Interactive comment on "On the linearity of the temperature response in Holocene: the spatial and temporal dependence" by Lingfeng Wan et al.

Anonymous Referee #1
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In this work, the authors make a first attempt to investigate the linearity of forced Holocene variability in dependence on the temporal and spatial scale – by using global surface temperature fields obtained from the TraCE-21ka paleo-climate CGCM simulation. The topic is interesting and the results seem to indicate that further research into this direction may provide further knowledge, that will be useful for both, (a) the interpretation of paleo records and (b) the attribution and detection field of research. Nonetheless, I have a number of concerns regarding the conceptual approach (see 'general comments') which, I think, need clarification before the conclusions, drawn by the authors, can be thoroughly evaluated. A few specific questions are listed at the end (see 'Specific comments').

1 General comments

(1) Throughout this work, it seems that the following two *different* questions are mixed up, which makes it basically impossible to evaluate the conclusions drawn from the results:

Q-1. How linear is the response to external forcing? If we denote the temperature resonse to the full external forcing, $F_{all}(t) = F_1(t) + F_2(t) + F_3(t) + F_4(t)$, by $T_R(F_{all}(t))$, the response to the individual forcings by $T_R(F_i(t))$ (with $i = 1, . . . , 4$), and the internal temperature variability of the five model simulations by $T_{I,all}$, $T_{I,1}$, $T_{I,2}$, $T_{I,3}$ *and* $T_{I,4}$, respectively, then the linearity of the response could be defined by the extent to which the statement

$$T_R(F_{all}(t)) = \sum_{i=1}^{4} T_R(F_i(t)) \qquad (1)$$

holds, and the linearity could be measured by the correlation between the forced response on the left and that on the right hand side of the above equation. In the manuscript, however, the correlation is computed (see Section 2.2) from the full 'forced plus internal' temperature variability, i.e. between $T_R\big(F_{all}(t)\big) + T_{I,all}(t)$ and $\sum_{i=1}^{4} T_R\big(F_i(t)\big) + T_{I,i}(t)$, Since this latter correlation is influenced by the signal-to-noise ratio, Var(T_R)/Var(T_I), a small correlation does not necessarily indicate the absence of linearity, because it could be that simply the signal-to-noise ratio is small, although the response is still perfectly linear. (One would need ensembles of model simulations for each of the five forcing scenarios, and then use the ensemble average in order to suppress the internal variability.) Hence, Q-1 cannot be answered by this approach (without additional information), unless one would always obtain correlations close to unity, which would indicate strong linearity.

Q-2. What is the relative importance of externally forced vs. internal variability, assuming the response were linear? To answer this question one could use the correlation computed from the full temperature variability, as done in the manuscript, but one had to assume the linearity which, however, is to be proven by this work, in particular, for different temporal and spatial scales.

Hence, the authors should clarify the above issues, and make explicit which of their results contributes to which one of the above two questions. This will also help to clarify the implications of the conclusions for various research fields.

Reply: We thank the reviewer for this comment. We agree with the reviewer completely on the definition of linearity. We apologize for our ambiguity in the original manuscript. Our focus is really on the slow evolution of temperature in the Holocene that is of comparable time scale to the forcing factors. We have made a major revision. First, we have rewritten all the sections except for section 3. In the revision, we clarified our single realization approach and its potential issues for assessing the linear response (in addition to clarification of the data and methods). Second, we have changed the title to: "Holocene temperature response to external forcing: Assessing the linear response and its spatial and temporal dependence"

(2) Even if we had ensembles available for each of the forcing scenarios, it would still be possible to obtain a large correlation coefficient although the response is only weakly linear (i.e., mostly non-linear), if the individual response to, for example, one of the forcings F_i is much larger than the responses to the remaining forcings, because in this case the full temperature variability might still be dominated by the response to the strong forcing (the non-linear interactions might still be relatively small). Thus, one would need to know the strength (e.g., in terms of variance) of the responses to the various individual forcings.

Reply: This raises an excellent point. The explained variance of each forcing factor is indeed very interesting. We think it deserves special attention. Due to the multiple time scales and the strong regional dependence here, however, a detailed study on the variance of each forced response would require much more analyses than in the current paper; it also tends to mix information with our basic information in the first paper here. Therefore, we will still focus on "if the linear response is valid" and will leave the study on "how much each forcing contribute" to a follow-up paper. As for the potential case of a response dominated by a single forcing, it is reasonable to consider this case as a good linear response to the dominant forcing, because the impact from other forcings are negligible anyway.

(3) In the Conclusions section it should be mentioned that, even if strong linearity for the given model simulations were proven, then this conclusion is valid only for the given range of forcing amplitudes as non-linearities may appear for stronger forcings.

Reply: Agreed. Comments are added in section 4. “It should therefore be kept in mind that the assessment could differ for different variables, in different models, for different periods and for different sets of forcing factors.” “The assessment will be also different if a different period is assessed, e.g. the last 21,000 years; with a large amplitude of climate forcing, the linear response may degenerate in the 21,000-year period.”

(4) How is it justified to estimate the variance of the internal variability at orbital and millennial time scales by the full variance at centennial and decadal variability (page 7, last paragraph)? That is, why should we have

$$\mathrm{Var}_{orb,mill}(T_{I,all}) \approx \mathrm{Var}_{cent+dec}(T_{I,all} + T_R(F_{all}))? \quad (2)$$

Even if we assume that $\mathrm{Var}_{cent+dec}(T_R(F_{all}))$ is small compared to $\mathrm{Var}_{cent+dec}(T_{I,all})$, this does not imply anything about the relation between $\mathrm{Var}_{cent+dec}(T_{I,all})$ and $\mathrm{Var}_{orb,mill}(T_{I,all})$. Maybe it could be helpful to investigate the power spectra of the temperature variability under the various forcing scenarios?

Reply: We apologize for the ambiguity here. Again, this is an approximation based on linear thinking. Since our forcing, orbital, ice sheet, meltwater and CO_2 are of time scales of millennial or longer (we don’t have solar variability and volcanic forcing!), we assume that the variability at centennial and decadal time scales are caused mostly by internal variability. This point is clarified now in the revision in subsection 3.3. Since our forcing factors are on millennial and orbital time scales, and the linear response is also largely valid for orbital and millennial variability, we use the variance of the orbital and millennial variability as a crude estimate of the linear response signal. Since there is no centennial and decadal forcing in our model and the response of centennial and decadal variability are not linear response, we use the variance of the sum of the centennial and decadal variability as a rough estimate for internal variability as the linear noise.

2 Specific comments

(5) It would be nice if the reasons for showing the linear error index Le were explicitly discussed, and what the implications of this index are for the linearity. And what is the added value of this index over the correlation coefficient?

Reply: The correlation represents the similarity of the ALL and SUM, but can’t

evaluate the absolute magnitude of the two responses. Even if two time series is perfectly correlated, their magnitudes can differ by an arbitrary constant. The linear error is to reflect the magnitude of the relative error between the ALL and SUM. More clarifications are added in the text in section 2.2 on this.

(6) Please, be a bit more explicit how the significance levels are computed. For example, how is the AR(1) fit done in case of the correlation, and what is the bootstrap design for the error index?

Reply: The bootstrap is greatly expanded in the rewritten section 2.2.