Point-by-point Response to Reviewers

Reviewer 1 (Michel Crucifix)

The present manuscript has undergone a first review round in Climate of the Past. Previous reviews had not formulated strong objections about the statistical treatment of the data, but were worried about poor presentation and lack of focus. The manuscript was rejected and resubmitted, which unfortunately resulted in depriving the current reviewers from the benefits of a point-by-point rebuttal to the first review. Upon inspection, most figures are the same, key equations are unchanged, but more room is given to the interpretation of the results, and the abstract is more informative and, to my opinion, better written.

Given these improvements, and given the fact that there is no major objection to the interest of the statistical approach (see however specific comments below), the study should be published. This said, the authors may want to seize the opportunity of this review to clarify or perhaps improve a few zones of discomfort that, I think, has somewhat hindered the reception of their recent works.

First, something needs to be done about graphics. There are many problems. To cite but a few: vertical axes with different tick marks have been superimposed (Figures 4 and 5), labels show aberrant disproportions caused by vertical stretching (many figures), numbers alignment is sometimes inconsistent (horizontal axis on Figure 6), horizontal ticks seem to have been added by hand, with shadows (Figure 5), or inconsistently aligned (Figure 2), and colour legends are missing (Figure 9).

Author Response: Thank you for the constructive comments. We have improved the figures according to your suggestions.

Second, it would be useful to have some estimates of the variance of estimators. Assuming a stationary signal, and given the amount of data at hand, which variance do we expect for the estimate of τc, or H, or even $C1$? Figure 10 seems indeed to make it clear that there is a significant difference between τc of phase 1 and 8; trends on Fig. 12 are much less obvious and having clearer ideas as to whether variations can be attributed to statistical sampling, and to non-stationarity (or at least, whether they are a sign that null-hypothesis of stationarity should be rejected), would be helpful.

Author Response: It is actually not trivial to talk about the “variance” of the exponents, as the exponents themselves are only really meaningful in a stochastic framework: there is not a deterministic parametric model that underlies the distribution. As explained in a new paragraph (in the discussion section), to go further would require the elaboration of a more precise stochastic model that could predict the expected variation of exponents from one realization to another. We have rephrased the text to talk about dispersion instead. We also expanded the discussion of Figure 12 as you suggest.

I leave it to the editor whether addressing these two comments in full is a hard request, especially the second one. This brings me to the point by point comments:

- *p. 2: "two extremes" : They are not "extremes". Perhaps write again: between daily and orbital time scales...*

Author Response: We changed “extremes” to “scales”

- *p. 2 l. 30: "temperature record" → "deuterium record" (especially for readers of Climate of the Past, they know that deuterium concentration and temperature are not the same)*

Author Response: Corrected

- *p. 2 l. 34: "The analysis of the dust record" → "The dust record"*

Author Response: Changed to “the dust data used here”

- *p. 3, l. 10: "can themselves be power laws" : be more specific about the conditions that generate power laws*

Author Response: Scale invariance of dynamics in time and in space lead to space-time statistics that are characterized by power laws. In this sentence we are referring to something rather different, to power laws in the tails of probability distributions. However, it turns out that several mechanisms have been proposed that link the two so that we could largely expect power law probability tails to emerge from space-time scaling dynamical processes.

- *p. 3, l. 18: "In particular, etc." this is not a sentence.*

Author Response: Corrected

- *p. 5, l. 8: "Since K(1)=0" add "by definition".*

Author Response: Changed to “Since K(1) = 0 is a basic property,…”

- *p. 5, l. 8: "While the mean to RMS ratio is intuitive" : the sentence is unclear. In what sense is C1 a ’mean to RMS’ ?*

Author Response: We have clarified the sentence: “While the mean to RMS ratio is an intuitive statistic, it does not give a direct estimate of C_1:…”

- *p. 5 Eq(9) : use standard notation lim$\Delta q \to 0$.*

Author Response: We have modified the equation and text accordingly.

- *p. 6, l. 13: full sentence needed after a semicolon ";"*

Author Response: Corrected

- *p. 8, l. 13: "The plot graphically conterposes two views of variability". I suspect you mean Figure 4, but yet what is meant by this sentence is still not clear to me.*

Author Response: Changed to “The variability shown in Figure 4 can be interpreted broadly or in detail.”

- *p. 8, l. 20: 100 ka is not a Milankovitch frequency (Milankovitch was unaware of 100-ka cycles, he focused on 40-ka cycles).*

Author Response: Changed to “orbital” throughout the text.

- *p. 8, overall, I found the lines 17-30 difficult to read. Consider whether it is possible to express the same message more concisely.*

Author Response: We tightened up the paragraph, it is shorter, hopefully it is clearer.

– p. 9, l. 5 "possibly (but not obviously) scaling". In order to be possible but not obvious, a strict definition of what scaling means is needed.

Author Response: We eliminate the "(but not obviously)". In this context, "scaling" means a power law behavior. Due to the shortness of the range and the poor statistics at theses long time scales, the situation is not clear.

- p. 10, l. 22: we compare, without 's'

Author Response: Corrected

- p. 11, l. 13: "p value of 0.12". See the above comment. We do not really know whether the distribution is normal. Do we know anything about a theoretical distribution, or perhaps one that could be simulated ?

Author Response: The problem is that we don't have a detailed stochastic model. Without one, we don't know how much variability is "normal" or "typical". Using the standard deviation is agnostic in the sense that it is simply a standard nonparametric characterization of the spread. The only place where we go one step further is to assign a p value = 0.12. We have added a comment on this.

- p. 13, l. 9: "low internal feedback" → "low response"

Author Response: Corrected

Reviewer 2

By using non-standard approaches, the authors analyze in this paper the 320.000 cm-long EPICA Dome C dust flux record published in Lambert et al., 2012. I cannot judge on the statistical techniques adopted for characterizing the cycles – that I leave to expert reviewers in this (mathematical/statistical) field.

Glacial-interglacial cycles that are present in the EPICA record are subdivided into 8 phases showing systematic variations of their statistical properties. The interpretation of the variability of four key indicators (H, C1, qD, A) provides some interesting paleoclimatic information. I have only some concern about the interpretation of A and H exponent and their link to the size of the Patagonian ice sheet (see below), as well as some minor comments/questions. If the statistical part is duly revised by an expert in the field, this paper is worth to be published in CP after some minor revisions.

Author Response: Thank you for the positive comments.

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Page 2, lines 17 to 29: they refer to figure 2 which is (according to the figure caption) redrawn from Lovejoy, 2017 – please reference to that paper in this paragraph.

Author Response: Added

Page 6, lines 25-27: if you consider every cm of core, also the dust record can be somewhat affected at depth. Is this something which should be mentioned here? Probably not. But keeping this in mind, please re-structure this first sentence and state that you just take published data from Lambert et al., 2012 and discuss them as they are.

Author Response: The worsening of the resolution with depth of the dust data is discussed in lines 27-30. In our opinion, the sentence is clear that the dust data is not a product of this publication.

Page 7 , line 1: dust CONCENTRATION measurements, please specify.

Author Response: Changed to "dust flux measurements"

Page 7, lines 1-4: dust production depends on the source "intensity" that includes also the areal extent of the source which is variable depending on the exposed continental shelf.

Author Response: Original text changed to "The amount of dust deposited in East Antarctica will depend on the size and vegetation cover of the source region…"

Page 7, Lines 5-9: dust depositional flux variability is also related to the hydrological cycle at low frequencies.. and to temperature…this explains the high correlation between dust and stable isotopes in ice cores. Please restructure this sentence.

Page 7, Lines 8-9: "at high frequency dust deposition variability depends on wind and hydrological cycle": which is the reference for this assumption? Dust concentration/flux depends on the hydrological cycle at different timescales… And wind (transport) influences mostly size rather than concentration. I think the sentence " at high frequency dust deposition variability depends on wind and hydrological cycle" is more a conclusion of your study, as written in lines 17-20, page 13 "At higher frequencies…[…] …dust deposition in Antarctica will be more sensitive to temporary atmospheric disturbances in the winds and hydrological cycle"

Page 7, Lines 9-10: As above, the sentence "..a single peak within a low background may reflect short-term atmospheric disturbance like drought over South America or low precipitation over the S.Ocean…" is more a conclusion of your work rather than a literature assumption. But in any case, why not an eruption? Why not an impurity (contamination) within the core? And at depth, why not a level where particles aggregates are present and to some extent perturb the signal? Is it certain that every spike registered in the core represents a climatic signal? It would be a huge work to analyze every sample where dust levels are above background, but I feel confident that many of these spikes can be attributed to these causes.

Author Response: Indeed, this sentence contains some results from our analyses. We have changed these sections to "High and low frequency variability in the dust flux record is likely driven by different processes. For examples, dust source conditions related to glaciers and vegetation cover may not have influenced high frequency variability due to their relatively slow rate of change. On the other hand, volcanic eruption or extreme events related to the hydrological cycle may produce high-frequency signals in the record." Indeed, analyzing every sample of the EPICA Dome C Continuous Flow Analysis data was a huge task and took F. Lambert over 2 years during his PhD. But thanks to this work we can be sure that most contamination peaks were cut out of the signal. Large volcanic eruptions usually saturated the dust signal and were cut out as well, but smaller more distant eruptions may produce a particle peak similar to a climatic event. Since we didn't check each dust peak for a corresponding sulphate peak, we have added in the text volcanoes as another possible high-frequency contributor. Aggregates did change the size distribution in the data quite a bit, but not the particle count. And since the dust flux record used here is a merged signal of particle concentrations and Ca and nssCa data, we feel confident that dust aggregates did not produce any sharp peaks.

Page 10, line 22: replace "compares" with "compare"

Author Response: Corrected

From Page 11, line 31, to page 12, line 24: the whole paragraph is very interesting as figure 12 is also interesting. But with so many acronyms or indices, would it be possible to write for example "DRIFT" over the first plot (H), "SPIKINESS" over the second (C1) …etc? And also, maybe draw a horizontal arrow in each plot, going from right to left with "TIME" written on it. And why not, close to each number 1,2,3… the informal name of the phase ("interglacial", "glacial maximum",….)? This just for clarification, and for helping this figure to give an immediate message to the reader; I think this is one of the most important figures in the paper, so put it into value.

Author Response: We have followed the suggestions and updated the plot.

Page 12, lines 26-29: I would not say that the precise climate significance of dust flux is hard to nail down. Rather, maybe you can find an elegant way to say that dust fluxes result from several synergic variables and dust flux alone does not allow distinguishing the contribution of each of these variables in detail.

Author Response: We agree and changed this sentence to "First, their dynamical interpretation is not unambiguous: because they depend on temperature, wind, and precipitation, and so are holistic climate indicators, dust flux variability is hard to attribute to a specific process."

Page 13, lines 5-8: is the broadness of the peak really indicating irregularities of the eccentricity-forced Milankovitch cycles or, as I think, you probably mean it is indicating the irregularities in the continental response, including sea level change and shelf exposure, vegetation, glacial activity…and so on?

