

In Zone 1, from 1,000 ka onward, we document a modest exchange of energy between the (eccentricity related) periodicities in the range from ~200 to 50 kyr (Fig. 8a). Notably, from ~500 ka onward, periodicities of ~80 to 50 kyr predominantly lose power, whereas those between ~200 and ~80 kyr mainly gain energy. Stronger exchanges are documented in Zone 2 (Fig. 8b). Especially the 40-kyr obliquity paced cycle gains energy between 3,500 and 900 ka in this zone, after which it loses energy from 900 ka onward. Also, the 50 to 60-kyr periodicity loses energy, especially from ~600 ka onward. Between ~1,100 and ~800 ka a single periodicity of ~87 kyr gains power. At ~750 ka this periodicity splits into an ~80-kyr periodicity and a 95-kyr eccentricity-paced cycle, which both gain energy. In general, interactions in Zone 2 change from (weakly) positive ($B^{im}(E\downarrow, E\downarrow, O\uparrow)$) to negative ($B^{im}(E\uparrow, E\uparrow, O\downarrow)$) during the MPT (~1,000 ka), indicative of a change from escalating energy to higher frequencies, to a cascading from higher frequencies. We note that energies exchanged through

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$B^{lm}(E, E, O)$ -interactions (Zone 2) are almost of equal amplitude as those of all other interactions combined. Zones 3 to 5 are mainly marked by increases in energy at the 40-kyr cycle, roughly from 2,500 ka onward (Fig. 8c to Fig. 8e). A smaller gain is documented in these zones (especially in Zone 3) at periodicities between ~70 and 50 kyr from ~1,300 ka onward, and in Zone 3 at the 95 kyr cycle from ~650 ka onward. Furthermore, we document decreases in energy at the ~30 to 27-kyr periodicities, the ~28 to 22-kyr periodicities, and the main precession periodicities of 24, 22 and 19-kyr, for Zones 3, 4, and 5, respectively. The switchover during the MPT, at about 750 ka, of energy loss at the 24-kyr cycle to a loss at the ~28-kyr periodicity is associated with $B^{lm}(O, E, P)$ -interactions (Zone 4) only. Zones 6 and 7 show weaker energy exchanges among the main astronomical periodicities. In Zone 6 the ~27 and 24-kyr cycles show a small gain between ~1,000 and 700 ka, and from ~700 ka onward, respectively. We only document very minimal direct fuelling of eccentricity-paced climate cycles by precession-paced climate cycles in this zone. Energy is lost in Zone 6 from 1,500 ka onward at the 19-kyr cycle. In Zone 7 relatively weak gains are present at the main obliquity and precession cycles from 1,500 ka onward. Energy losses of Zone 7 are located mainly at sub-precession periodicities of ~15 to <13 kyr (Fig. 8f and Fig. 8g). No energy exchanges of similar magnitude are documented in Zone 8 (Fig. 8h).

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Overall, the zonal integrations are marked by negative triad interactions, and thus reveal a cascade of energy from the (sub-) precession bandwidth, to the obliquity and (ultimately) eccentricity bandwidths through several successive interactions (Fig. 8). These analyses further show that many periodicities fulfil a dual role that often remains stable through time, namely: they serve simultaneously and persistently as energy provider and receiver. For example, the 24-kyr precession-paced cycle gains energy through $B^{lm}(P, E, P)$ - and $B^{lm}(P, O, P)$ -interactions, as documented in Zones 6 and 7, meanwhile losing energy through $B^{lm}(O, O, P)$ - and $B^{lm}(O, E, P)$ -interactions, as can be observed in Zones 4 and 5. As an exception to this persistent behaviour of many lower periodicities in transferring energy to higher periodicities, the role of the 40-kyr obliquity-paced cycle evolves, but only in $B^{lm}(E, E, O)$ -interactions (Zone 2). In Zones 3, 4, 5, and 7 the 40-kyr obliquity-paced climate cycle (predominantly) partakes in triad interactions as a difference frequency, and it continuously receives energy, mainly from precession. However, during the MPT (from ~900 ka onward) in Zone 2, when the 40-kyr periodicity partakes in interactions solely as a sum frequency (f_3) of two eccentricity components ($f_1 + f_2$), it starts providing energy to the eccentricity bandwidth. The net (i.e., total) result of these opposing roles for the 40-kyr obliquity paced cycle after the MPT is that energy losses overtake the gains (Fig. 5b and Fig. S3). However, the initial decrease of the integrated “obliquity” zone, between ~1,000 and 600 ka, is linked to the ~28-kyr periodicity, not the 40-kyr periodicity (Fig. 5b).

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3.4.2. Energy conservation

Computations of energy conservation, hereafter also referred to as “conservativity”, indicate that both the total, as well as the zonal, energy gains in triad interactions are commensurate with losses, given our assumed coupling coefficient. The

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dominance of negative interactions (i.e., positive values for $f_1 + f_2$, and negative values for f_3) is indicative of a transfer within the Earth's climate-cryosphere system of insolation energy to lower frequencies. Energy is not well conserved in triad interactions documented by Zone 8. This is probably the result of the low number of interactions located in Zone 8 (Fig. 4), and the small area of this zone (Fig. 3).

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3.4.3. Energy exchanges involving precession-, obliquity-, or eccentricity-paced climate cycles

To highlight the separate roles of the precession, obliquity and eccentricity bandwidths during nonlinear interactions within the climate-cryosphere system, we recombine (i.e., sum) the bispectral zones of Figure 8. To do so, we include those zones that contain either one, two or three frequency-components of a particular bandwidth (Fig. 3 and Fig. 10). The general pattern of an energy cascade is robust, but it becomes clearer that precession cycles do not contribute much to the fuelling of eccentricity-paced climate variability directly (Fig. 10a-c). The 40-kyr periodicity during the Middle and Late Pleistocene gains energy from precession dominated interactions, but loses energy in obliquity and eccentricity dominated interactions (Fig. 10a-c). A comparison of conservativities of the recombined zones that include at least one precession, obliquity, or eccentricity component, indicates that approximately similar amounts of energy are exchanged in (triad) interactions involving obliquity, as in those involving eccentricity (Fig. 10d).

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4. Suitability of bispectral applications for palaeoclimatology

4.1. Limitations

4.1.1. Age uncertainty and signal to noise ratio

20 By combining more than fifty individual benthic foraminiferal $\delta^{18}\text{O}$ records, the LR04 stack simultaneously decreases the age uncertainty and increases the ratio of signal-to-noise considerably in comparison to any record individually (Lisiecki and Raymo, 2005). Reduced age uncertainties and improved signal-to-noise ratios of stacked records may be obtained through the computation of averaged sedimentation rates in combination with statistical "binning" (Lisiecki and Raymo, 2005), a probabilistic approach using a hidden Markov model (Ahn et al., 2017), or, as is the case in this reanalysis of the (1-kyr resampled) LR04 stack, by statistical "smoothing" (Chaudhuri and Marron, 1999). For successful application of advanced bispectral analyses techniques, good age control and (very) high signal to noise ratios are needed, because the distributions of only small amounts of variance over time and frequency are considered (Hasselmann et al., 1963; King, 1996). For consecutive computational steps that use bispectra (i.e., the bispectrum and the integration over the bispectrum), increasingly

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better constrained and noise-freer data is required, because each step zooms in further on these small amounts of variance. The bispectra of climate cycles presented here show clear interactions (Fig. 4), and the integrations of the bispectrum reveal qualitatively consistent patterns of energy transfers for analysis with varying window lengths (Fig. 5b and Supp. Fig. 1). Therefore, we argue that the LR04 stack has sufficiently accurate age calibrations, and a high enough signal-to-noise ratio for the successful application of advanced bispectral analysis.

4.1.2. Resolvability in time and frequency domains

Similar to the power spectrum, the bispectrum is marked by a trade-off between resolutions in the time versus frequency domain (Fig. S3). Bispectral results become more significant, and can yield greater degrees of freedom, if a larger number of similarly shaped cycles, or waveforms, can be included for analysis (i.e., a lengthening of the window). For example, studies on natural nearshore waves or those on flume-generated waves (periodicities of seconds to minutes), use hour-long stable time series with high sampling resolutions and wave numbers (de Bakker et al., 2014). Bispectral analyses of such datasets are well resolved in the frequency domain. Furthermore, they permit the computation of robust confidence levels on the results, and the selection of statistical settings that yield high degrees of freedom (e.g., (Elgar and Guza, 1985; de Bakker et al., 2015)). However, for the Pliocene and Pleistocene record, similar statistical objectives are unattainable, because no two climate cycles are alike and because of the low number of ~110-kyr cycles in the Pleistocene (King, 1996; Lisiecki, 2010). As a result of this absence of “ergodicity” (i.e., a stable waveform through space and time), which characterises the unique Pliocene-Pleistocene climatic evolution as captured by the LR04 stack, relatively short window-lengths are preferred to obtain well-resolved results in the time domain. The Hann windowing function clarifies the interactions in both the bispectrum and the integration over the bispectrum. Similarly to previous bispectral studies on the palaeoclimate archive (Hagelberg et al., 1991; King, 1996), we cannot define confidence levels for the bispectral analysis of the LR04 stack, because the number of cycles per window is too small. However, the bispectral couplings among climate cycles documented here are robust for varying bispectral settings (Fig. 5, Fig. 8, and Supp. Fig. S3 to Supp. Fig. S5).

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4.1.3. Quantifying cycle geometries of nonergodic records

In contrast to the relatively more robust bispectral results (see previous section), absolute geometry values are more strongly affected by the nonergodic nature of the Pliocene-Pleistocene record. Choices of (i) window length, (ii) method of detrending the time series, and (iii) the application of a windowing function or not, can have profound effects on the robustness of the results, (van Peer, 2018). These, somewhat arbitrary choices affect both ways of quantifying geometries similarly (i.e., using the central moments or higher order spectra (Fig. 2, Supp. Fig. 2, and Supp. Fig. S3) (van Peer, 2018).

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The sensitivity of geometry computations to data processing techniques was not previously appreciated in full, and may have resulted in over-optimistic confidence levels for cycle geometry values computed on an Oligocene and early Miocene climate record (Liebrand et al., 2017). We note that the uncertainty in skewness and asymmetry becomes larger for cycles with values near the mean ($= 0$). The relatively high values of about -1 or $+1$, similar to those computed on the LR04 stack of the Middle and Late Pleistocene, are much less sensitive to data processing methods, and have been independently computed on at least three occasions (Fig. 2) (Hagelberg et al., 1991; King, 1996; Lisiecki and Raymo, 2007). Remarkably, kurtosis values between $\sim 3,000$ and ~ 350 ka even change sign between computations that do apply a Hann window (resulting in leptokurtic cycles; Fig. 2) and those that do not (yielding platykurtic cycles that reach values of -1.0 during the Pleistocene; not shown). By comparison, skewness and asymmetry values appear much more robust to various methods of data processing than kurtosis. This greater sensitivity of kurtosis to windowing is not surprising considering that fourth moment statistics (and the trispectrum) zoom in on even smaller amounts of variance in comparison to the third central moments and the bispectrum (Collis et al., 1998).