Author Response: Correct, we changed this to "The broadness of this peak already indicates the irregularity of the Earth system response to the eccentricity-forced orbital cycles."

Page 13 lines 15 to 20: this is an important consideration and conclusion of this paper that needs to be hemphasized a bit more.

Author Response: We expanded the corresponding section in the conclusion and added a sentence in the abstract.

Page 14, lines 1-2: after saying on page 12 that it is complicated to associate the dust flux increase or decrease to one variable, you are now associating high dust supply during phases 6-7 to the size of the Patagonian ice cap. That is not correct, first of all because dust influx to Antarctica does not depend solely on source production. Yet, even considering only dust availability at the source (source production), and only solely glacial dust sources, then you must consider that dust production is not one-to-one related to the size of the Patagonian ice sheet, but to the intensity of all glacial and periglacial processes potentially involved in dust production, which change in time, of course. Therefore, not only glacial *processes related to the size of the ice sheet (involving erosional processes related to the movement of ice and pressure on the underlying surface leading to great amounts of erosion) are involved. Also transport and deposition of glacial debris and formation of tills, outwash sediments (glacio-fluvial process), glaciolacustrine deposits, glacioeolian deposits, can act as dust sources; in addition, dust can derive also from periglacial processes related to nivation, frost action, mass wasting, fluvial processes and eolian processes that are enhanced by freeze drying of surface sediments, scarce vegetation cover and exposure to strong winds. So I think it is too simplistic to relate the A and H exponent to the size of the Patagonian ice sheet… please consider the possibility to relate these indices to the intensity of glacial and periglacial processes in South America.*

Author Response: We agree that was too simplistic and we removed that passage from the abstract. We have changed that section to "The higher amplitudes in phases 6 and 7 indicates that dust supply became abundant then. Since the Argentinean continental shelf was still submerged at that moment and the outwash plains not yet fully extended, the higher dust emissions may have been due to a transformation in vegetation cover about 30 kyr after glacial inception, possibly accompanied by changes in glacial and periglacial processes in the Andes."

About interpretation of C1 and qD exponents, related to short-term events. You cite possible short-term disturbances in the atmosphere. Why you do not consider volcanic eruptions? Probably because of the short-term atmospheric disturbance of these events? Or because you do not have a corresponding sulphate signal in the core? Please clarify introducing one or more sentences before the conclusion paragraph.

Author Response: Identifying volcano eruptions using the sulphate record alone is tricky because many large sulphate peaks do not have a corresponding dust peak. This means that even if you do have matching dust and sulphate peaks, it could be an eruption or a coincidence. Unfortunately, tephras in the EDC ice core were measured only at very low resolutions to get an idea about eruption frequencies, and we do not have that data available to unequivocally identify single events. But you are right, this means we cannot exclude volcanoes. A paragraph was added explaining this. "

"Finally, we could mention volcanoes. Volcano eruptions usually saturated the dust measuring device and were mostly cut from the record. Using the sulphate record to identify eruptions is tricky because many large sulphate peaks do not have a corresponding dust peak. This means that even if you do have matching dust and sulphate peaks, it could be an eruption or a coincidence. Therefore, the influence of volcanic variability on the results cannot be completely eliminated, although our key results are fairly robust with respect to the phase of the cycle and are therefore unlikely to be influenced by volcanic eruptions."

Reviewer 3

In their paper Lovejoy and Lambert present an application of fluctuation analysis to a high-resolution reconstruction of dust fluxes in central Antarctica over the last 800 ka. This type of analysis with a record that is this highly resolved is new and generally merits the publication of the manuscript.

That being said, there are a number some minor and a number of major concerns that need to be addressed before the manuscript can be published in Climate of the Past.

I cannot speak to the correctness of the statistical analysis as I am not an expert on fluctuation analysis. That being said, I can probably speak from the point of view of a large fraction of the potential readers of a Climate of the Past paper: I found the description of the method reasonably approachable. Personally, I would have liked to get more intuition for the method, its results and its possible applicability to other paleoclimate records.

Author Response: Many thanks for the positive comment. Although the fluctuation analysis presented here has been previously used on some mostly modern climate records (temperatures, various climate indices, precipitation, CO2 concentrations etc.), none of these are as classically "paleo" or intermittent/spiky as the EDC dust fluxes. In the following publication we will make a comparison between various ice core dust and temperature records and in that one we will definitely talk about wider-scale application and potential geographical disparities. Here, it is more of a proof-of-concept paper.

Overall, the paper could be improved greatly by adding more discussion of the methods, their results and their interpretation. At present the paper focuses a lot on the listing of the results of the different statistical analysis. This takes away from the potential interest of both the method and the results to the wider paleo-climate community.

Author Response: We agree. However, as stated above, this is the first application of this method to a dataset of this kind and we want to focus here on the statistical and mathematical correctness and robustness of the method. In the subsequent paper we intend to talk more broadly about application for the broad community.

One major concern is, that the manuscript is lacking a clear description of the data set that has been used, the way it was generated, and how this affects the analysis presented here. I know that this description is given in the original publication. Nevertheless, I think this is a vital point given that Lambert et al. (2012) state to keep the generation of the dust flux reconstruction in mind when interpreting its variance as it is affected by the assumptions and corrections that were involved.

Author Response: We added a new paragraph (2.4) in the methods section that describes the dataset and it's features in relation with our analysis: "The dust flux data used in this study is based on a linear combination of insoluble particles, calcium, and non-sea-salt calcium concentrations (Lambert et al., 2012). Because missing data gaps in the three original datasets were linearly interpolated prior to the PCA, high frequency variability can sometimes be underestimated in short sections that feature a gap in one of the three original datasets. This occurs in about 25% of all dust flux data points, although half of those are concentrated in the first 760 m of the core (0-43 kaBP), when an older less reliable dust measuring device was used. Below 760m these occurrences are evenly distributed and do not affect our analysis. Due to the sometimes slightly underestimated variability, the analysis shown here is a conservative estimate."

One other major concern that I have is, that the interpretation of the, I think interesting, results unfortunately seems to be ad-hoc and not very thoroughly argued. From my reading of the previously rejected version of this paper and its reviews this point has only been improved marginally. To strengthen *the manuscript and make it more suitable for Climate of the Past, I hope the authors extent the discussion of the results both in comparison with other studies and in terms of their paleo-climatological interpretation. The discussion and the results sections are completely lacking any information on the uncertainties of the obtained results. The large variability of the results between the different glacial/interglacial cycles indicates to me, that the results might not be very robust. One further observation that I made is that in many Figures the authors omit error or variability indications "for the sake of clarity" which I think is a poor choice. Additionally, the analysis is hinged upon a number of assumptions that are not justified in the text. Specifically, the slopes used for the breakpoint analysis and the range of time scales used for the fluctuation analysis. The influence of these choices on the results needs to be shown and clearly discussed.*

Author Response: We have added a new paragraph in the conclusion section about uncertainty estimates:

"The results presented in this paper are largely empirical characterizations of a relatively less known source of climate data: dust fluxes. Dust flux statistics defy standard models: they require new analysis techniques and better physical models for their explanation. These reasons explain why our results may appear to be rough and approximate. Readers may nevertheless wonder why we did not provide standard uncertainty estimates. But meaningful uncertainties can only be made with respect to a theory and we have become used to theories that are deterministic, whose uncertainty is parametric, and that arises from measurement error. The present case is quite different: our basic theoretical framework is rather a stochastic one, it implicitly involves a stochastic "earth process" that produces an infinite number of statistically identical planet earths of which we only have access to a single ensemble member. From this single realization, we neglected measurement errors and estimated various exponents that characterized the statistical variability over wide ranges of time scale, realizing that the exponents themselves are statistically variable from one realization to the next. In place of an uncertainty analysis, we therefore quantified the spread of the exponents (which themselves quantify variability). In the absence of a precise stochastic model we cannot do much better."

Specific comments:

Abstract:

P1 L17: The dataset has only a maximum resolution of 5 years. How can fluctuations on the one-year time scale be resolved. Please rephrase.

Author Response: We changed this sentence to "The temporal resolution ranges from annual at the top of the core to 25 years at the bottom ,…"

P1 L24-27: The logic of this sentence is not clear. Please rephrase.

Author Response: Changed to "In other words, our results suggest that glacial maxima, interglacials, and glacial inceptions were characterized by relatively stable atmospheric conditions, but punctuated by frequent and severe droughts, whereas the mid-glacial climate was inherently more unstable."

P1 L27f: Why do they suggest this?

Author Response: We're putting in plain text what is stated in technical words in the previous sentence.

Introduction:

P2 L12-15: Please state that you refer to the temperature proxy time series, and also mention that this is a proxy, not a temperature in the strict sense.

Author Response: Changed to "Fig. 1 shows this visually for the EPICA Dome C Antarctic ice core temperature proxy (5787 measurements in all);"

P2 L23-27: Please consider explaining why the macro weather to climate transition timescale is important.

Author Response: We added this: "The macroweather-climate transition scale marks a change of regime where the dominant high frequency processes associated with weather processes and reproduced by GCMs in control runs gives way to a new regime where the variability is dominated by either the responses to external forcings or to new, slow internal sources of variability."

Method:

P3 L15: ...the spectrum is the Fourier transform...

Author Response: corrected

P3 L24-26: It is unclear why due to scale invariance, the results from the dust fluxes can be transferred to the temperature proxies if they are affected by different climatic mechanisms.

Author Response: Scale invariance is a symmetry under time dilations. In a dynamical regime in which two different components such as temperature and dust are strongly coupled, each may have different scaling properties, but both should respect the scale symmetry including the transition scale at which the symmetry breaks down. It is only these aspects that can be "transferred".

We have changed the text to reflect these ideas.

P3 L26: The analysis in the "future publication" should more thoroughly be discussed here, especially as you present results from it, or alternatively only be mentioned in the outlook. In my experience these "future publications" unfortunately often do not manifest themselves.

Author Response: The key analyses – the systematic comparison of temperature and dust as a function of time scale - have already been performed but this paper is already long enough. We prefer to discuss them in a separate paper.

P4 L 24: extra comma between "compare" and "the"

Author Response: corrected

Results:

The data set description should be in the Data + Methods section and more time should be spent describing the dataset used as pointed out above

Author Response: We moved the data description to a new sub-chapter 2.4 and describe the data in detail as suggested. The new chapter 2.4 now reads "The dust flux data used in this study is based on a linear combination of insoluble particles, calcium, and non-sea-salt calcium concentrations (Lambert et al., 2012). Because no-data gaps in the three original datasets were linearly interpolated prior to the PCA, high frequency variability can sometimes be underestimated in short sections that feature a gap in one of the three original datasets. However, the amount of points that feature

Unlike water isotopes that diffuse and lose their temporal resolution in the bottom section of an ice core at high pressures and densities, the relatively large dust particles diffuse much less and have been used to estimate the dust flux over every centimetre of the 3.2 km long EPICA core (298,203 measurements, (Lambert et al., 2012)). The temporal resolution of this series varies from 0.81 years to 11.1 yrs (the averages over the most recent and the most ancient 100 kyrs respectively). The worst temporal resolution of 25 years per centimeter occurs around 3050 m depth, with the result that at that resolution, there are virtually no missing data points in the whole record (Fig. 1)." The last paragraph in this text was moved from sub-chapter 3.1.

P6 L27: With the strong emphasis that is put on the number of datapoint and their sampling frequency throughout the manuscript, the numbers should add up. Generally, this information is not strictly necessary for the paper and could be safely removed. Especially in light of the fact that the dust flux reconstruction is a combination of a multitude of measurements and ice flow modelling.

Author Response: The rest are data gaps due to cleaning of the raw data (contamination, missing ice, etc.). We changed the text to "298,203 valid measurements"

P7 L1: Water isotopes cannot be assigned to one particular atmospheric variable either, even though they are often used to reconstruct Temperature. Please consider rephrasing

Author Response: We changed that phrase to "Polar dust flux measurements cannot be assigned to one particular atmospheric variable, like temperature for the water isotopes."

P7 L5ff: Please consider mentioning the recent publications by Markle et al (2018) and Schüpbach et al. (2018) that deal with this the relation of en-route washout and aerosol deposition on the ice sheets more quantitively.

Author Response: Yes, maybe a distinction between short and large timescales is in order here. We changed these sentences to "At any given moment, the amount of dust deposited in East Antarctica will depend on the size and vegetation cover of the source region (mostly Patagonia for East Antarctic dust

(Delmonte et al., 2008)), on the amount of dust available in the source region (can depend on the presence of glaciers), on the strength of the prevailing winds between South America and Antarctica, and the strength of the hydrological cycle (more precipitation will wash out more dust from the atmosphere (Lambert et al., 2008)). Over large scales it is thought that temperature-driven moisture condensation may be the major process driving low-frequency variability (Markle et al., 2018), although that may not globally be the case (Schüpbach et al., 2018)."

P7 L19: There is no red line in the Figures.

Author Response: Yes, we removed the word "red".

P8 L7f: The unit of the spectral amplitude of the log-transformed fluxes is wrong.

Author Response: Thank you, corrected.

P8 L10f: Consider moving the comparison of the spectral densities with the results of the fluctuation analysis to after their introduction or to the discussion section.

Author Response: Comparison moved to the discussion of Figure 5.

P8 L13ff: This list of scaling exponents is completely irrelevant here and should be removed for clarity.

Author Response: Sentence removed

P8 L16f: It is unclear why this supports the use of dust as a proxy for atmospheric variability. Please clarify.

Author Response: We argued that temperature and dust variability are of the same statistical type yet with significant differences. The former makes it likely that the dust signal is a real climate signal while the latter shows that it has different information. We eliminated the original sentence and replaced it with one similar to the above.

P8 L 25-30: The results presented here are very hard to follow. Please consider reformulating.

Author Response: We have modified the paragraph.

P9 L1: Whether the Haar fluctuations of the dust flux have simple interpretations is not shown in the Figure but is rather a matter of taste and should be left to the reader to decide. Please reformulate or remove this statement.

Author Response: Changed to "Haar fluctuations allow some direct interpretations…"

P9 L6f: After spending a lot of time describing and interpreting the spectral analysis in the previous section the description here is rather short. As the fluctuation analysis is the main focus of this study the authors should spent more time describing their results and leading the uninitiated reader through them.

Author Response: Yes, this is a good point, thanks. We replaced this first paragraph by three paragraphs giving a fuller explanation of the figure.

P9 L14: “positive definite” seems to be the wrong phase here, consider replacing with “always positive” or similar.

Author Response: *Corrected.*

P9 L18: Why is it important that the value is similar to those obtained from other ice cores. Please move all comparisons to other studies to the discussion clarify the interpretation and relevance.

Author Response: We have removed that section in the revised manuscript.

P9 L20: This statement is somewhat superfluous: If the dust fluxes are not log-normally distributed due to the occurrence of large spikes, their logarithm will not be normally distributed.

Author Response: Yes. But due to the prevalence of log-transforming dust data in climate science, we would like to stress that statement.

P9 L24: Please move the interpretation and the relevance for tipping point analysis to the discussion and consider expanding on this point if you think it is an important application.

Author Response: We have moved this sentence to the discussion. Discussion of this point would be a (actually several) paper in itself and is not central to this paper. But we feel that it is an important point to make for the tipping point community, many of which will read this paper.

P9 L30f: Strictly, the statement that the scaling spectrum is the underlying behavior is an assumption and has not been shown in this study. Even though this a reasonable assumption, consider rephrasing.

Author Response: Yes, we only show that the data are consistent with this hypothesis. No more can be done. We added a sentence to this effect.

P10 L4f: Please clearly state that any of the stacking approaches assume that all the glacial cycles and their sub phases are realizations of the same underlying process.

Author Response: We added “…, assuming that the major underlying processes were constant over the last 800,000 years” to that phrase.

P10 L21ff: The start of this paragraph makes the reader expect spectra averaged over the different cycles. Please consider rephrasing and more extensively introducing the Figure.

Author Response: We have modified this paragraph as suggested.

P10 L29ff: The Figure indicates a slope of 0.35, not 0.25 as mentioned in the text. More importantly it is entirely unclear why the authors choose to set the slopes before and after the transition as constants and then only fit the transition time scale. The effect of the chosen values on the presented results needs to be clearly discussed and the uncertainty of the results are missing completely from the text. I strongly urge the authors to do a proper breakpoint and error analysis.

Author Response: The value 0.25 was a typographic error, the value used was actually 0.35 as indicated in fig. 9 (not 10 as indicated in the text). The hypothesis here was that there were two regimes, each characterized by a different exponent each of which was estimated from the ensemble statistics. Given the hypothesis, the analysis only needed to estimate the scale at which the low frequency process exceeded the high frequency one. Therefore, we found the break point that minimized the RMS deviation from the bi-power law behaviour. We have added material explaining this better.

P11 L5-9: As the other method is arguably only slightly more objective than the breakpoint inference by hand, please consider removing this entire section.

Author Response: We are dealing with a system with strongly nonclassical statistical characteristics: scaling and strongly non-Gaussian fluctuations. We do not have a precise stochastic model of the dynamics, the aim of this study is to yield a first, initial empirical characterization that could in the future provide the basis of a more precise model that would allow us to justify the estimation of uncertainty limits.

P11 L20f: I suggest that the authors remove a couple of lines from the figure for clarity instead of the uncertainties or find a different way of visualizing the results.

Author Response: Perhaps the problem is that the dashed lines were mistaken for uncertainty limits of the solid lines? We have added a phrase in the text to clarify this. There was also a reference two lines earlier to fig. 10 that should have been fig. 9. This correction may also help understand fig. 11.

P11 L23: Please state why this exact time range was chosen.

Author Response: As indicated in the text, the idea here was to estimate exponents from fixed ranges rather than ranges that varied depending on somewhat uncertain estimates of τ_c. The range of time scales was chosen so that in most phases, most of the range of time scales was in the climate regime ($H>0$), hence the lower limit of $\Delta t>500$ year. The upper limit of $\Delta t < 3000$ years was chosen because at longer scales, the statistics were less reliable: for phases 12500 years longer, there are only 4 disjoint intervals available and for larger Δt there are fewer. The range choice was a compromise that aimed at quantifying systematic phase to phase changes (the solid lines connecting the points) as well as the cycle to cycle dispersion of the exponents at each phase (the error bars).

P12 L10: There is nothing black in Figure 12, please correct.

Author Response: Corrected with additional information.

P12 L18-21: Consider moving this sentence to the beginning of the paragraph to make it easier for the reader to follow the results.

Author Response: Moved as suggested

Discussion:

P13 L7-9: You do not perform any significance analysis, so please reformulate this statement. Also please discuss why the lack of power around the obliquity cycle is surprising.

Author Response: We replaced the precise word "significantly" by the vaguer term "barely" which is adequate for our purpose here. It is true that the absence of a 41kyr cycle is no longer surprising, we have modified the sentence accordingly.

P13 L28-32: Is there any supporting evidence for the glacier variability?

Author Response: We added the reference Sugden et al., 2009 and Garcia et al., 2018 on South American glacial glacier variability.

P14 L1: If A reflects the amplitude of the variance a change indicates only a change in the variability, not in their overall abundance, please reformulate.

Author Response: Since dust is positive definite, a higher variability amplitude will produce a higher average. We added more explanation to this sentence: "Since the Argentinean continental shelf was still submerged at that moment and the outwash plains not yet fully extended, the higher dust emissions may have been due to a transformation in vegetation cover about 30 kyr after glacial inception, possibly accompanied by changes in glacial and periglacial processes in the Andes."

P14 L14-17: Please discuss this statement in the context of recent proxy and model studies that indicate fast Southern Hemisphere circulation changes during DO events. (Markle et al. 2016, Buizert et al. 2018, Pedro et al. 2018).

Author Response: We have expanded this paragraph, which now reads "The recuperation of vegetation cover would be more gradual, though, resulting in a saw-tooth shape of the dust spike that we do not observe in the data. Similarly, it has been suggested that rapid climate change in the Northern Hemisphere (e.g. Dansgaard/Oeschger events) would have synchronously changed the Southern Hemisphere atmospheric circulation and wind belts (Buizert et al., 2018; Markle et al., 2017). This could again have quickly changed the source or transport conditions, but would again have resulted in a saw-tooth shaped peak, either by steady regrowth of vegetation in the dust source areas, or as climate conditions in the north Atlantic gradually return to stadial (Pedro et al., 2018)."

P14 L24: Only ice cores have the intrinsic property that they become less resolved with increasing depth and thus age. Sediment cores are not affected by this. Please reformulate.

Author Response: In this sentence we only address the low (compared to the dust record used here) resolution of existing paleotemperature timeseries, not the change in resolution. We changed "temperature reconstructions" to "Pleistocene temperature reconstructions" to clarify.

P14 L30: The paper does not show that the data neither over-samples nor smoothes. Please either add this to the paper or remove this statement.

Author Response: We deleted that last sentence and changed the previous one to "…, we therefore took advantage of the unique EPICA Dome C dust flux dataset with 1 cm resolution measuring 320,000 cm, whose worst time resolution over the whole core is 25 years."

P15 L24: The reduction to the characterization of the different phase is a decision that the authors take, not an intrinsic property or the result of some analysis. Please reformulate this statement.

Author Response: We changed this sentence to "We addressed the task of statistically characterizing the cycles by primarily characterizing the phases' variability exponents H, C_1, q_D and amplitude A."

P15 L25: Missing dot between "A" and "We"

Author Response: Corrected

P15 L 26f: How is the variability of the dust flux at Dome C connected to the "activity" of the Patagonian ice sheet. Please extent and clarify this argument.

Author Response: We removed the mention to the Patagonian ice sheet and changed this sentence to "However, the low amplitude of dust variability during glacial inceptions indicates that vegetation cover and dust production processes did not significantly change until ~30 kyr after glacial inception."

Figures:

6b: The units for the axis are wrong.