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4.2. Coupling coefficient

For ocean waves, the relationship between the integration of the bispectrum and absolute nonlinear energy transfers is determined by a coupling coefficient, which is based on the second-order Boussinesq theory (Herbers and Burton, 1997; Herbers et al., 2000). This Boussinesq approximation describes how energy is transferred from the primary ocean waves to higher and lower frequency components by near-resonant nonlinear triad interactions. A coupling coefficient is needed (i) to ensure the conservation of energy within a single triad interaction (i.e., all energy lost at a certain frequency is compensated by an energy gain at the other two frequencies, and vice versa), and (ii) to obtain absolute energy transfers, which are directly comparable with the changes in the power spectrum when the waves propagate toward the coast (Herbers et al., 2000; de Bakker et al., 2014). Here, we apply a similar approach to palaeoclimate time series, and demonstrate that energy conservation can be ensured assuming a simple coupling coefficient that multiplies the energy transfers at each specific sum frequency after integration, with that specific frequency (f_s) (see Section 2.4.1). Comparison of energy gains and losses, both the integration over the entire imaginary part of the bispectrum, as well as for each bispectral zone separately, shows that sum (S_{n^+} , see Eq. 7) and difference (S_{n^-} , see Eq. 8) interactions balance each other, which is indicative of energy conservation, and qualitatively trustworthy signals in energy transfers (Fig. 9). However, the second step toward absolute energy transfers is not developed here (see qualitative comparison of spectral and bispectral results in Fig. 6), because a more fundamental understanding is needed of the many climatologic and oceanographic, biologic, sedimentologic and lithologic processes that affect the (globally integrated) benthic foraminiferal $\delta^{18}\text{O}$ record.

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5. Discussion

5.1. Revisiting the problem of the ice ages

The common denominator of (i) the 40-kyr problem of the Pliocene and Early Pleistocene, (ii) the ~110-kyr problem of the Middle and Late Pleistocene, and (iii) the (nonlinear) origins of many climate cycles without primary astronomical pacemaker (e.g., the ~28-kyr and 80-kyr periodicities), is the mismatch in the distribution of power spectral density between benthic foraminiferal $\delta^{18}\text{O}$ and records and insolation (Raymo and Nisancioglu, 2003; Lisiecki, 2010; Lourens et al., 2010). Furthermore, (iv) during the mid-Pleistocene transition, this discrepancy increases significantly (Lisiecki and Raymo, 2005; Clark et al., 2006; Huybers, 2007), by the shifting of spectral power of climate records from the 40-kyr, via the ~56-kyr and the ~80-kyr, to the ~110-kyr periodicity (Rial and Anaclerio, 2000; Lourens et al., 2010; Viaggi, 2018). In light of the bispectral results obtained in this study, we revisit the problem of the ice ages (Agassiz, 1840; Milankovitch, 1941; Hays et al., 1976), in search for the most parsimonious explanation(s) for the origins and evolution of climate cycles with and without astronomical periodicities. The answer that we distil from the bispectra of Pliocene and Pleistocene climate cycles, in greater clarity than before (Hagelberg et al., 1991; Rial and Anaclerio, 2000; Lourens et al., 2010), is that these problems (i.e., *i* to *iv*) are intrinsically linked by the (many) increasingly nonlinear responses of Earth's climate-cryosphere system to insolation forcing (Pisias et al., 1990; Yiou et al., 1994). Namely, the bispectra confirm (Hays et al., 1976; Hagelberg et al., 1991), that energy is transferred from the lower to the higher periodicities from at least 2,500 ka onward. More importantly, they show (i) how this happens, i.e., through which nonlinear triad interactions (Fig. 4), and (ii) how their involvement in transferring energy evolves through time (Fig. 5, Fig. 8, and Fig. 10), especially during MPT.

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20 5.2. The origins of climate cycles: toward a better mechanistic understanding

To advance the understanding of the origins of Pliocene and Pleistocene climate cycles, we explore the potential climatic and cryospheric processes that cause the documented nonlinear triad interactions and energy transfers. Similar to the difficulties in linking spectral peaks of benthic foraminiferal $\delta^{18}\text{O}$ records, which describe the integrated effect of deep water temperatures and cryosphere (Hays et al., 1976; Elderfield et al., 2012; Rohling et al., 2014; Crowhurst et al., 2018), to particular mechanisms, it is not straightforward to identify (singular) causes for bispectral peaks (Fig. 4). Should we attempt to link every individual triad interaction to a specific climatic or cryospheric process? Or, alternatively, should we search for one, or a perhaps a couple of, mechanism(s) that can explain multiple, or even all, nonlinear couplings at once? A comparison to nearshore ocean waves, and the interactions that characterize them, as they break in the surf zone, may be informative (Herbers et al., 2000; de Bakker et al., 2014, 2016). The strength and number of nonlinear interactions of breaking waves is strongly determined by the shallowing of the sea floor and the ways in which the wave base interacts with

the beach (de Bakker et al., 2015; de Bakker et al., 2016). Thus, a single mechanism, i.e., the interaction between a wave and the beach, causes multiple nonlinear triad interactions.

In contrast to the single cause for multiple nonlinear triad interactions among ocean waves, we tentatively link two mechanisms to energy transfers among climate cycles, because we observe two key features in the total and zonal integrations over the bispectrum: (i) a persistent fuelling of obliquity-paced climate cycles by precession-paced climate cycles, which we link to astronomical forcing of atmospheric and oceanic circulation (“the climatic precession motor”) (Section 5.2.1), and (ii) a change of obliquity-paced climate cycles from net energy sink into net energy source, which we link to a resonance of cycles of crustal sinking and rebounds with the eccentricity modulation of precession, after NH land-ice mass loading passed a critical threshold during the MPT (“the cryospheric obliquity motor”) (Section 5.2.2.).

5.2.1. The climatic precession motor

First, we speculate that the fuelling of the ice ages by (mainly) the precession periodicities is largely linked to the low-to-mid latitude monsoons (Rutherford and D’Hondt, 2000; Berger et al., 2006; Bosmans et al., 2015a). The (sub-) tropical zones, where most insolation is received, constitute Earth’s heat engine, and their monsoons function as both a source of moisture and as a meridional teleconnection to build up large polar (land-) ice volumes. The interactions between NH land-ice volumes and the African and Asian monsoons are well documented for large parts of the Pliocene and Pleistocene (deMenocal, 1995; Sun et al., 2006; Cheng et al., 2016). The monsoonal (i.e., atmospheric) redistribution of heat and moisture was most probably aided by orbital scale variability in the strength of Atlantic (i.e., oceanic) meridional overturning circulation (AMOC) (Lisiecki et al., 2008; Ziegler et al., 2010; Elderfield et al., 2012). Based on the bispectral results, we infer that during the Pliocene and Early Pleistocene this predominantly monsoonally-driven precession motor fuels the 40-kyr obliquity-paced ice age cycles, aided by more linear climatic-cryospheric responses resulting from variability in insolation at this periodicity, either cross-equatorially integrated (Huybers and Tziperman, 2008; Bosmans et al., 2015b), or locally at higher latitudes. The precession motor is especially strongly expressed in $B^{lm}(O, O, P)$ -interactions (Zone 5, Fig. 8e). However, uncertainty remains over the exact moisture sources for the Middle-to-Late Pleistocene ice volume fluctuations (e.g., (Werner et al., 2001; de Boer et al., 2012)).

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5.2.2. The cryospheric obliquity motor

Second, we hypothesize that the role reversal of obliquity paced climate cycles (from net energy receiver to net energy provider) across the MPT, in both $B^{lm}(E, E, O)$ -interactions (Fig. 4) and total net energy transfers (Fig. 5b and Fig 8b), is

linked to a resonance of (auto-) cycles of crustal sinking and a delayed rebound with ~110-kyr long (allo-) cycles of eccentricity modulated precession. In this hypothesis, crustal sinking is caused by the passing of a land-ice mass-loading threshold, and the delayed response to both eccentricity modulated precession and obliquity, is caused by the inertia of bulky, continental-sized ice sheets, which causes a nonlinear positive albedo-feedback, and hence, a hysteresis-pathway for glacial-interglacial cycles (Calov and Ganopolski, 2005; Abe-Ouchi et al., 2013). This obliquity motor evolves during and after the MPT, from a lower harmonic of two to three obliquity-paced cycles (Fig. 6b) (Ridgwell et al., 1999; Huybers and Wunsch, 2005), into a resonance (i.e., phase coupling) with the ~110-kyr eccentricity paced cycles, mainly the ~95-kyr component (Fig. 6a) (Hagelberg et al., 1991; Tziperman et al., 2006; Abe-Ouchi et al., 2013). We speculate that the resonance of crustal sinking and a delayed isostatic rebound of the lithosphere-asthenosphere, also shifted the sensitivity of the climate-cryosphere system to insolation forcing at higher latitudes, where the relative influence of obliquity compared to precession is greater than at lower latitudes. We conjecture that this shift in sensitivity to obliquity-forcing was especially enhanced during glacial terminations (Huybers and Wunsch, 2005; Huybers, 2011; Konijnendijk et al., 2015).

5.3. Climatic and tectonic boundary conditions

If our interpretations of the bispectra are indeed supportive of an astronomically-paced monsoonal/AMOC “push” of (heat and) moisture, followed by a resonant lithospheric-asthenospheric “pull” through a cooling of the NH polar region after an initial phase of ice growth, for sufficiently large glacial land-ice masses to form, then they would highlight the importance of delayed isostatic rebound and the shape (i.e., extent, volume, and especially height) of the NH ice sheets in determining the pattern and geometry of climate-cryosphere variability observed for the Middle and Late Pleistocene (Roe, 2006; Bintanja and van de Wal, 2008; Abe-Ouchi et al., 2013). However, these mechanisms do not explain why large-scale NH glaciations only developed during the Late Pliocene and Pleistocene, and not during earlier time-periods with comparable astronomical configurations. The Pliocene-Pleistocene climatic-cryospheric evolution can therefore not be seen outside a context of long-term, multi-Myr changes in climatic and tectonic boundary conditions, not captured by the bispectral analysis presented here. For example, the “arc” in spectral power (~3,500–2,000 ka) (Fig. 5c), which is largely absent in bispectral power (Fig. 5b), and which precedes the strong glaciations of the MPT and Late Pleistocene, may be indicative of these gradually changing boundary conditions. We speculate that this arc reflects a change in the response time of the cryosphere, potentially through glacial landscaping by earlier (smaller) glacial cycles (Clark and Pollard, 1998; Tabor and Poulsen, 2016) or through changing tectonic/geographic boundary conditions and associated oceanic circulation patterns (Haug and Tiedemann, 1998; Kender et al., 2018). These developments could have preconditioned the Earth’s system for the full-blown bipolar glacial cycles of the Middle and Late Pleistocene. These cycles were amplified by variability in radiative forcing on astronomical time scales and set against a background of an all-Cenozoic-time low $P_{\text{CO}_2^{\text{atm}}}$ (Shackleton, 2000; Beerling and Royer, 2011; Martinez-Boti et al., 2015; Chalk et al., 2017).