Author Response: Yes, thanks, we also fixed the far-right expression for the number distribution. We have now indicated the units in the caption.

11: I suggest that the authors remove a couple of lines from the figure for clarity instead of the uncertainties or find a different way of visualizing the results.

Author Response: We responded to this suggestion above.

Spiky Fluctuations and Scaling in High-Resolution EPICA Ice Core Dust Fluxes

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Abstract.

Atmospheric variability as a function of scale has been divided in various dynamical “regimes” with alternating increasing and decreasing fluctuations: weather, macroweather, climate, macroclimate, megaclimate. Although a vast amount of data is available at small scales, the larger picture is not well constrained due to the scarcity and low resolution of long paleoclimatic time-series. Using statistical techniques originally developed for the study of turbulence, we analyse the fluctuations of a centimetric resolution dust flux time-series from the EPICA Dome C ice-core in Antarctica that spans the past 800,000 years. The temporal resolution ~~is 5 years over~~ranges from annual at the ~~last 400 kyrs, and~~top of the core to 25 years ~~over~~at the ~~last 800kyrs~~bottom, enabling the detailed statistical analysis and comparison of eight glaciation cycles, and the subdivision of each cycle into eight consecutive phases. The unique span and resolution of the dataset allows us to analyze the macroweather and climate scales in detail~~, i.e. fluctuations with periodicities from 1 year to 100,000 years. ~~.

We find that the interglacial and glacial maximum phases of each cycle showed particularly large macroweather to climate transition scale τ_c (around 2 kyrs), whereas mid-glacial phases feature centennial transition scales (average of 300 ~~yr~~yrs). This suggests that interglacials and glacial maxima are exceptionally stable when compared with the rest of a glacial cycle. The Holocene (with $\tau_c \approx 7.9$ kyrs) had a particularly large τ_c but it was not an outlier when compared with the phase 1 and 2 of other cycles.

We hypothesize that dust variability at larger (climate) scales appears to be predominantly driven by slow changes in glaciers and vegetation cover, whereas at small (macroweather) scales atmospheric processes and changes in the hydrological cycles are the main drivers.

For each phase, we quantified the drift, intermittency, amplitude, and extremeness of the variability. Phases close to the interglacials (1, 2, 8) show low drift, moderate intermittency, and strong extremes, while the “glacial” middle phases 3-7 display strong drift, weak intermittency, and weaker extremes. In other words, our results suggest that~~ despite the large climatic changes occurring during glacial-interglacial transitions,~~ glacial maxima, interglacials, and glacial inceptions were characterized by relatively stable atmospheric conditions, but punctuated by ~~more~~ frequent and severe droughts, ~~than during~~whereas the ~~more unstable~~ mid-glacial ~~conditions. The low amplitude during phases 6-8 also suggests that the Patagonian ice sheet~~climate was ~~not yet fully developed before 30 kyr after glacial inception.~~inherently more unstable.

1 Introduction

Over the late Pleistocene, surface temperature variability is strongly modulated by insolation, both at orbital (Jouzel et al., 2007), and daily time scales. In between these two ~~extremes~~scales, temperature variability has been shown to scale according to power-law relationships, thus evidencing a continuum of variability at all frequencies (Huybers and Curry, 2006). However, although a vast amount of high-resolution data exists for modern conditions, our knowledge of climatic variability at glacial-interglacial time scales is usually limited by the lower resolution of paleoclimatic archive records, thus restricting high frequency analyses during older time sections. Previous analyses using marine and terrestrial temperature proxies from both hemispheres suggest a generally stormier and more variable atmosphere during glacial times than during interglacials (Ditlevsen et al., 1996; Rehfeld et al., 2018).

One of the difficulties in characterizing climate variability is that ice core paleo-temperature reconstructions rapidly lose their resolutions as we move to the bottom of the ice column. Fig. 1 shows this visually for the EPICA Dome C Antarctic ice core temperature proxy (5787 measurements in all); the curve becomes noticeably smoother as we move back in time. In terms of data points, the most recent 100 kyr period has more than 3000 points (≈30-year resolution) whereas the most ancient 100 kyr period has only 137 (≈730 year resolution). This implies that while the most recent glacial-interglacial cycle can be perceived with reasonable detail, it is hard to compare it quantitatively to previous cycles - or to deduce any general cycle characteristics.

Fluctuation analysis (Lovejoy, 2017; Lovejoy and Schertzer, 2013; Nilsen et al., 2016), gives a relatively simple picture of atmospheric temperature variability (Fig. 2). The figure shows a series of regimes each with variability alternately increasing and decreasing with scale. From left to right we see weather scale variability, in which fluctuations tend to persist, building up with scale - they are unstable - increasing up to the lifetime of planetary structures (about 10 days), followed by a macroweather regime with fluctuations tending to cancel each other out, decreasing with scale, displaying stable behaviour. In the last century, anthropogenically forced temperature changes dominate the natural (internal, macroweather) variability at about 10- 20 years. In pre-industrial periods the lower frequency climate regime starts somewhere between 100 and 1000 years (the macroweather-climate transition scale τ_c~~).~~) indicating that new long frequency processes become dominant. The macroweather-climate transition scale marks a change of regime where the dominant high frequency processes associated with weather processes (and reproduced by GCMs in control runs) gives way to a new regime where the variability is dominated by either the responses to external forcings or to new, slow internal sources of variability. Further to the right of Fig. 2, we can see the broad peak associated with the glacial cycles at about 50kyrs (half the 100 kyr period) and then at very low frequencies, the megaclimate regime again shows increasing variability with scale. In between the climate and megaclimate regimes, the fluctuations decrease with scale over a relatively short range from about 100 kyrs to 500 kyrs. However, the temperature fluctuations shown in Fig. 2 display average behavior, which can potentially hide large variations from epoch to epoch. In this paper, we use a uniquely long and high-resolution paleo dataset to analyze the macroweather and climate scales in detail.

We focus on the EPICA Dome C dust flux record, which has a 55 times higher resolution than the ~~temperature~~deuterium record, including high resolution over even the oldest cycle (Lambert et al., 2012, Fig. 1). Antarctic dust fluxes are well correlated with temperature at orbital frequencies (Lambert et al., 2008; Ridgwell, 2003). But the fluxes are also affected by climatic conditions at the source and during transport (Lambert et al., 2008; Maher et al., 2010). ~~The analysis of~~ The dust ~~record presented~~data used here can therefore be thought of as a more “holistic” climatic parameter that includes not only temperature changes~~,~~ but describes atmospheric variability as a whole (including wind strength and patterns, and the hydrological cycle).

2 Method

In order to proceed to a further quantitative analysis of the types of statistical variability, and of the macroweather-climate transition scale, we need to make some definitions. A commonly used way of quantifying fluctuations is the Fourier analysis. It quantifies the contribution of each frequency range to the total variance of the process. However, the interpretation of the spectrum is neither intuitive, nor straightforward (section 2.3). The highly non-Gaussian spikiness – for both dust flux and its logarithm (e.g. Fig. 3b, c), implies strong – but stochastic - Fourier space spikes~~;~~. Indeed, (Lovejoy, 2018) found that the probability distribution of spectral amplitudes can themselves be power laws. This has important implications for interpreting spectra, especially those estimated from single series (“periodograms”): if the spectral amplitudes are highly non-Gaussian, then we will typically see strong spectral spikes ~~that are~~whose origin is purely random ~~in origin~~. This makes it very tempting to attribute quasi-oscillatory processes to what are in fact random spectral peaks. It therefore makes sense to consider the real (rather than Fourier) space variability (fluctuations). The problem here is that the spectrum is a second order statistical moment (the spectrum ~~in~~is the Fourier transform of the autocorrelation function). While second order moments are sufficient for characterizing the variability of Gaussian processes, in the more general and usual case - especially with the highly variable dust fluxes - we need to quantify statistics of higher orders~~.~~, in particular, the higher order statistics that characterize the extremes. Here, we will use two simple concepts to describe variability and intermittency (or spikiness) of the data.

The theoretical framework that we use in this paper is that of scaling, multifractals, the outcome of decades of research attempting to understand turbulent intermittency. Intermittent – spiky transitions – characterized by different scaling exponents for different statistical moments - turns out to be the generic consequence of turbulent cascade processes. Although the cascades are multiplicative, the extreme probabilities generally turn out to be power laws (Mandelbrot, 1974; Schertzer and Lovejoy, 1987) - not log-normals (as was originally proposed by (Kolmogorov, 1962)). The analyses are based on scaling regimes and their statistical characteristics. Because scaling is a symmetry (in this case invariance of exponents under dilations in time), in a dynamical regime in which two different components - such as temperature and dust - are strongly coupled parts of the system, each may have different scaling properties, but both should respect the scale symmetry including the transition scale at which the symmetry breaks down. Therefore, the broad conclusions of our dust flux analyses – scaling regimes and their break points, stability/~~instabilty~~instability - are expected to be valid for the more usual climate parameters including the temperature. Although it is beyond our present scope, we will explore the scale by scale relationship between EPICA dust fluxes and temperatures in a future publication.

2.1 Haar Fluctuations

The basic tool we use to characterize variability in real space is the Haar fluctuation, which is simply the absolute difference of the mean over the first and second halves of an interval:

$$\Delta F(\Delta t) = \frac{2}{\Delta t}\int_{t-\Delta t/2}^{t} F(t')dt' - \frac{2}{\Delta t}\int_{t-\Delta t}^{t-\Delta t/2} F(t')dt' \tag{1}$$

We can characterize the fluctuations by their statistics. For example, by analyzing the whole dataset using intervals of various lengths, we can thus define the variability as a function of scale (i.e. interval length). If over a range of time scales Δt, there is no characteristic time, then this relationship is a power law, and the mean absolute fluctuation varies as:

$$\left\langle \left| \Delta F(\Delta t) \right| \right\rangle \propto \Delta t^{H} \tag{2}$$

where "< >" indicates ensemble average, here an average over all the available disjoint intervals. A positive H implies that the average fluctuations increase with scale. This situation corresponds to unstable behavior identified with the climate regime. In contrast, when H is negative, variability converges towards a mean state with increasing scale. This is the situation found in the stable macroweather regime. Haar fluctuations are useful for the exponent range $-1<H<1$ which is valid for the dust series – and indeed for almost all geodata analyzed to date.

More generally, we can consider other statistical moments of the fluctuations, the "generalized structure functions", $S_q(\Delta t)$:

$$S_q(\Delta t) = \left\langle \left| \Delta F(\Delta t) \right|^q \right\rangle \propto \Delta t^{\xi(q)} \tag{3}$$

If the fluctuations are from a Gaussian process, then their exponent function is linear: $\xi(q) = qH$. More generally however, ξ(q) is concave and it is important to characterize this, since the nonlinearity in ξ(q) is due to intermittency, i.e. sudden, spiky transitions (for more details on Haar fluctuations and intermittency we refer to (Lovejoy and Schertzer, 2012)). We therefore decompose $\xi(q)$ into a linear and a nonlinear (convex) part $K(q)$, with $K(1)=0$:

$$\xi(q) = qH - K(q) \tag{4}$$

so that $K(q)$ =0 for quasi-Gaussian processes. Since the spectrum is a second order moment, the spectrum of a scaling process at frequency ω is a power law:

$$E(\omega) \approx \omega^{-\beta} \tag{5}$$

where the spectral exponent $\beta = 1+\xi(2) = 1+2H - K(2)$; $K(2)$ is therefore sometimes termed the "intermittency correction".

2.2 Intermittency

A simple way to quantify the intermittency is thus to compare~~,~~ the mean and Root Mean Square (RMS) Haar fluctuations:

$$S_1(\Delta t) = \left\langle \left| \left(\Delta F(\Delta t) \right) \right| \right\rangle \propto \Delta t^{\xi(1)} = \Delta t^{H} \tag{6}$$

$$S_2(\Delta t)^{1/2} = \left\langle \left(\Delta F(\Delta t) \right)^2 \right\rangle^{1/2} \propto \Delta t^{\xi(2)/2} = \Delta t^{H-K(2)/2} \tag{7}$$

with ratio:

$$S_1(\Delta t) / S_2(\Delta t)^{1/2} = \left\langle \left| \Delta F(\Delta t) \right| \right\rangle / \left\langle \left(\Delta F(\Delta t) \right)^2 \right\rangle^{1/2} \propto \Delta t^{K(2)/2} \tag{8}$$

where we estimate $S(\Delta t)$ using all available disjoint intervals of size Δt. These expressions are valid in a scaling regime. Since the number of disjoint intervals decreases as Δt increases, so does the sample size, hence the statistics are less reliable at large Δt.

For theoretical reasons (Lovejoy and Schertzer, 2013; Schertzer and Lovejoy, 1987), it turns out that the intermittency near the mean (q=1) is best quantified by the parameter $C_1 = K'(1)$. Since $K(1) = 0$ is a basic property, it turns out that for log-normal multifractals, (approximately relevant here) the ratio exponent $K(2)/2 \approx C_1$.

While the mean to RMS ratio is an intuitive~~,~~ statistic, it does not give a direct estimate of C_1: a more accurate estimate of C_1 uses the intermittency function $G(\Delta t)$:

~~(9)~~

$$G(\Delta t) = \lim_{\Delta q \to 0} \langle \Delta F \rangle \left[\frac{\langle \Delta F^{1-\Delta q} \rangle}{\langle \Delta F^{1+\Delta q} \rangle} \right]^{1/(2\Delta q)} \propto \Delta t^{\xi(1)-\xi'(1)} = \Delta t^{C_1} \tag{9}$$

(this is exact in the limit Δq->0) whose exponent is C_1. The intermittency exponent C_1 quantifies the *rate* at which the clustering near the mean builds up as a function of the range of scales over which the dynamical processes act; it only partially quantifies the spikiness. For this, we need other exponents, in particular the exponent q_D that characterizes the tails of the probability distributions. This is because scaling in space and/or time generically gives rise to power law probability distributions (Mandelbrot, 1974; Schertzer and Lovejoy, 1987). Specifically, the probability (*Pr*) of a random dust flux fluctuation ΔF exceeding a fixed threshold s is:

$$\Pr\left(\Delta F > s\right) \approx s^{-q_D}; \quad s >> 1 \tag{10}$$

Where the exponent q_D characterizes the extremes, for example, $q_D \approx 5$ has been estimated for wind or temperature (Lovejoy and Schertzer, 1986) and for paleotemperatures (Lovejoy and Schertzer, 2013) whereas $q_D = 3$ for precipitation (Lovejoy et al., 2012). A qualitative classification of probability distributions describes classical exponential tailed distributions (such as the Gaussian) as "thin tailed", log normal (and log-Levy) distributions as "long-tailed", and power law distributions as "fat tailed". Whereas thin and long tailed distributions have convergence of all statistical moments, power distributions only have finite moments for orders $q<q_D$.

2.3 How Fluctuations help interpret spectra

Although spectra may be familiar, their physical interpretations are nontrivial, a fact that was underscored in (Lovejoy, 2015). In a scaling regime – a good approximation to the macroweather and climate regimes discussed here – the spectrum is a power law form (eq. 5) where the spectral exponent β characterizes the spectral *density*. Although β tells us how quickly the variance changes per frequency *interval*, its physical significance is neither intuitive nor obvious. Integrating the spectrum over a frequency range is already easier to understand~~:~~; it is the total variance of the process contributed by the range. Therefore, we already see that β-1 (the exponent of the integrated spectrum) is more directly relevant than β. But even to understand this, we need to consider whether over a range of frequencies the process is dominated by either high or low frequencies. For this, we can compare the total variance contributed by neighboring octaves. For a power law spectrum, the variance ratio of one octave to its neighboring higher frequency octave is $2^{1-\beta}$. From this, we see that $\beta > 1$ yields a ratio $2^{1-\beta} < 1$ implying low frequency dominance whereas when $\beta < 1$, we have $2^{1-\beta} > 1$ and high frequency dominance.

But what does low frequency or high frequency "dominance" mean physically? For this, it is easier to consider the situation in real space using fluctuations; the simplest relevant fluctuations ~~being~~are the Haar fluctuations ΔF discussed in section 2.2 that ~~varies~~vary with time interval Δt as $\Delta F \approx \Delta t^H$. We saw that the exponents in real and spectral space were simply related by $\beta = 1+2H - K(2)$ where $K(2) > 0$ due to the spikiness (intermittency). This formula leads to two important conclusions. First, if we ignore intermittency (putting $C_1 = 0$, hence $K(2) = 0$) and assume that the mean fluctuations scale with the same exponent as the RMS fluctuations, then $H = (\beta - 1)/2$ showing again that it is the sign of $\beta - 1$ that is fundamental: $\beta > 1$ implies $H > 0$ hence fluctuations grow with scale and the process "drifts" or "wanders", it is unstable.

Conversely β < 1 implies $H < 0$ hence fluctuations decrease with scale and the process "cancels", "converges", it is "stable". The second conclusion is that if intermittency is strong (here we typically have $C_1 \approx 0.1$, $K(2) \approx 0.2$), then the relationship between the second and first order statistical moments is a little more complex so that for example, with these values and a β ≈ 0.9 we would have high frequencies dominating the variance (β < 1) but low frequencies dominating the mean ($H > 0$).

2.4 Dust Flux Data

The dust flux data used in this study is based on a linear combination of insoluble particles, calcium, and non-sea-salt calcium concentrations (Lambert et al., 2012). Because missing-data gaps in the three original datasets were linearly interpolated prior to the PCA, high frequency variability can sometimes be underestimated in short sections that feature a gap in one of the three original datasets. This occurs in about 25% of all dust flux data points, although half of those are concentrated in the first 760 m of the core (0-43 kaBP), when an older less reliable dust measuring device was used. Below 760m these occurrences are evenly distributed and do not affect our analysis. Due to the sometimes slightly underestimated variability, the analysis shown here is a conservative estimate (Lambert et al., 2012). Because missing data gaps in the three original datasets were linearly interpolated prior to the PCA, high frequency variability can sometimes be underestimated in short sections that feature a gap in one of the three original datasets. This occurs in about 25% of all dust flux data points, although half of those are concentrated in the first 760 m of the core (0-43 kaBP), when an older less reliable dust measuring device was used. Below 760m these occurrences are evenly distributed and do not affect our analysis. Due to the sometimes slightly underestimated variability, the analysis shown here is a conservative estimate.

~~3 Results~~

~~3.1 Looking at the data~~

Unlike water isotopes that diffuse and lose their temporal resolution in the bottom section of an ice core at high pressures and densities, the relatively large dust particles diffuse much less and have been used to estimate the dust flux over every centimetre of the 3.2 km long EPICA core (298,203 valid measurements, (Lambert et al., 2012)). The temporal resolution of this series varies from 0.81 years to 11.1 yrs (the averages over the most recent and the most ancient 100 kyrs respectively). The worst temporal resolution of 25 years per ~~centimeter~~centimetre occurs around 3050 m depth, with the result that at that resolution, there are virtually no missing data points in the whole record (Fig. 1).

~~Dust~~3 Results

3.1 Looking at the data

Polar dust flux measurements cannot be assigned to one particular atmospheric variable, like temperature for the water isotopes. At any given moment, the amount of dust deposited in East Antarctica will depend on the size and vegetation cover ~~at~~of the source region (mostly Patagonia for East Antarctic dust (Delmonte et al., 2008)), on the amount of dust available in the source region (can depend on the presence of glaciers), on the strength of the prevailing winds between South America and Antarctica, and the strength of the hydrological cycle (more precipitation will wash out more dust from the atmosphere (Lambert et al., 2008)). Over large scales it is thought that temperature-driven moisture condensation may be the major process driving low-frequency variability (Markle et al., 2018), although that may not globally be the case (Schüpbach et al., 2018)~~At low frequencies the dust variability will be driven by conditions at the source (presence of glaciers, vegetation cover), which is primarily driven by Southern Hemisphere temperature, explaining the high correlation between dust and temperature in ice cores. At high frequencies however, dust and temperature are decoupled and dust variability will be driven by changes in wind and the hydrological cycle.~~. High and low frequency variability in the dust flux record is likely driven by different processes. For examples, dust source conditions related to glaciers and vegetation cover may not have influenced high frequency variability due to their relatively slow rate of change. On the other hand, volcanic eruption or extreme events related to the hydrological cycle may produce high-frequency signals in the record. A single dust peak within a low background may therefore reflect a short-term atmospheric disturbance like an eruption or drought over South America or low precipitation over the Southern Ocean. The analysis presented here focuses heavily on the occurrence of dust fluctuations, the physical interpretation of which will depend on the scale of the phenomenon.

Fig. 3a shows a succession of 10 factors of 2 “blowdowns” (upper left to lower right at 11 different resolutions). In order to avoid smoothing, the data was “zoomed” in depth rather than time, but the point is clear: the signal is very roughly scale invariant, at no stage is there any sign of obvious smoothing, and the quasi-periodic 100 kyr oscillations is the only obvious time scale (we quantify this below). In comparison with more common paleoclimate signals such as temperature proxies~~,~~ - which are apparently smoother but with spiky transitions - the dust flux itself is already quite spiky. However, it also displays spiky transitions. In Fig. 3b we show the absolute change in dust flux and one can visually see the strong spikiness associated with strongly non-Gaussian variability: the intermittency. At each resolution, the solid~~ red~~ line indicates the maximum spike expected if the process was Gaussian, and the upper dashed lines the expected level for a (Gaussian) spike with probability 10^{-6}. Again, without sophisticated analysis, we can see that the spikes are wildly non-Gaussian, frequently exceeding the 10^{-6} level even though each segment has only 290 points, with the spikiness being nearly independent of resolution.

Taking the logarithms of the dust flux is a common practice since it reduces the extremes and makes the signal closer to the temperature and other more familiar atmospheric parameters. We therefore show the corresponding spike plot for the log transformed data (fig. 3c). Although the extreme spikes are indeed less extreme (see also fig. 6a, b), we see that the transformation has not qualitatively changed the situation with spikes still regularly exceeding (log) Gaussian probability levels of 10^{-5} and occasionally 10^{-8}.