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6. Outlook

We present considerable advances in the understanding of the origins of many Pliocene and Pleistocene climate cycles by applying advanced bispectral analysis to the palaeoclimate archive. However, several lines of research remain to be explored

5 further. Here we list those that we think need more attention in the future:

- For nearshore ocean waves, energy transfers are (almost) fully described by the imaginary part of the bispectrum (Norheim et al., 1998), because (i) nearshore ocean waves are dominated by asymmetric wave forms, and (ii) the imaginary part of bispectra in general describe the majority of energy that is transferred. Supporting evidence for the importance of the imaginary part in describing most all energy exchanges, comes from spectral analysis of synthetically skewed and asymmetric time series (not shown). These spectra indicate that the higher harmonic peaks of skewed cycles are about an order of magnitude smaller than those of asymmetric cycles, especially for the higher harmonic peaks ($3f$, $4f$, ...) than the first higher harmonic ($2f$). Despite these profound implications of bispectral theory, and the high asymmetry values of Middle and Late Pleistocene climate cycles, a more comprehensive understanding of the real part of the bispectrum may still be important for palaeoclimate studies, because it may enable a(n) (even) more complete description of nonlinear energy exchanges (Supp. Fig. 1 and Supp. Fig. 2, e.g. see the ~28-kyr periodicity).

- Although we obtain conservative energy exchanges with our simple, assumed coupling coefficient, other studies are needed to scale these exchanges to the power spectrum (Fig. 6). Coupled climate-cryosphere models or energy balance models may be used to derive or approximate coupling coefficients for palaeoclimates that are based on “first principles” (i.e., a physicochemical understanding of the Earth system). Alternative coupling coefficients may also change how the bispectrum redistributes energy from climate cycles with low to those with high periodicities. However, we note that energy conservation should be respected for every triad interaction in the bispectrum.

- The proposed relationships between nonlinear triad interactions as documented in bispectra of climate cycles and specific processes of the climate-cryosphere system (i.e., monsoons, isostatic rebound), remain speculative. Therefore, we cannot currently separate these mechanisms from, for example, deep ocean carbon storage (e.g., (Willeit et al., 2015; Ganopolski et al., 2016; Farmer et al., 2019)), AMOC shutdown and subsequent overshoot (Liu et al., 2009; Wu et al., 2011), and/or changing sediment cover (Clark and Pollard, 1998; Ganopolski and Caloy, 2011; Willeit et al., 2019) and the amplifying effects these processes may have had on the strength of the 40 and ~110 kyr cycles. More data (analysis) and/or (isotope-enabled) modelling studies are needed to support our preferred hypotheses. For example, global climate models could potentially help identify which mechanisms correspond to the specific frequency interactions we observe in the LR04 stack.

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• There is a limited amount of information that can be extracted from studying a single climate record. Bispectral analysis of other (Pliocene-Pleistocene) records may shed more light on the dynamics of Earth’s palaeoclimate system. Not only in elucidating the couplings among astronomical frequencies, but also across the spectral continuum, and in relation to $1/f$ “red” noise and/or the power-law scaling of energy exchanges between frequencies (see Section 2.4.1. and Section 4.2) (Hasselmann, 1976; Bak et al., 1987, 1988; Bak, 1996; Pelletier, 1998; Huybers and Curry, 2006). Examples of interactions that could be studied in this way are those between astronomical and sub-astronomical (e.g., millennial) climate cycles (Hagelberg et al., 1994; von Dobebeck and Schmieder, 1999; Wara et al., 2000; Da Silva et al., 2018), and among cycles with centennial, decadal and annual durations (King Hagelberg and Cole, 1995). We emphasize that other applications of bispectral analysis techniques to the palaeoclimate archive should be limited to records with good age control and high signal-to-noise ratios (see Section 4.1.1).

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7. Conclusions

In this interdisciplinary study, we present a new, higher-order spectral analysis and interpretation of Pliocene and Pleistocene climate cycles as present in the well-established LR04 globally-averaged benthic-foraminiferal $\delta^{18}\text{O}$ stack. We use the nonsinusoidal properties of these climate cycles as a proxy for the strength of nonlinear couplings and interactions among different components of the climate system that operate on distinctly separate time scales. These advanced bispectral analysis techniques were initially developed by the nearshore ocean wave community, and are adapted here to make them more applicable for paleoclimatic research purposes. They show, more thoroughly than before, the complexities of the Pliocene-Pleistocene climatic-cryospheric evolution, namely that (i) a nonlinear energy exchanges resulted in the distinct asymmetric cycle geometries of the Middle and Late Pleistocene, (ii) energy is transferred across the power spectrum, among astronomical and nonastronomical climate cycles, (iii) precession-paced climate cycles fuelled both the 40-kyr and ~110-kyr ice age cycles of the Pliocene and Pleistocene, (iv) the role of obliquity-paced climate cycles changes from being a net sink into a net source of energy during the MPT (i.e., in $B^{lm}(E, E, O)$ -interactions), and (v) after the MPT these obliquity-paced climate cycles helped fuel the ~110-kyr eccentricity-paced climate cycles of the Middle and Late Pleistocene.

We postulate two distinct fuelling mechanisms for the ice ages to explain the energy cascade of negative interactions from climate cycles with high frequencies to those with low frequencies: (i) a continuous Pliocene-Pleistocene climatic “precession motor” and (ii) a Middle-to-Late Pleistocene cryospheric “obliquity motor”. We interpret the dominant precession fuelling of obliquity paced climate cycles (i.e., the precession motor) as evidence of a low latitude, (sub-) tropically-driven climate system, in which the monsoons (and oceanic meridional overturning circulation) transport heat and

moisture to higher latitudes. We argue that the role-reversal of obliquity-paced climate cycles during the MPT is indicative of the passing of a land-ice mass-loading threshold, after which (auto-) cycles of crustal sinking and (delayed) rebound start to resonate, i.e., become frequency- and phase-coupled, with the ~ 110 -kyr [long](#) (allo-) cycles of eccentricity-modulated precession. This resonance may have caused a shift of sensitivity to (obliquity-paced) insolation changes at increasingly higher latitudes, especially during terminations.

Despite the fact that the evidence for energy transfers agrees well with previously published mechanisms, their unprecedentedly detailed description here is an important step forward toward a more comprehensive solution of the problem of the ice ages, because they largely reconcile the mismatch in power spectral density between records that capture the combined effects of deep-sea temperatures and land-ice volumes and those of insolation. Furthermore, we can now state with greater certainty than before, that the geometry (i.e., asymmetry) of the Pliocene and Pleistocene ice ages is in very close agreement with [Neo-Milankovitchism: i.e.](#), the classical Milankovitch Theory of astronomical climate forcing and its [Neo-Milankovitchistic](#) derivatives/superlatives that give greater weight to Earth's internal nonlinear responses. If the bispectral energy transfers documented here indeed track the entire redistribution of power over the spectrum, then the 40-kyr problem of the Pliocene and Early Pleistocene, and the ~ 110 -kyr problem of the Middle and Late Pleistocene are reduced to finding appropriate coupling coefficients that describe the relevant climatic and cryospheric mechanisms—a promising outlook.

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Figure 1. Sinusoidal properties of Pliocene and Pleistocene climate cycles (after (Lisiecki and Raymo, 2005; Lisiecki, 2010)). **(a)** Resampled LR04 benthic foraminiferal $\delta^{18}\text{O}$ stack. **(b)** Wavelet analysis of the LR04 stack after detrending. [Black contours represent 95% significance.](#) **(c)** Absolute amplitude modulations and filters of the ~ 110 -kyr, 40 kyr, and ~ 21 -kyr [periodicities.](#) **(d)** Phase evolution of the ~ 110 -kyr [periodicity](#) with respect to eccentricity (Laskar et al., 2011a; Laskar et al., 2011b), computed over a 750 kyr sliding window. Blackman-Tukey phase estimates are only shown when coherent (Paillard et al., 1996). We focus the phase interpretation on the time interval with high amplitude ~ 110 -kyr [long](#) cycles (transparent area). Phase relationship of $\delta^{18}\text{O}$ to obliquity is not shown because it is assumed during the astronomical tuning

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process. Phase to precession is not shown because globally averaged $\delta^{18}\text{O}$ records have no single response to either NH or SH precession forcing. **(e)** Obliquity, eccentricity and precession solutions. The 1.2-Myr obliquity, and 2.4-Myr and 405-kyr eccentricity cycles numbers are given. Black and white bars correspond to normal and reversed magnetochrons of the 2012 geologic time scale (Hilgen et al., 2012).

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Figure 2. Nonsinusoidal properties of Pliocene and Pleistocene climate cycles (after (Lisiecki and Raymo, 2007)). **(a)** As in Figure 1. **(b)** Evolution of glacial-interglacial cycle skewness, **(c)** asymmetry, and **(d)** kurtosis (i.e., normalised third and fourth central moments). Computed over a 668 kyr sliding window, using standard (teal and blue lines) and bispectral (pink lines) methods. Cycle kurtosis is computed with the standard method only, because no trispectra were computed. Inset boxes show corresponding cycle shapes. The arrow shows the direction of time, g = glacial, ig = interglacial. We focus the geometry interpretation on the time interval with the greater amplitude variability in the record (transparent area). **(e)** As in Figure 1.

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Figure 3. Bispectral zonation scheme. Bispectra of climate cycles can be subdivided into 15 zones that reflect unique combinations of three frequency bandwidths. Orange, purple, and blue lines represent the main astronomical frequencies of the eccentricity, obliquity and precession cycles, respectively (this colour coding is consistent throughout the paper). The grey shaded areas represent suborbital periodicities. Difference frequencies f_1 and f_2 are plotted along the horizontal and vertical axes, respectively. Sum frequencies f_3 (i.e., $f_1 + f_2$) are depicted as diagonal lines in the bispectrum. See also Table A1 for a list of bispectral zones, and Table A2 for a list of triad types among the main astronomical frequencies in the bispectrum.