3.2 Spectra

Figure 4 shows various spectral analyses (for the corresponding fluctuation analyses, see fig. 5). There is a clear periodicity at about (100 kyrs)$^{-1}$. In the double power law fit (line plot), the transition frequencies are a little lower: ω_0 = (160 kyr)$^{-1}$ (flux) and ω_c = (145 kyr)$^{-1}$ (log flux), although a Gaussian fit near the max gives a spike at (94±9 kyrs)$^{-1}$. Note that it is actually a little bit "wide" (two peaks) hence it is not perfectly periodic, and the amplitude is only about a factor 4 above the background. In comparison, the amplitude of the annual temperature frequency peak is several thousand times above the background (depending on the location) and is narrower (not shown).

Since this is a log-log plot, power laws appear as straight lines. We show in the figure the fits to the bi-scaling function

$$E(\omega)=\frac{a}{\left(\omega/\omega_0\right)^{\beta_h}+\left(\omega/\omega_0\right)^{\beta_l}} \qquad (11)$$

that smoothly transitions between a spectrum with $E(\omega)\approx\omega^{-\beta_h}$ at $\omega > \omega_0$ and $E(\omega)\approx\omega^{-\beta_l}$ at $\omega < \omega_0$. The figure shows the regressions with β_l ~~= ~~= -2.5, β_h = 1.7, and a = 7.5 (mg/m^2/yr)^{2}yr, $\omega_0 \approx$ (145kyrs)$^{-1}$ for the fluxes, and a = 0.375 ~~(mg/m^2/~~yr~~)^{2}yr^{-1}~~, $\omega_0 \approx$ (160kyrs)$^{-1}$ for the logarithms of fluxes. According to the figure, the high frequency climate regime scaling continues to about (300 yrs)$^{-1}$ before flattening to a very high frequency scaling ($\beta_m \approx 0.8$) "macroweather" regime (Lovejoy and Schertzer, 2013)~~. Note that this spectral transition scale is close but not identical to the transition scale estimated in real space with fluctuation analysis (around 250 years, Fig. 5).~~. The scaling exponents β_h = 1.7 and β_m =0.8 corresponding to the climate and macroweather regime respectively, may be compared with the values 2.1 and 0.4 for the EPICA paleotemperatures discussed in a future publication (compare however the red and black curves in Fig. ~~2). A review covering nearly 2 dozen β_h estimates for temperature proxies from both hemispheres was given in (see especially table 11.4; also). Although the dust and temperature exponents are not identical - implying scale-varying correlations – these results do support the use of dust as a proxy for atmospheric variability.~~2). These results show that temperature and dust variability are of the same statistical type so that it is likely that the dust signal is a real climate signal - yet the significant differences in their exponents shows that it has a different information content.

~~The plot graphically counterposes two views of the~~ The variability~~. Although we clearly see a~~ shown in Figure 4 can be interpreted broadly or in detail. A clear feature is the spectral maximum at around (100 kyrs)$^{-1}$~~,~~. The broad bispectral scaling model (eq. 11) of the peak already accounts for 96% of the spectral energy (variance) leaving only 4% for the (extra) contribution from the (near) (100kyrs)$^{-1}$ ~~Milankovitch~~orbital frequency~~. If it is argued that~~ (using the logarithm of the flux ~~is more physically relevant (blue spectrum) the situation is barely changed.~~changes little). Alternatively, ~~we may take~~with a narrow ~~spectral spike model that approximates the spectral spike near (100 kyr)$^{-1}$ as a~~ Gaussian shaped ~~profile. With this~~spectral spike model, the spike is localised at (94±9 kyrs)$^{-1}$ and contributes a total of 31% of the total variance. However, not all of this is above what we would expect from a scaling background; the exact amount depends on how the background is defined. For example, over the range from the 6th to the 11th highest frequencies in this discrete spectrum (from (133 kyrs)$^{-1}$ to (72 kyrs)$^{-1}$), in comparison to the background over this range, there is an enhancement of about 80% due to the strong peaks (the enhancement is about 100% for the 7th to the 12th frequencies). This means although the (94±9 kyr)$^{-1}$ peak represents 31% of the total variability over the range from (800 kyrs)$^{-1}$ to (25 yrs)$^{-1}$, it is only about 15% above the "background" (note that only 5% of the total variance is between (25yrs)$^{-1}$ and (1 kyr)$^{-1}$). We did not do the corresponding analysis for the (41 kyr)$^{-1}$ obliquity frequency since the figure 4 shows visually that it is barely discernable above the background.

The overall conclusion is that the background represents between 85% and 96% of the total variance.

3.3 Haar Fluctuation Analysis

Figure 5 shows ~~that~~the Haar fluctuations ~~have simple~~comparing their statistics for both dimensional and nondimensional cycles as well as for the mean and RMS fluctuations (bottom and top set of curves respectively). To start, let's consider the direct interpretations of the fluctuations in terms of the variability of the dust flux. Recall that when the fluctuations increase with scale, they represent typical differences whereas when then decrease with scale, they represent typical anomalies (deviations from long term mean values). For example, typical variations over a glacial-interglacial cycle (half cycle ≈ 50 kyrs) are about ±3mg/m^2/yr (i.e. a range of 6mg/m^2/yr, the dashed horizontal line~~). From the~~) whereas typical variations at the 250-year minimum are ≈ ±0.5mg/m^2/yr.

The macroweather, climate, and macroclimate regimes noted in fig. 4 are also clearly visible. In figure 5, we can clearly see ~~there is a~~the short regime with $H<0$ (up to about 250 yrs), a scaling regime with $H>0$ (up to glacial-interglacial periods ~~(≈50 kyrs) and finally a long time scale decrease in variability that is possibly (but not obviously) scaling. As expected, the regimes correspond to those indicated in Fig. 4 with the relation β=1+ξ(2) where~~ $\cancel{E(\omega) \approx \omega^{-\beta}}$ ~~and represent macroweather, climate, and macroclimate, respectively~~≈50 kyrs) and finally a long-time (possibly scaling) decrease in variability. The spectral and real-space statistics are linked via the relation $\beta=1+\xi(2)$ (see eqs. 4, 5). Starting with the high frequency macroweather regime, the exponents $H= -0.05$, $K(2) \approx 0.10$ correspond to $\beta= 0.8$ (fig. 4) and the real space macroweather - climate transition scale ($\tau_c \approx$ 250 yrs) is close to the spectral transition scale ($1/\omega_c \approx$ 300 years, Fig. 4). In the middle (climate) regime, the top (RMS) curves (slope 0.33) implies $\xi(2) = 0.66$, $\beta = 1.66$ which is close to the corresponding exponent in fig. 4 (β_h =1.7). Finally, at the longest (macroclimate) scales, the low frequency part of the spectrum in figure 4 (β_l = -2.5) implies that the fluctuation exponent $H \approx (\beta_l - 1)/2 = -1.75$. However, this is less than the minimum detectable by Haar analysis (H = -1); therefore, we expect the far-right slope to equal -1 (as shown by a reference line). To correctly estimate this steep slope, one must use other definitions of fluctuations. We could also note that the climate- macroclimate transition time scale is broad and a little shorter than the value spectral value $1/\omega_c$ estimated in fig. 4.

Beyond confirming the results of the spectral analysis and allowing for direct interpretations of the fluctuation values in terms of typical fluxes, Haar analysis also quantifies the intermittency from the convergence of the RMS and mean statistics at larger and larger time scales (see the clear difference in slopes shown in the climate regime: 0.38 versus 0.33). This underlines the limitation of spectral analysis discussed earlier, the fact that it is a second order statistic that is only a partial characterization of the variability. Finally, the figure also clearly shows that whether the cycles are defined in dimensional or nondimensional time that the statistical characterizations (including the exponents) are virtually unaffected.

Fig. 6a shows the fluctuation probabilities of the entire 800 kyr series at 25-year resolution~~.~~ (here the fluctuations are simply taken as absolute differences at 25-year resolution). We see that the large fluctuations (the tail) part of the distribution is indeed quite linear on a log-log plot with exponents $q_D \approx 2.75$ and 2.98 in time and depth respectively (both ~~fit~~from fits to the extreme 0.1% of the distributions). To get an idea of how extreme these distributions are, consider the depth distribution with $q_D = 2.98$. With this exponent, dust flux fluctuations 10 times larger than typical fluctuations occur only $10^{2.98} \approx 1000$ times less frequently. In comparison, for a Gaussian, they would be $\approx 10^{23}$ times less likely; they would never be observed.

While the dust fluxes are always positive~~ definite~~ and so cannot be Gaussians, the increments analyzed here could easily be approximately so. Nevertheless, a common way of trying to tame the spikes is by making a log transformation of fluxes. Fig. 4 already showed that this did not alter the spectrum very much; here it similarly has only a marginal effect. For example, Fig. 6b shows that the extreme tails on the log dust flux distribution has q_D =3.60 in time (25yrs) and 4.59 in depth (at 1cm resolution~~; this is close to the value $q_D \approx 5$ reported for both GRIP and Vostok paleotemperatures in).~~). The log-transformed variable still displays huge extremes with the extreme log flux corresponding to a log-Gaussian probability of 10^{-30} and 10^{-50} (time, depth respectively). Whether or not taking logarithms yields a more climate relevant parameter, it does not significantly change the problem of intermittency or of the extremes.

~~These power law fluctuations are so large that according to the classical assumptions, they would be outliers. While Gaussians are mathematically convenient and can be justified when dealing with measurement errors, in atmospheric science thanks to the scaling, very few processes are Gaussian. This has important applications in tipping point analysis, where noise induced tipping points are generally studied using well behaved white or Gaussian noise .~~

3.4 Phases

Scaling is a statistical symmetry. In our case, it means that *on average* the statistics at small, medium and large scales are the same in some way. The difficulty is that on a single realization – such as that available here, a single core from a single planet earth – the symmetry will necessarily be broken. For example, in the spectrum Fig. 4, in each of the proposed scaling regimes, scaling only predicts that the actual spectrum from this single core will vary about the indicated straight lines that represent the ensemble behaviour. Since this variability is strong, we made the potential scaling regimes more obvious by either averaging the spectrum over frequency bins (the red and blue spectra) – or by breaking the series into shorter parts and averaging the spectra over all the parts, effectively treating each segment as a separate realization of a single process (green). In any event, all that any empirical analysis can show is that the data are consistent with the scaling hypothesis.

This already illustrates the general problem: in order to obtain robust statistics we need to average over numerous realizations – and since here we have a single series, the best we can do is to break the series into disjoint segments and average the statistics over them~~.~~, assuming that the major underlying processes were constant over the last 800,000 years. Yet at the same time, in order to see the wide-range scaling picture (which also helps to more accurately estimate the scaling properties/exponents), we need segments that are as long as possible. The compromise that we chose between numerous short segments and a small number of long ones was to break the series into 8 glacial-interglacial cycles, and each cycle into 8 successive phases. As a first approximation, we defined eight successive 100kyr periods (hereafter called "segments", Fig. 7, top set), corresponding fairly closely to the main periodicity of the series. As we discussed, the spectral peak is broad implying that the duration of each cycle is variable – the cycles are only "quasi-periodic". It is therefore of interest to consider an additional somewhat flexible definition of cycles defining them as the period from one interglacial to the next (hereafter called "cycle", Fig. 7, bottom set). The break points were taken at interglacial optima: 0.4, 128.5, 243.5, 336, 407.5, 490, 614, 700, 789 kyrs BP, i.e. 96.9±18.7 kyrs per cycle. Using ~~this~~the latter definition, the cycles were nondimensionalized so that nondimensional time was defined as the fraction of the cycle, effectively stretching or compressing the cycles by ±19%.

With either of these definitions, we have 8 segments or cycles, each with 8 phases. Note that in our nomenclature, phase 1 and 8 are the youngest and oldest phases, respectively, and that time flows from phase 8 to phase 1. Fig. 8 shows the phase by phase information summarized by the average flux over each cycle including the dispersion of each cycle about the mean (for the segments in the top set, and the cycles in the bottom set). We see that the variability is highest in the middle of a cycle and lowest at the ends.

~~Since~~ The spectra ~~in Fig. 4~~ showed that there were wide scale ranges that are on average scale invariant – power laws ~~–~~ – and in Fig. 4 quantified the glacial – interglacial cycle. We are thus interested in characterizing the scaling properties over the different phases~~.~~ of the cycle; for this we turn to real space statistics. In Fig. 9 we ~~compares~~compare the statistics averaged over cycles and the statistics averaged over phases. The figure shows that the phase to phase differences are much more important than the cycle to cycle differences. $\left\langle \left| \left(\Delta F(\Delta t) \right) \right| \right\rangle$ ~~Particularly noticeable are the phase to phase differences in the average fluctuations (lower left).~~ (lower left).

From the global statistics (e.g. Figs. 4, 5), it is clear that in each glacial-interglacial cycle there are two regimes, so that before characterizing the structure functions by their exponents (e.g. $H = \xi(1)$ for the mean fluctuations), we have to determine the macroweather-climate transition time scale τ_c whose average (from Fig. 4, 5) is 250-300 years.

One way of estimating the transition scale τ_c is to make a bilinear fit of $\log_{10}S_1(\Delta t)$ (i.e. Haar with $q = 1$, the mean absolute fluctuation) with the mean slopes -0.05 (small Δt) and slope +0.~~25~~35 (large Δt; the values were chosen because they are roughly the H estimates from the average over all the cycles) (Fig. ~~10).~~9). The hypothesis here was that there were two regimes, each characterized by a different exponent each of which was estimated from the ensemble statistics. Therefore, the analysis only needed to estimate the scale at which the low frequency process exceeded the high frequency one. Bilinear fits were made for each phase of each segment (blue) as well as for each phase of each cycle (black). For each phase there were thus 8 transition scales, which were used to calculate the mean and its standard deviation, (shown here as representative black arrows). From the figure we see that at first (phases 8-3) the transition scale is relatively short (250-400 yr), but that it rapidly moves to longer (1 – 2 kyrs) scales for the final phases 2 and 1. The average transition scale over all phases is around 300 years.

The figure shows that our results are robust since the results are not very different using dimensional and nondimensional time (segments and cycles). Comparing the blue and black curves, we see that in all cases the late phases have much larger τ_c than the early and middle phases. Also shown in Fig. 10 (dashed) is a plot of the break points estimated by a more subjective method that attempts to visually determine a break point on $\log S_1 - \log \Delta t$ plots. Again, we reach the same conclusion with quantitatively very similar results: a transition of millennia for phases 1 and 2, and a few centuries in the middle of the cycle. The cycle average value ($\tau_c \approx 300$ years) is therefore not representative of the latest phases where τ_c is many times larger (glacial maxima and interglacials). The Holocene has an even larger transition scale (τ_c = 7.~~9kyrs~~9 kyrs, marked by an X in Fig. 10), but it lies just outside the standard deviation of the first nondimensional phases (red arrows in Fig. 10). Although the Holocene value of τ_c is the largest in phase 1, it corresponds to 1.55 standard deviations above the mean with (assuming a Gaussian variability) a corresponding p value of 0.12, roughly the expected extreme of a sample of 8; it is therefore not a statistical outlier.

Alternatively, rather than fixing a phase and determining the variation of the mean fluctuation and intermittency function (Fig. ~~10~~9), we can consider the variation of the Haar fluctuations at fixed time scales and see how they vary from phase to phase (Fig. 11). The figure shows the phase to phase variation of Haar fluctuations at 50, 100, 200, 400, 800, 3300, 7000 years scales (bottom to top~~).~~; the dashed and solid lines alternate to demarcate the different curves, they are not uncertainties). Over the macroweather regime (up to about 400 ~~to~~ 800 years) the fluctuations tend to cancel so that the variability is nearly independent of time scale. In contrast, once we reach the longer scales in the climate regime (~~800~~up to 7000 years), the fluctuations increase noticeably as the time interval Δt is increased. For every time scale, there is a clear cyclicity (left to right), with fluctuation amplitudes largest in the middle phases. We note that the cycle to cycle variability is fairly large; about a factor of 2 (for clarity the error bars indicating this cycle to cycle spread were not shown).

Finally, we describe for each phase the drift tendency and the intermittency, as well as fluctuation amplitude and extremeness of the data. In Figure 12 we show the result on the nondimensional phases of the range 500 years < Δt < 3000 years, (upper left and right; the range was chosen to be mostly in the climate regime, i.e. with $\Delta t > \tau_c$, and it was fixed so as to avoid any uncertainty associated with the algorithm used to estimate τ_c). Recall that the fluctuation exponent $H > 0$ quantifies the rate at which the average fluctuations increase with time scale. Similarly, the exponent C_1 characterizes the rate at which the spikiness near the mean (the intermittency exponent) increases with scale. We see (upper left) that H is fairly high in the early phases with H reaching small value in the later phases (with H actually a little bit negative on average in phase 1 due to the large τ_c value). C_1 on the other hand (upper right) decreases a bit in the middle the phases. The error bars show that there is quite a lot of cycle to cycle variability.

If H quantifies the "drift" and C_1 the "spikiness", then Fig. 12 shows that the early phases have high drift and medium spikiness, the middle phases have high drift and lower spikiness, while phases 1-2 have low drift but medium spikiness. To understand this better, consider the transition time scales in Fig. 10. The youngest 2 phases with the low drift and spikiness are also the phase with the longest transition scales. This means that the rate at which the variability builds up is small and that it only builds up over a short range of scales (from τ_c to roughly Δt = 50 kyrs, the half cycle duration, this can be checked on Fig. 9 that shows the phase by phase structure functions and intermittency functions). Conversely, phases 3 and 4 with high drift and high intermittency also have a smaller τ_c so that both the fluctuations and spikiness build up faster (Fig. 11) and over a wider range of scales (Fig. 10).

Another useful characterisation of the phases is to directly consider the flux variability at a fixed reference scale, taken here as the 25-year resolution; quantifying the amplitude of the variability of each segment by its standard deviation A at 25yr time scale (Fig. 12, lower left). This is not the *difference* between neighbouring values or ~~fluctuation~~fluctuations (as in figure 11), it is rather the variability of the series itself at 25-year resolution. For each of the phases, we have 8 estimates (one from each cycle); these are used to calculate the mean (~~black~~central solid line) and standard deviation shown by the error bars showing the cycle to cycle dispersion of the values. We can see that the amplitude of the 25 yr scale fluctuations is about four times higher in the middle of the ice age (phase 4) than at interglacial (phase 1). The figure clearly shows the strong change of variability across the cycle.

Whereas C_1 characterizes the intermittency near the mean, we have seen that the probability exponent q_D characterizes the extreme spikiness. ~~Fig. 12 lower right, compares q_D phase by phase. Recalling that small q_D implies more extreme extremes, we see that the extremes are stronger in the beginning and end of the cycle, and somewhat less pronounced in the middle phases of the cycle (note the overall mean is 2.62±0.42, this can be compared to the value q_D =3.60 for the overall log transformed data, fig. 6b). Notice that for phase 8, q_D=2.03 (the mean); this is close to the value q_D = 2 below which the extremes are so strong that the variance (and hence spectrum) does not converge.~~ An extreme (low) exponent q_D phase implies that most of the time the changes in flux are small, but occasionally, there are huge transitions. Conversely, a high (less extreme) q_D implies that there is a wider range of different flux changes so that most of the changes tend to be in a restricted range. Fig. 12 lower right compares q_D phase by phase. The smaller the value of q_D, the extreme fluctuations are more and more extreme relative to the typical ones. Therefore, from the figure, we see that the extremes are stronger in the beginning and end of the cycle, and somewhat less pronounced in the middle phases of the cycle (note the overall mean is 2.62±0.42, this can be compared to the value q_D =3.60 for the overall log transformed data, fig. 6b). Notice that for phase 8, q_D = 2.03; this is close to the value q_D = 2 below which the extremes are so strong that the variance (and hence spectrum) does not converge. Summarizing, we can now categorize the phase by phase spikiness as: extremes strong, and medium spikiness (phases 1, 2, 8), and extremes intermediate and low spikiness (phases 3-7). For the cycle to cycle estimates (not shown), the value q_D =2.75±0.41, seems to be fairly representative of all the cycles, although there is a slight tendency for q_D to decrease for the older cycles implying that they may have been a bit more extreme than the recent ones.

4 Discussion

An attractive aspect of dust fluxes is that they are paleo indicators with unparalleled resolutions over huge ranges of temporal scales. However, they come with two difficulties. First, their ~~physical~~dynamical interpretation is not ~~clear: while~~unambiguous: they depend on temperature, wind, and precipitation~~, and so are holistic climate indicators, the precise climate significance of~~: dust flux variability is hard to ~~nail down~~attribute to a specific process, it is a holistic climate indicator. Second, their appearance as a sequence of strong spikes is unlike that of any of the familiar proxies. Indeed, we argued that their highly spiky (intermittent) nature (i.e. with C_1>0) is outside the purview of conventional statistical frameworks including autoregressive, moving average or more generally of quasi-Gaussian or even quasi log-Gaussian processes.

Due to the dominance of the continuum~~,~~ (spectral, background) variability, physical interpretations must be based on an understanding of climate variability as a function of scale. We ~~will~~ first consider overall analyses over the whole dust flux series, and then focus on the phases. The spectral analysis (Fig. 4) is the most familiar and for the dust fluxes, it is qualitatively similar to previous results obtained with temperature data, although temperature spectra with anything approaching the resolution of Fig. 4 are only possible over the most recent glacial cycle. The most striking spectral feature is the peak over the background at 100 kyr periodicity. The broadness of this peak already indicates the irregularity of the Earth system response to the eccentricity-forced ~~Milankovitch~~orbital cycles. ~~More surprising is~~ The (near) absence of obliquity frequencies at 41 kyr~~.~~ is notable and is consistent with the corresponding analysis of paleotemperatures. Although there is definitely power in that frequency range, it is ~~not significantly~~barely larger than the background continuum, suggesting a low ~~internal feedback~~response to that forcing. Finally, our high-resolution data allows us to discern two different power-law regimes, one at low frequencies with an exponent β = 1.7, and one at high frequencies with exponent β = 0.8, with the transition between the two at around 300 years.