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25 **Figure 4.** Bispectra of Pliocene and Pleistocene climate cycles. **(a)** Imaginary part of a bispectrum of the Middle and Late Pleistocene “~110-kyr world”. Age range = 668 ka to 0 ka. Resampling resolution = 1 kyr⁻¹. Window length = 668 kyr. Block length = 668 kyr. Number of blocks = 1. Number of merged frequencies = 1. Frequency resolution = 1.50 Myr⁻¹. Degrees of freedom = 2. Min value = -598 %³ kyr⁻². Max value = 51 %³ kyr⁻². **(b)** Imaginary part of a bispectrum of the mid-Pleistocene transition “~80-kyr world”. Age range = 1,318 ka to 650 ka. Resampling resolution = 1 kyr⁻¹. Window length = 668 kyr. Block length = 668 kyr. Number of blocks = 1. Number of merged frequencies = 1. Frequency resolution = 1.50 Myr⁻¹. Degrees of freedom = 2. Min value = -57 %³ kyr⁻². Max value = 73 %³ kyr⁻². **(c)** Imaginary part of a bispectrum of the Pliocene and Early Pleistocene “40-kyr world”. Age range = 3,000 ka to 1,000 ka. Resampling resolution = 1 kyr⁻¹. Window length = 2,000 kyr. Block length = 2,000 kyr. Number of blocks = 1. Number of merged frequencies = 3. Frequency

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resolution = 1.50 Myr⁻¹. Degrees of freedom = 6. Min value = -21 ‰ kyr⁻². Max value = 23 ‰ kyr⁻². Note the different scaling of the z-axes between panel (a), (b), and (c). These bispectra are computed on the LR04 stack (see Methods) (Lisiecki and Raymo, 2005).

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Figure 5. (a) Input → (b) “black box” climate → (c) output. (a) Power spectral density of mean summer insolation (June 21 to September 21) at 65°N (Paillard et al., 1996; Laskar et al., 2004). (b) Conservative net energy transfers (i.e., after correction using the coupling coefficient) during the Pliocene and Pleistocene, computed by integrating over the entire imaginary part of the bispectrum (see Methods). Input data is the resampled LR04 stack (Lisiecki and Raymo, 2005). (c) Power spectral density of the LR04 stack (Lisiecki and Raymo, 2005). For all three panels, the window length is 668 data points (668 kyr), and the step-size between partially overlapping windows is 50 data points (50 kyr). We used no blocks and did not merge frequencies. These settings yield a frequency resolution of 1.50 Myr⁻¹, and 2 degrees of freedom.

15 **Figure 6.** Qualitative linkage of power spectra of insolation to those of climate, via energy exchanges among climate cycles. (a) 334 ka. (b) 834 ka. (c) 1334 ka. (d) 1834 ka. (e) 2334 ka. Computational settings as in Figure 5. Note that the scaling between the spectra and energy exchanges is arbitrary. Energy exchanges of the LR04 stack can be scaled to the power spectrum of the LR04 by using multiplication factors of 430, 250, 360, 230, and 250 for panels (a) to (e), respectively.

20 **Figure 7.** Integration of the spectra and bispectra per astronomical bandwidth (top right). (a) Totally and zonally integrated spectra of mean summer insolation (June 21 to September 21) at 65°N. (b) Totally and zonally integrated bispectra of the LR04 stack. (c) Totally and zonally integrated spectra of the LR04 stack. Computational settings as in Figure 5.

25 **Figure 8.** Disentangling “black box” climate. Conservative net energy transfers during the Pliocene and Pleistocene over specific zones in the imaginary part of the bispectrum (see Methods). Computational settings as in Figure 5b. (a) Zone 1. (b) Zone 2. (c) Zone 3. (d) Zone 4. (e) Zone 5. (f) Zone 6. (g) Zone 7. (h) Zone 8. See also Figure 3 and Table A1.

30 **Figure 9.** Conservation of energy in triad interactions located in the imaginary part of Pliocene and Pleistocene bispectra. Conservativity of (a) the entire bispectrum (Fig. 5b), and (b) Zone 1, (c) Zone 2, (d) Zone 3, (e) Zone 4, (f) Zone 5, (g) Zone

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6, **(h)** Zone 7, and **(i)** Zone 8, of the bispectrum (Fig. 8). Energy gains or losses of f_1 and f_2 (dashed lines) are computed using Eq. 8, those of f_3 (single coloured lines) following Eq. 7, and their differences (black lines) by Eq. 6.

5 **Figure 10.** Summed zonal integrations of the imaginary part of Pliocene and Pleistocene bispectra. Net energy transfers are computed by **(a)** summing Zones 1, 2, 3, 4, and 6 for triad interactions corresponding to the “eccentricity” bandwidth, **(b)** summing Zones 2, 3, 4, 5, and 7 for triad interactions corresponding to the “obliquity” bandwidth, and **(c)** summing Zones 4, 5, 6, 7, and 8 for triad interactions corresponding to the “precession” bandwidth. Panel **(d)** shows the corresponding summed zonal conservation of energy, with eccentricity in orange, obliquity in purple, and precession in blue.

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Appendices

10 **Table A1.** Bispectral zones defined in this study. These zones correspond to Fig. 3. The boundaries between the climate cycle zones are (arbitrarily) defined at periodicities of ~56.2, ~28.1 and 12.5 kyr (i.e., frequencies of 17.8, 35.6 and 80.0 Myr⁻¹).

20 **Table A2.** Bispectral triads among climate cycles with astronomical frequencies. One, two, or three astronomical frequencies can partake in triad interactions, and we refer to these coordinates in the bispectrum as single, double or triple junctions, respectively. These triads reflect positions in the frequency-frequency domain where nonlinear energy transfers among astronomically-paced climate cycles are likely to occur. They correspond to the crossing points of the coloured lines in Fig. 3. In case of a “single junction”, an astronomically-paced cycle exchanges energy with itself, or with a frequency that is only slightly offset. Single junctions where $f_1 = f_2$ constitute the first higher harmonic interactions. Double or triple junctions are “combination tones” of two or three different astronomically-paced cycles (Pestiaux et al., 1988; Hagelberg et al., 1991; King, 1996; Rial and Anaclerio, 2000).

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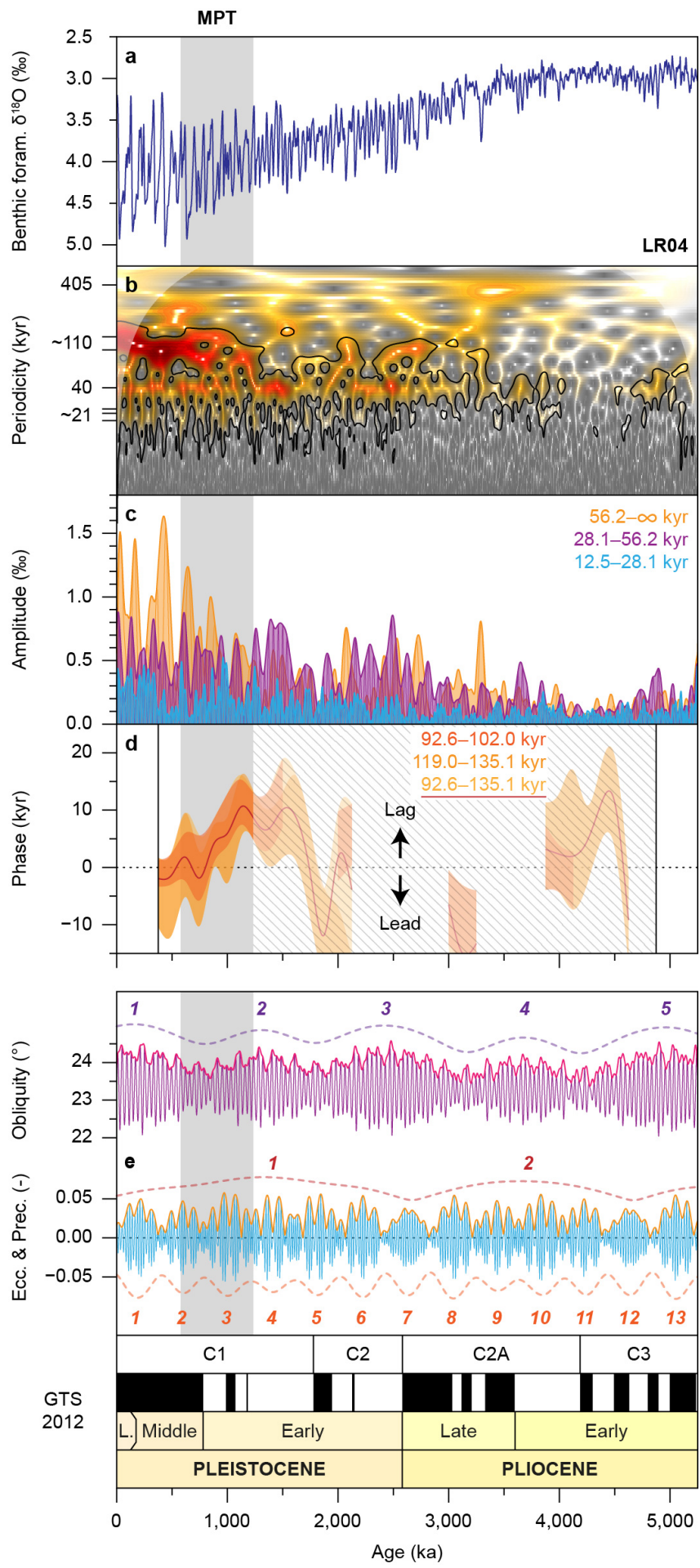
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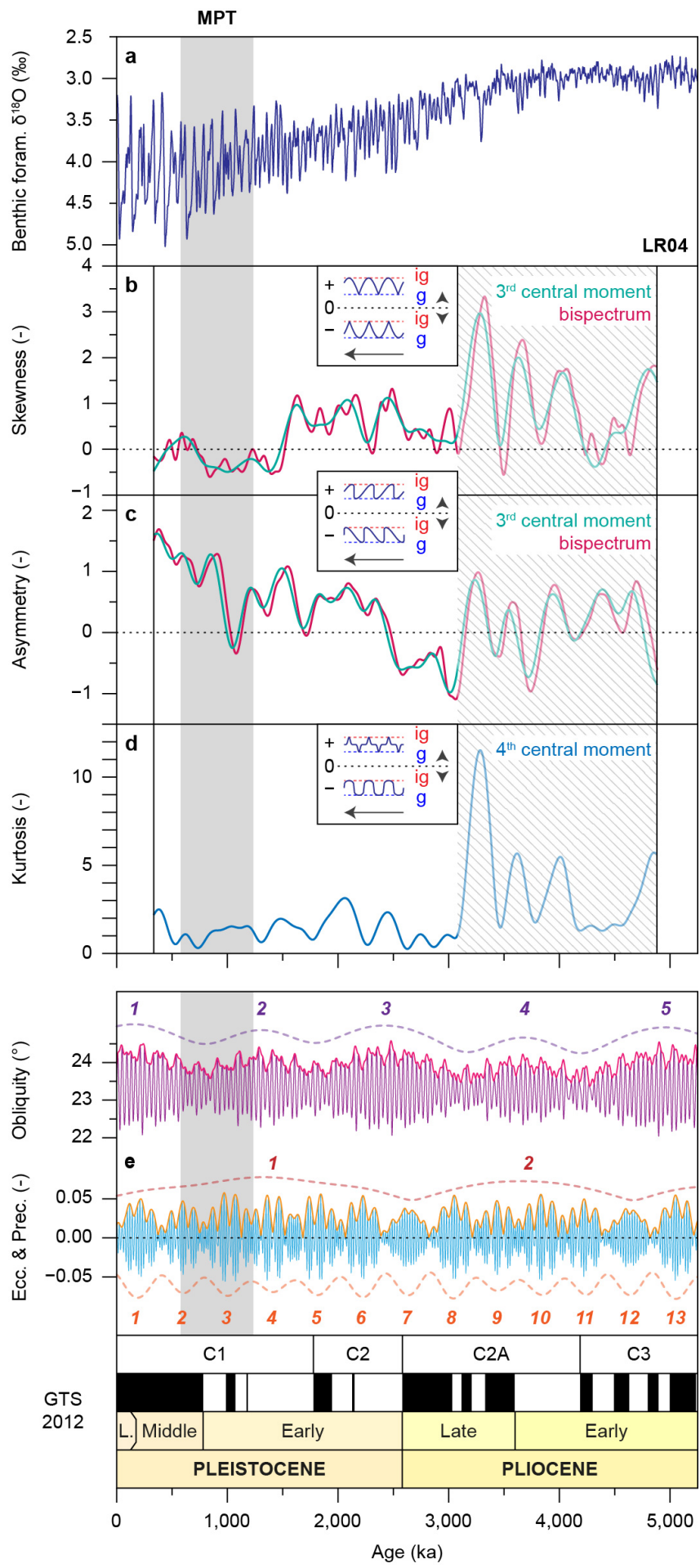
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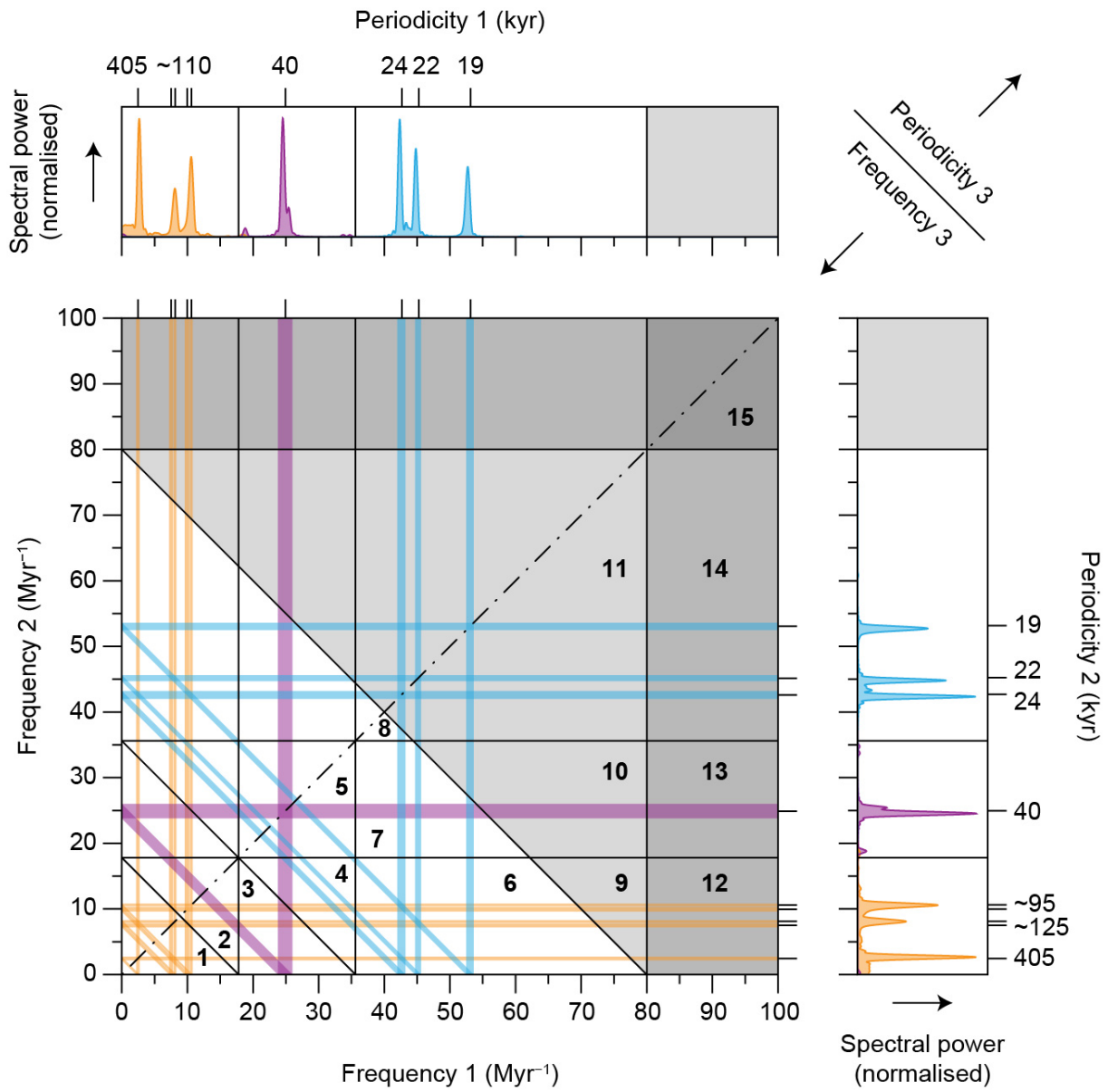
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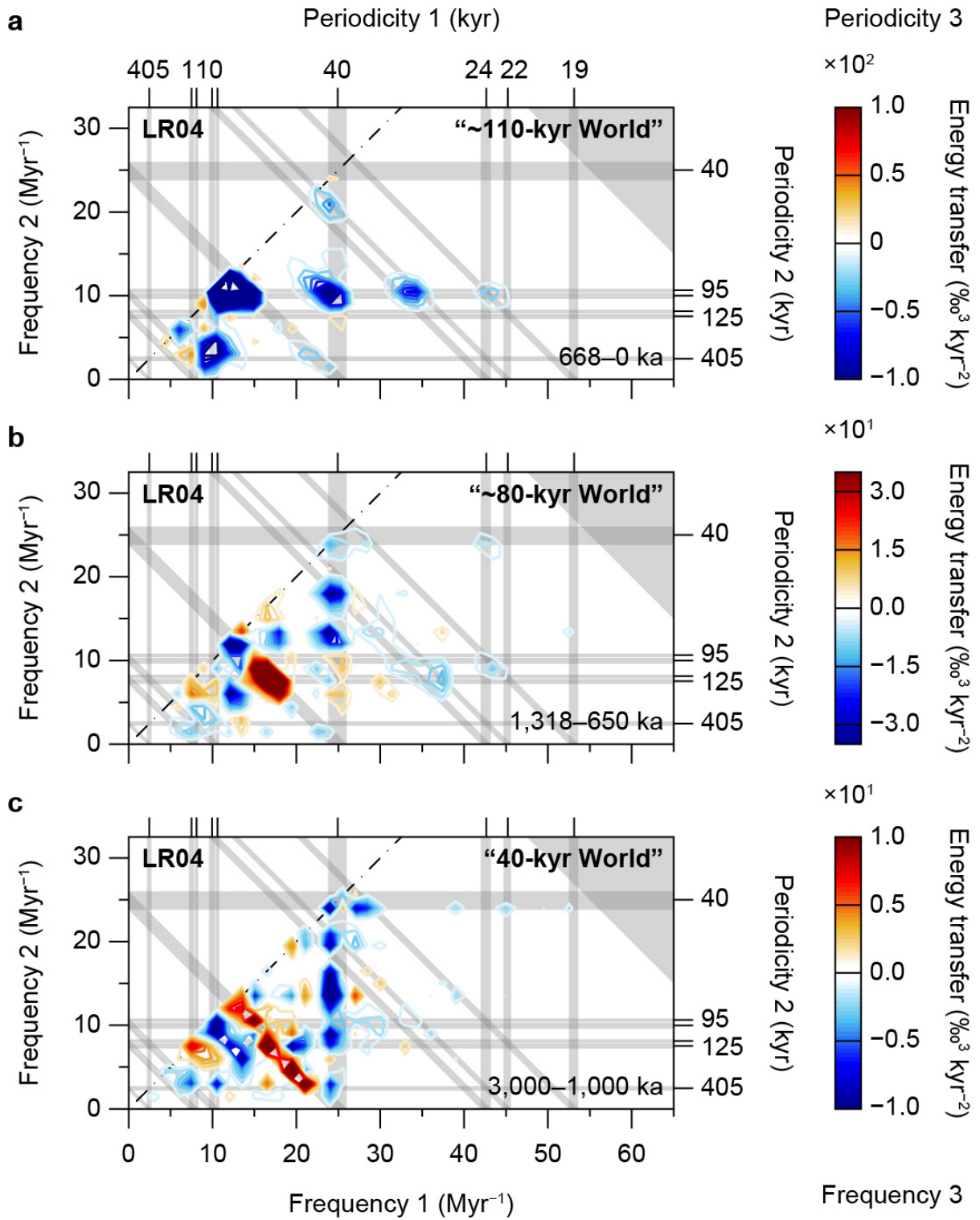
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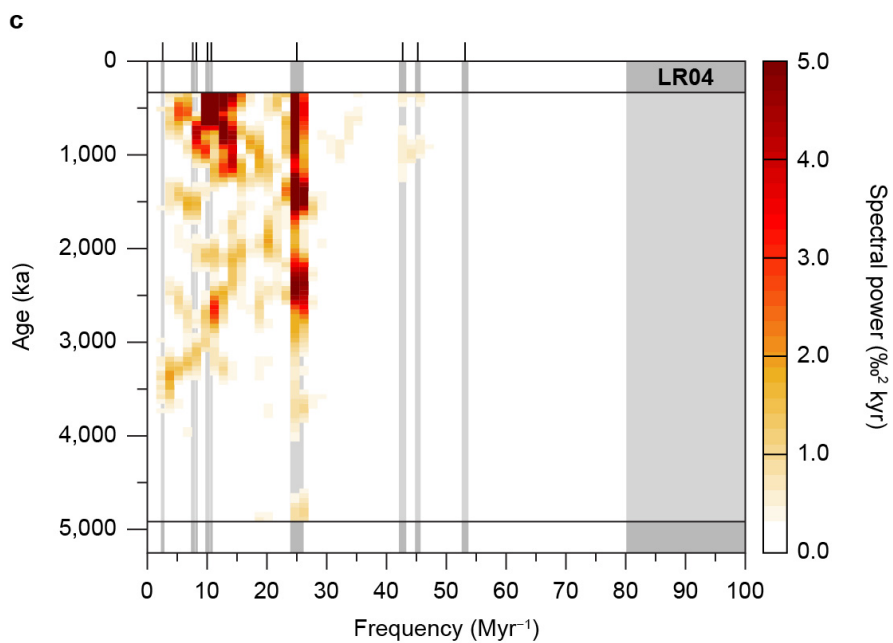
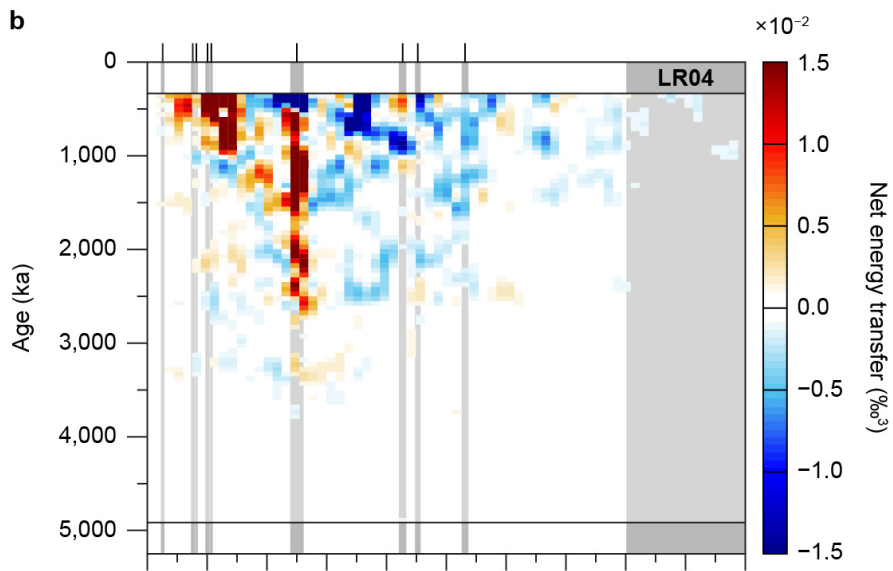
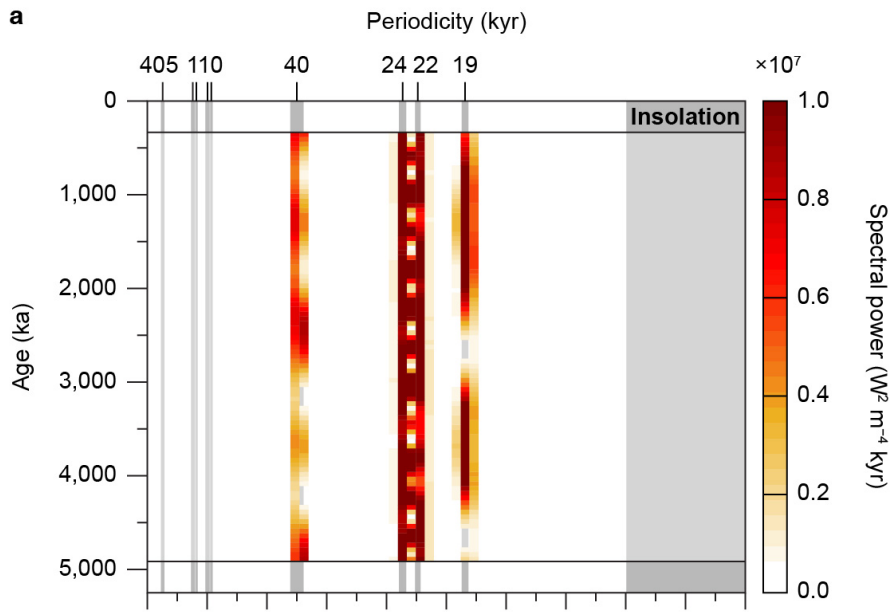
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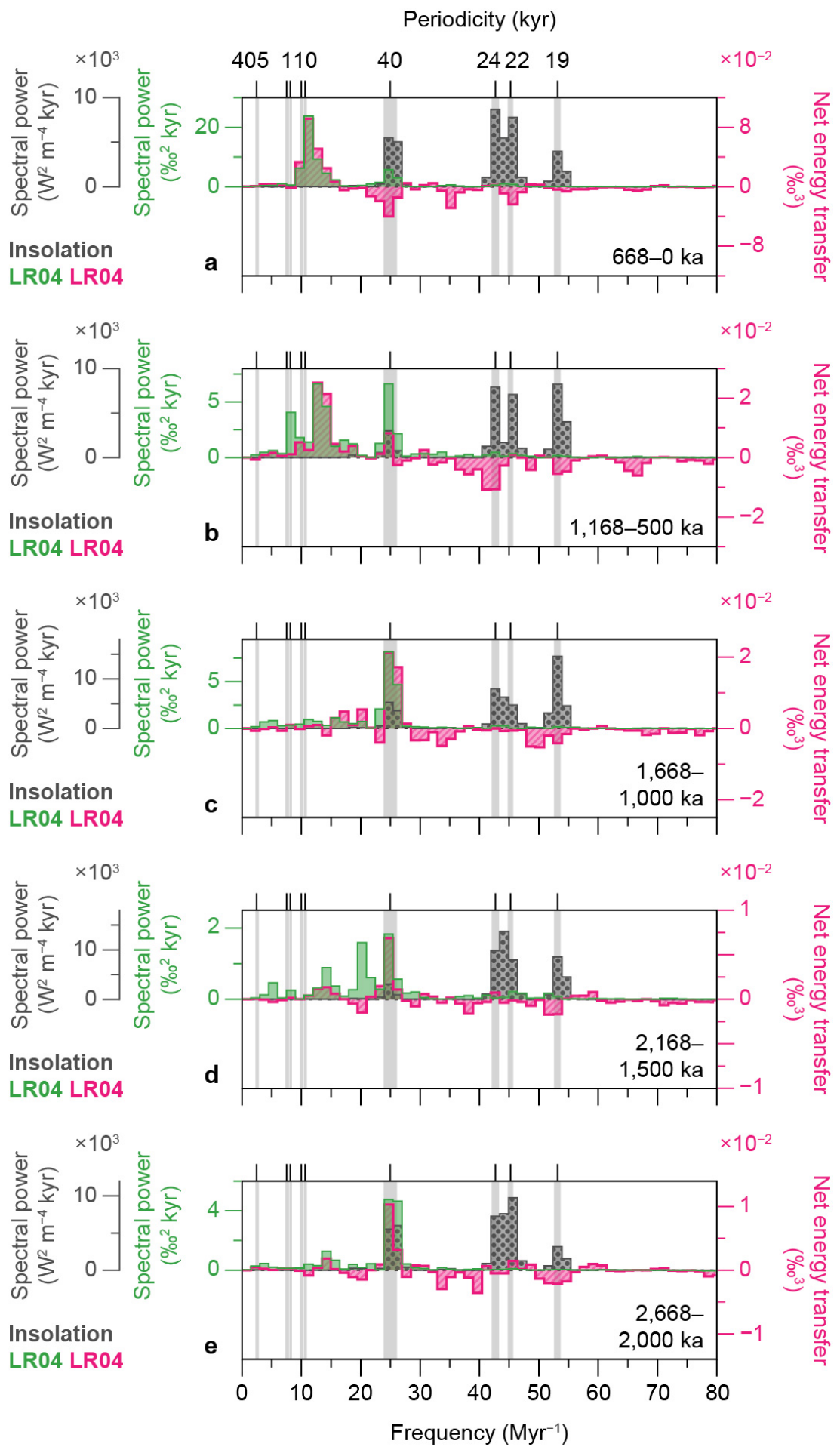


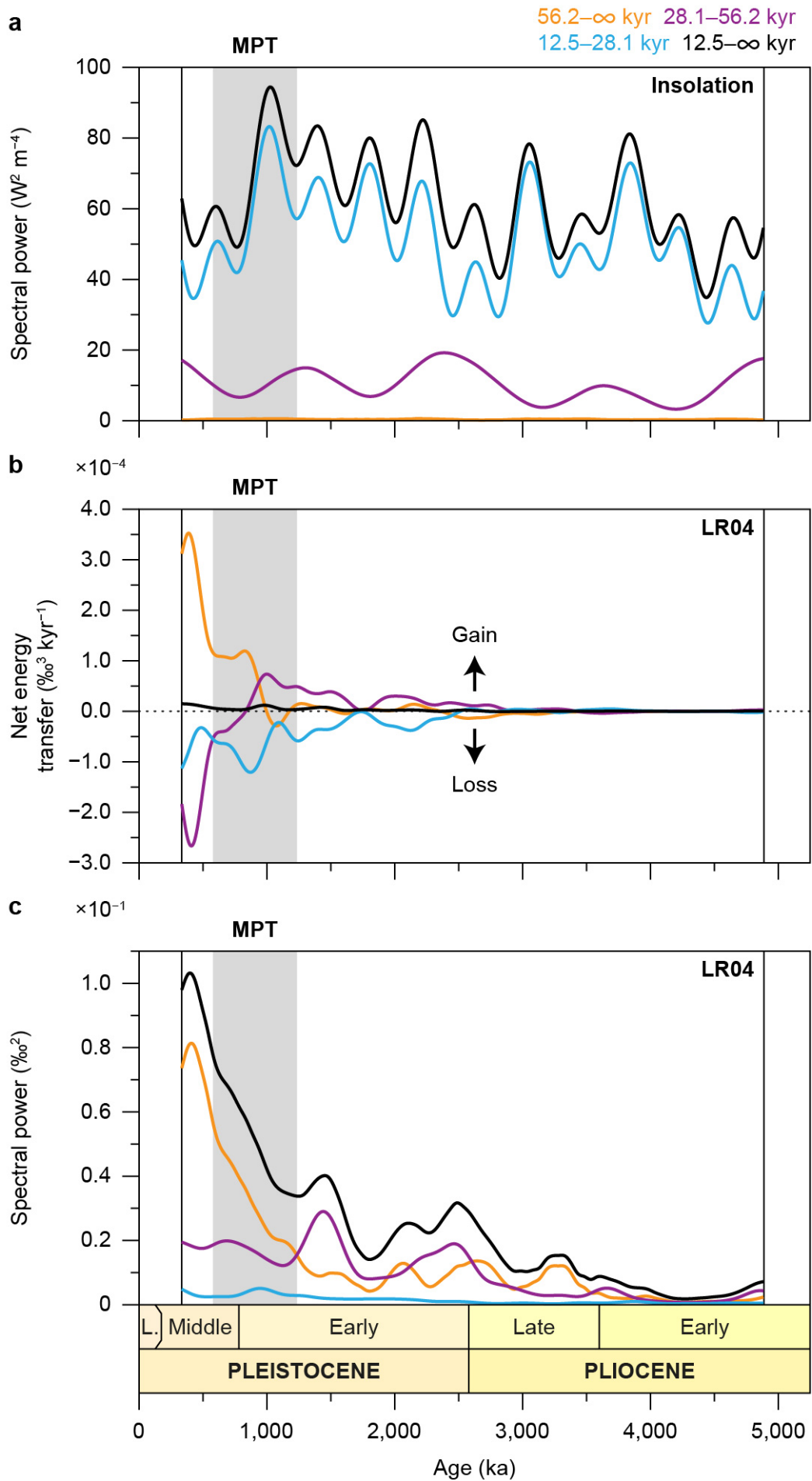


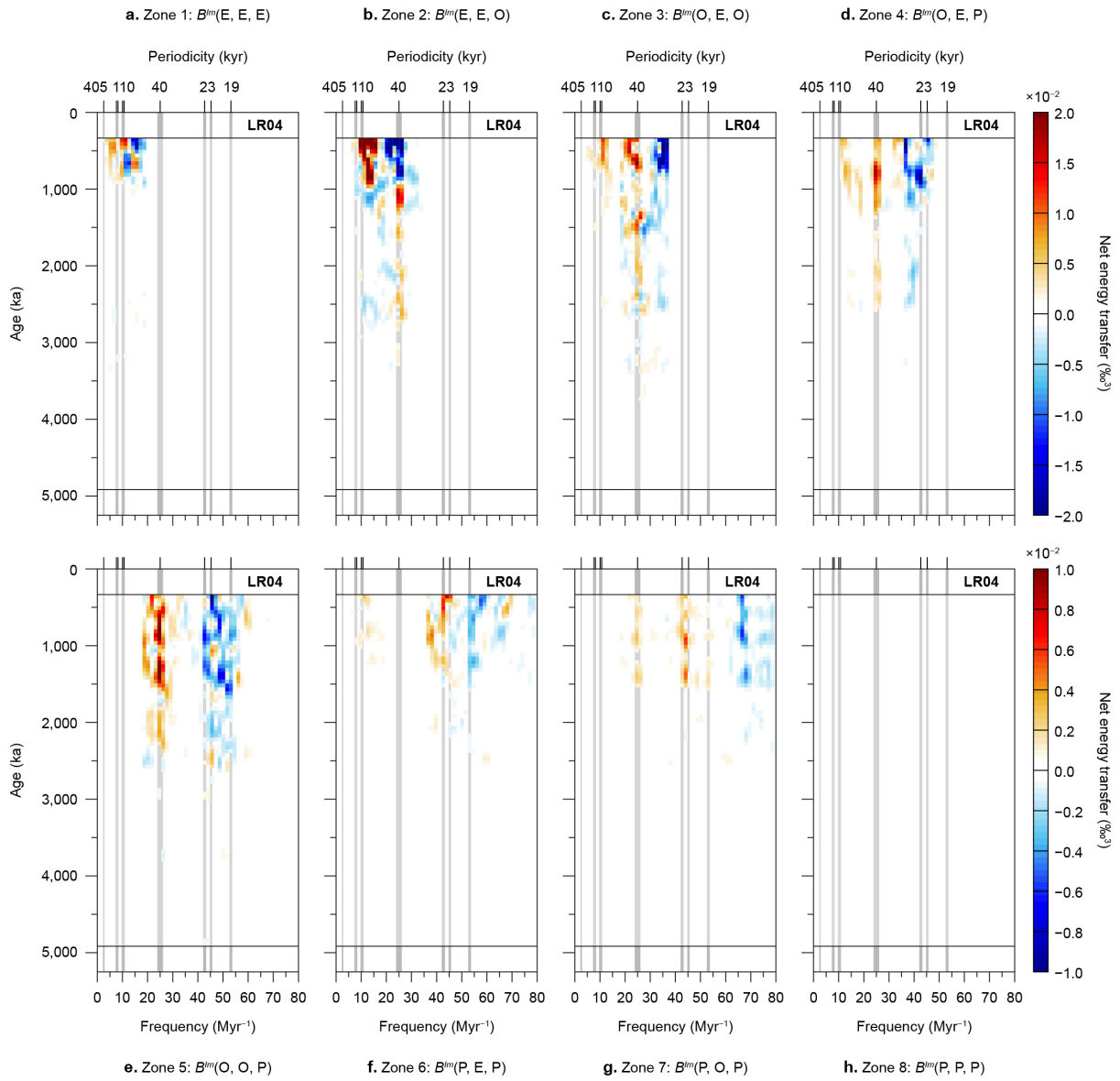


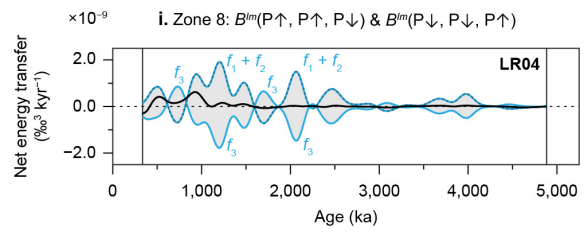
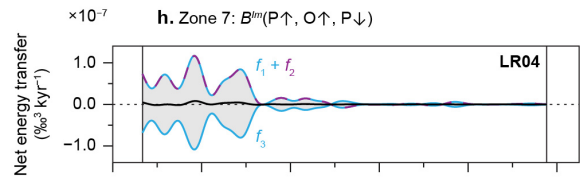
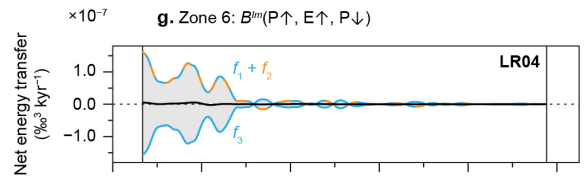
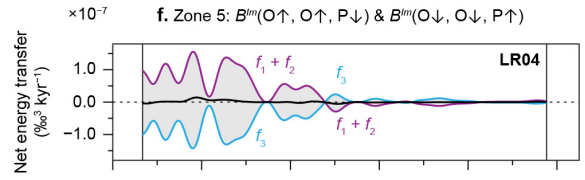
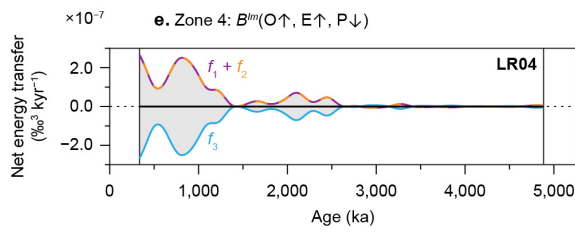
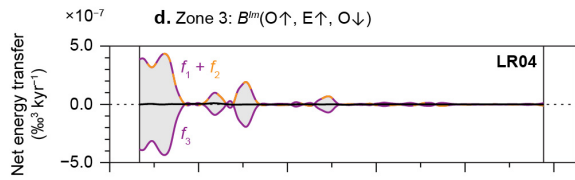
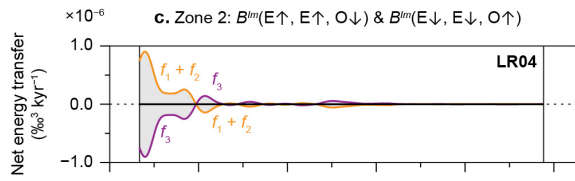
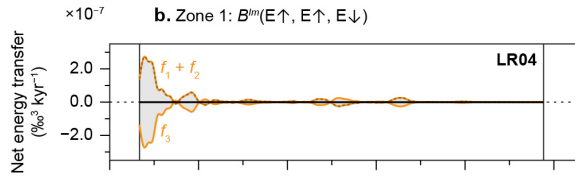
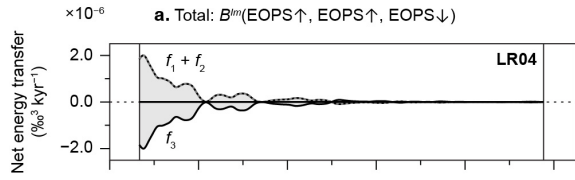


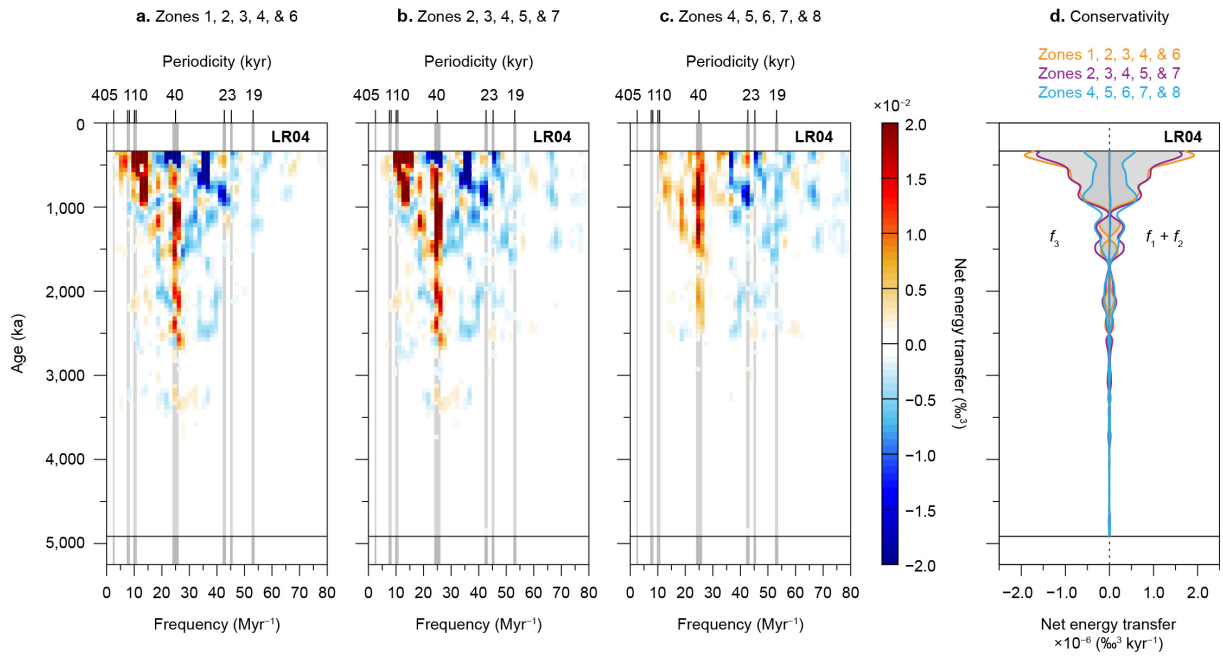












Bispectral zone	Periodicity 1 (difference)	Periodicity 2 (difference)	Periodicity 3 (sum)	Bispectral notation
1	Eccentricity	Eccentricity	Eccentricity	$B(E, E, E)$
2	Eccentricity	Eccentricity	Obliquity	$B(E, E, O)$
No zone	Eccentricity	Eccentricity	Precession	$B(E, E, P)$
No zone	Eccentricity	Eccentricity	Suborbital	$B(E, E, S)$
3	Obliquity	Eccentricity	Obliquity	$B(O, E, O)$
4	Obliquity	Eccentricity	Precession	$B(O, E, P)$
No zone	Obliquity	Eccentricity	Suborbital	$B(O, E, S)$
No zone	Obliquity	Obliquity	Obliquity	$B(O, O, O)$
5	Obliquity	Obliquity	Precession	$B(O, O, P)$
No zone	Obliquity	Obliquity	Suborbital	$B(O, O, S)$
6	Precession	Eccentricity	Precession	$B(P, E, P)$
7	Precession	Obliquity	Precession	$B(P, O, P)$
8	Precession	Precession	Precession	$B(P, P, P)$
9	Precession	Eccentricity	Suborbital	$B(P, E, S)$
10	Precession	Obliquity	Suborbital	$B(P, O, S)$
11	Precession	Precession	Suborbital	$B(P, P, S)$
12	Suborbital	Eccentricity	Suborbital	$B(S, E, S)$
13	Suborbital	Obliquity	Suborbital	$B(S, O, S)$
14	Suborbital	Precession	Suborbital	$B(S, P, S)$
15	Suborbital	Suborbital	Suborbital	$B(S, S, S)$

Bispectral zone(s)	$B_f(f_1, f_2, f_3)$ (Myr⁻¹)	$B_p(p_1, p_2, p_3)$ (kyr)	Triad type
1	$B_f(2.5, 0.0, 2.5)$	$B_p(405, \infty, 405)$	Single junction
1	$B_f(2.5, 2.5, 4.9)$	$B_p(405, 405, 203)$	Single junction
1	$B_f(5.5, 2.5, 8.0)$	$B_p(181, 405, 125)$	Double junction
1	$B_f(8.0, 0.0, 8.0)$	$B_p(125, \infty, 125)$	Single junction
1	$B_f(8.0, 2.5, 10.5)$	$B_p(125, 405, 95)$	Triple junction
1	$B_f(8.0, 8.0, 16.0)$	$B_p(125, 125, 63)$	Single junction
1	$B_f(10.5, 0.0, 10.5)$	$B_p(95, \infty, 95)$	Single junction
1	$B_f(10.5, 2.5, 13.0)$	$B_p(95, 405, 77)$	Double junction
1, 2	$B_f(10.5, 8.0, 18.5)$	$B_p(95, 125, 54)$	Double junction
2	$B_f(10.5, 10.5, 21.0)$	$B_p(95, 95, 48)$	Single junction
2	$B_f(14.5, 10.5, 25.0)$	$B_p(69, 95, 40)$	Double junction
2, (3)	$B_f(17.0, 8.0, 25.0)$	$B_p(59, 125, 40)$	Double junction
3	$B_f(22.5, 2.5, 25.0)$	$B_p(44, 405, 40)$	Double junction
3	$B_f(25.0, 0.0, 25.0)$	$B_p(40, \infty, 40)$	Single junction
3	$B_f(25.0, 2.5, 27.5)$	$B_p(40, 405, 36)$	Double junction
3	$B_f(25.0, 8.0, 33.0)$	$B_p(40, 125, 30)$	Double junction
3, 4	$B_f(25.0, 10.5, 35.5)$	$B_p(40, 95, 28)$	Double junction
4, 5	$B_f(25.0, 17.3, 42.3)$	$B_p(40, 58, 24)$	Double junction
5	$B_f(25.0, 19.8, 44.8)$	$B_p(40, 51, 22)$	Double junction
5	$B_f(25.0, 25.0, 50.0)$	$B_p(40, 40, 20)$	Single junction
5	$B_f(27.8, 25.0, 52.8)$	$B_p(36, 40, 19)$	Double junction
4	$B_f(31.8, 10.5, 42.3)$	$B_p(31, 95, 24)$	Double junction
4	$B_f(34.3, 8.0, 42.3)$	$B_p(29, 125, 24)$	Double junction
4	$B_f(34.3, 10.5, 44.8)$	$B_p(29, 95, 22)$	Double junction
6	$B_f(36.8, 8.0, 44.8)$	$B_p(27, 125, 22)$	Double junction
6	$B_f(39.8, 2.5, 42.3)$	$B_p(25, 405, 24)$	Double junction
6	$B_f(42.3, 0.0, 42.3)$	$B_p(24, \infty, 24)$	Single junction
6	$B_f(42.3, 2.5, 44.8)$	$B_p(24, 405, 22)$	Triple junction
6	$B_f(42.3, 8.0, 50.3)$	$B_p(24, 125, 20)$	Double junction
6	$B_f(42.3, 10.5, 52.8)$	$B_p(24, 95, 19)$	Triple junction
7	$B_f(42.3, 25.0, 67.3)$	$B_p(24, 40, 15)$	Double junction
11	$B_f(42.3, 42.3, 84.6)$	$B_p(24, 24, 12)$	Single junction
6	$B_f(44.8, 0.0, 44.8)$	$B_p(22, \infty, 22)$	Single junction
6	$B_f(44.8, 2.5, 47.3)$	$B_p(22, 405, 21)$	Double junction
6	$B_f(44.8, 8.0, 52.8)$	$B_p(22, 125, 19)$	Triple junction
6	$B_f(44.8, 10.5, 55.3)$	$B_p(22, 95, 18)$	Double junction
7	$B_f(44.8, 25.0, 69.8)$	$B_p(22, 40, 14)$	Double junction
11	$B_f(44.8, 42.3, 87.1)$	$B_p(22, 24, 11)$	Double junction
11	$B_f(44.8, 44.8, 89.6)$	$B_p(22, 22, 11)$	Single junction
6	$B_f(50.3, 2.5, 52.8)$	$B_p(20, 405, 19)$	Double junction
6	$B_f(52.8, 0.0, 52.8)$	$B_p(19, \infty, 19)$	Single junction
6	$B_f(52.8, 2.5, 55.3)$	$B_p(19, 405, 18)$	Double junction
6	$B_f(52.8, 8.0, 60.8)$	$B_p(19, 125, 16)$	Double junction
6	$B_f(52.8, 10.5, 63.3)$	$B_p(19, 95, 16)$	Double junction
7	$B_f(52.8, 25.0, 77.8)$	$B_p(19, 40, 13)$	Double junction
11	$B_f(52.8, 42.3, 95.1)$	$B_p(19, 24, 11)$	Double junction
11	$B_f(52.8, 44.8, 97.6)$	$B_p(19, 22, 10)$	Double junction
11	$B_f(52.8, 52.8, 105.6)$	$B_p(19, 19, 9)$	Single junction

Supplementary Information to:

Bispectra of climate cycles show how ice ages are fuelled

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Supplementary Figure S1. Examples of the real part of the bispectrum. These examples correspond to the imaginary parts show in Fig. 4 in the main document. **(a)** Real part of a bispectrum of the Middle and Late Pleistocene “~110-kyr world”. Min value = $-146 \text{‰}^3 \text{ kyr}^{-2}$. Max value = $397 \text{‰}^3 \text{ kyr}^{-2}$. **(b)** Real part of a bispectrum of the mid-Pleistocene transition “~80-kyr world”. Degrees of freedom = 2. Min value = $-108 \text{‰}^3 \text{ kyr}^{-2}$. Max value = $112 \text{‰}^3 \text{ kyr}^{-2}$. **(c)** Real part of a bispectrum of the Pliocene and Early Pleistocene “40-kyr world”. Min value = $-63 \text{‰}^3 \text{ kyr}^{-2}$. Max value = $14 \text{‰}^3 \text{ kyr}^{-2}$. Note the different scaling of the z-axes between panel (a), (b), and (c). Computational settings as in Figure 4 (see main document). These 15 bispectra are computed on the LR04 stack (see Methods) ([Lisiecki and Raymo, 2005](#)).

Supplementary Figure S2. Integration over the real part of the bispectrum. This Figure corresponds to Figure 5b in the main document. Computational settings as in Figure 5b (see main document).

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Supplementary Figure S3. Conservative net energy transfers during the Pliocene and Pleistocene, computed by integrating over the entire imaginary part of the bispectrum (see Methods). Input data is the resampled LR04 stack ([Lisiecki and Raymo, 2005](#)). **(a)** Window length = 500 data points (500 kyr), step-size = 50 data points (50 kyr). No blocks. No frequency merging. 25 Frequency resolution = 2.00 Myr^{-1} . Degrees of freedom = 2. **(b) Asymmetry corresponding to panel (a).** **(c)** As in Figure 5b (see main document). Window length = 668 data points (668 kyr), step-size = 50 data points (50 kyr). No blocks. No frequency merging. Frequency resolution = 1.50 Myr^{-1} . Degrees of freedom = 2. **(d) Asymmetry corresponding to panel (c).** **(e)** Window length = 1,000 data points (1,000 kyr), step-size = 50 data points (50 kyr). No blocks. No frequency merging. 30 Frequency resolution = 1.00 Myr^{-1} . Degrees of freedom = 2. **(f) Asymmetry corresponding to panel (e).**

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Supplementary Figure S4. Conservative net energy transfers during the Pliocene and Pleistocene over specific zones in the imaginary part of the bispectrum (see Methods). Computational settings as in Figure S3a (i.e., window length = 500 data points). **(a)** Zone 1. **(b)** Zone 2. **(c)** Zone 3. **(d)** Zone 4. **(e)** Zone 5. **(f)** Zone 6. **(g)** Zone 7. **(h)** Zone 8. See also Figure 3 and Table A1 (main document).

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Supplementary Figure S5. Conservative net energy transfers during the Pliocene and Pleistocene over specific zones in the imaginary part of the bispectrum (see Methods). Computational settings as in Figure S3c (i.e., window length = 1,000 data points). **(a)** Zone 1. **(b)** Zone 2. **(c)** Zone 3. **(d)** Zone 4. **(e)** Zone 5. **(f)** Zone 6. **(g)** Zone 7. **(h)** Zone 8. See also Figure 3 and Table A1 (main document).

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References

Lisiecki, L. E., and Raymo, M. E.: A Pliocene-Pleistocene stack of 57 globally distributed benthic $\delta^{18}\text{O}$ records, *Paleoceanography*, 20, <https://doi.org/10.1029/2004PA001071>, 2005.