In section 2.3, we discussed some of the difficulties inherent in interpreting spectra and showed that the exponent of the integrated spectrum β-1 is more directly relevant than ~~β.~~β (ignoring intermittency, this is the same as the wandering/cancelling criterion H>0 or H<0). Applying this understanding to the dust exponents, we see that in macroweather, there is a weak high frequency dominance (1-β ≈ 0.2 > 0) whereas the climate regime is dominated by low frequencies (1-β ≈ -0.7 < 0). A plausible physical explanation is that over long periods of time (at ~~large~~climate regime scales), the amount of dust in the SH atmosphere is driven by changes in glacier and vegetation coverage, which is itself forced by SH temperature change. There is therefore a very strong correlation between dust and temperature at climatic scales~~. At higher frequencies in the macroweather regime~~ (indeed, elsewhere we show that the correlation increases from very low to very high over the entire climate regime from 500 to 50 kyrs). At higher (macroweather) frequencies, temperature oscillations are too fast to overcome the inertia of ice sheet and vegetation responses~~.~~, dust and temperature correlations are very low. Instead, dust deposition in Antarctica will be more sensitive to temporary atmospheric disturbances in the winds and the hydrological cycle.

To interpret the analysis by phase of the dust record (Fig. 12) one must understand the significance of A and of the exponents H, C_1, and q_D in the context of dust deposition. The H exponent and the amplitude A are directly linked to ~~oscillations/~~mean fluctuations~~.~~ and values, A being the standard deviation ($<F^2>^{1/2}$) of the dust flux variability at a fixed (here, 25 year) time scale whereas H determines the rate at which the flux fluctuations ($(<\Delta F(\Delta t)^2>^{1/2})$) change with time scale Δt. We saw that ~~the~~a positive H-exponent signifies a tendency to "drift", ~~meaning that~~whereas when $H< 0$, the dust ~~oscillations will~~fluctuations tend to cancel each other out and the record will cluster around a mean value. In contrast, $H > 0$ indicates that the dust fluxes will not cluster around a mean value, in essence, the process wanders and does not stay constant~~.~~, it appears to be unstable. The low H numbers during phases 1 and 2 (interglacial and glacial maximum) indicate a very constant, stable climatic state, with Patagonian dust production being either very low during interglacials (low glacier activity, large vegetation cover) or very high (Patagonian ice cap fully grown, large outwash plains on the Argentinean side). In contrast, the high H and amplitude A values during the mid-glacial may have been due to strong variability in glacier extent during that time~~, and therefore~~ (García et al., 2018; Sugden et al., 2009), and therefore a very variable dust supply (see also Fig. 11 that shows how the amplitude of the fluctuations at different time scales varies with the phase). The glacial inception (phases 7 and 8) features low A but a high H exponent. This implies that the mean dust level was highly variable, but the dust supply was still low, thus not allowing for large amplitude fluctuations. The higher amplitudes in phases 6 and 7 indicates that dust supply became abundant then~~, which suggests that the Patagonian ice cap had already grown to a substantial size about 30 kyr after glacial inception~~. Since the Argentinean continental shelf was still submerged at that moment and the outwash plains not yet fully extended, the higher dust emissions may have been due to a transformation in vegetation cover about 30 kyr after glacial inception, possibly accompanied by changes in glacial and periglacial processes in the Andes.possibly accompanied by changes in glacial and periglacial processes in the Andes.

The exponents C_1 and q_D are associated with the intermittency, or spikiness of the data. ~~While~~ C_1 is a measure of the sparseness (or degree of clustering) of the mean-level spikes (i.e. whose amplitudes contribute most to the mean~~,~~ spike level), it is equal to one minus the fractal dimension of the set of spikes that exceed the mean level ($D_1 = 1- C_1$). q_D characterizes how extreme ~~these~~the most extreme spike values are. The dust flux record is generally more intermittent (with sparser, more clustered spikes, larger C_1) in phases 8, 1, and 2 (glacial inception, interglacial, glacial maximum) than in the mid-glacial, with also more extreme spike values~~. Since~~ (lower q_D). These power law fluctuations implied by the ~~C_1~~low values of q_D are so large that according to the classical assumptions, they would be outliers. While Gaussians are mathematically convenient and ~~q_D~~can be justified when dealing with measurement errors, in atmospheric science thanks to the scaling, very few processes are Gaussian. This has important applications in tipping point analysis, where noise induced tipping points are generally studied using well behaved white or Gaussian noise (e.g. Dakos et al., 2012)

The exponents ~~are calculated from the derivative of the signal~~characterize the variability of the dust signal over a wide range of scales. To understand the two scaling regimes, it may be helpful to recall that the ice core dust signal depends on both the variability of the dust source and that of the overall climate system. For example, a spike in the dust source and a fast change in the system state (e.g. Dansgaard-Oeschger ~~event~~-DO- events in the NH) ~~will~~could both produce a similar signal. However, ~~such~~in the SH, fast changes in system state – such as the DO events in the NH - apparently do not occur in the SH where the corresponding ~~signal to NH DO events is~~signals are more triangular and gradual in shape. ~~We therefore interpret the C_1 and q_D exponents as purely indicative of spikes in the dust signal. A short large spike (<100 years)~~High frequency variations in dust deposition ~~cannot be associated to~~ (at scales in the macroweather regime) are thus likely to be dominated by dust source dynamics rather than ice sheet changes ~~which~~that have generally larger reaction times. ~~Its origin is therefore~~ One hypothesis is that the transition time scale τ_c is the scale at which the source variability – that decreases with scale (H<0) - becomes less than the system variability that increases with scale (H>0). The macroweather variability is therefore likely due to dominated by vegetation and/or atmospheric changes. Large-scale natural fires could alter the landscape in a very short time, allowing for more dust uptake by the winds and a sudden rise in atmospheric dust. The recuperation of vegetation cover would be more gradual, though, resulting in a saw-tooth shape of the dust spike that we do not observe in the data. Similarly, it has been suggested that rapid climate change in the Northern Hemisphere (e.g. DO events) would have synchronously changed the Southern Hemisphere atmospheric circulation and wind belts (Buizert et al., 2018; Markle et al., 2017). This could again have quickly changed the source or transport conditions but would again have resulted in a saw-tooth shaped peak, either by steady regrowth of vegetation in the dust source areas, or as climate conditions in the north Atlantic gradually return to stadial (Pedro et al., 2018).

Finally, we could mention volcanoes. Volcano eruptions usually saturated the dust measuring device and were mostly cut from the record. Using the sulphate record to identify eruptions is tricky because many large sulphate peaks do not have a corresponding dust peak. This means that even if you do have matching dust and sulphate peaks, it could be an eruption or a coincidence. Therefore, the influence of volcanic variability on the results cannot be completely eliminated, although our key results are fairly robust with respect to the phase of the cycle and are therefore unlikely to be influenced by volcanic eruptions.

Although the spikes occur at all scales (see fig. 3), the most likely explanation for the (shorter) macroweather scale dust spikes is disturbances in the atmosphere, involving either the winds or the hydrological cycle (or both at the same time). The obvious candidate for a perturbation that would lead to increased dust in the atmosphere is drought. We will therefore interpret macroweather dust spikes as multiannual to multidecadal, multicentennial drought events in southern South America. With this interpretation, we can conclude that glacial maxima, interglacials, and glacial inceptions were characterized by more frequent and more severe drought events than during the mid-glacial. During glacial maxima, such extreme dust events could have contributed to Southern Hemisphere deglaciation by significantly lowering ice sheet albedo at the beginning of the termination (Ganopolski and Calov, 2011). In contrast, more frequent dust events could have contributed to glacial inception through negative radiative forcing of the atmosphere.

Since the C_1 and q_D exponents characterize the abruptness of changes in the signal, a spike and a fast change in the system state (e.g. DO event in the NH) will both produce a similar signal. However, such fast changes in system state do not occur in the SH where the corresponding signal to NH DO events is more triangular and gradual in shape. We therefore interpret the C_1 and q_D exponents as purely indicative of spikes in the dust signal. A short large spike ($<\tau_c$) in dust deposition cannot be associated with ice sheet changes which have generally larger reaction times. Its origin is therefore likely due to vegetation and/or atmospheric changes. Large-scale natural fires could alter the landscape in a very short time, allowing for more dust uptake by the winds and a sudden rise in atmospheric dust. The recuperation of vegetation cover would be more gradual, though, resulting in a saw-tooth shape of the dust spike that we do not observe in the data. Similarly, it has been suggested that rapid climate change in the Northern Hemisphere (e.g. DO events) would have synchronously changed the Southern Hemisphere atmospheric circulation and wind belts (Buizert et al., 2018; Markle et al., 2017). This could again have quickly changed the source or transport conditions, but would again have resulted in a saw-tooth shaped peak, either by steady regrowth of vegetation in the dust source areas, or as climate conditions in the north Atlantic gradually return to stadial (Pedro et al., 2018).

The most likely explanation for a dust spike is therefore a short-term disturbance in the atmosphere, involving either the winds or the hydrological cycle (or both at the same time). The obvious (but not exclusive) candidate for a perturbation that would lead to increased dust in the atmosphere is drought. As a first approximation we will therefore interpret short dust spikes as multiannual to multidecadal drought events in southern South America. With this interpretation, we can conclude that glacial maxima, interglacials, and glacial inceptions were characterized by more frequent and more severe drought events than during the mid-glacial. During glacial maxima, such extreme dust events could have contributed to Southern Hemisphere deglaciation by significantly lowering ice sheet albedo at the beginning of the termination (Ganopolski and Calov, 2011). In contrast, more frequent dust events could have contributed to glacial inception through negative radiative forcing of the atmosphere.

5 Conclusions

Until now, a systematic comparison of the different glacial-interglacial cycles has been hindered by a limitation of the most common paleoclimate indicators – the low resolution of Pleistocene temperature reconstructions from ice or marine sediment cores. Due to this intrinsic characteristic, the older cycles are poorly discerned; we gave the example of EPICA paleo temperatures whose resolution in the most recent cycle was 25 times higher than the resolution in the oldest one. In this paper, we therefore took advantage of ~~a~~the unique EPICA Dome C dust flux dataset with 1 cm resolution measuring 320,000 cm~~. The most recent four cycles were discerned at 5 year~~, whose worst time resolution ~~throughout (20,000 points per cycle) and the entire record of eight glaciations could be resolved at~~over the whole core is 25 years~~, and this, without signs of over-sampling or smoothing.~~.

Dust fluxes are challenging not only because of their high resolutions, but also because of their unusually high spikiness (intermittency) and their extreme transitions that occur over huge ranges of time scales. Standard statistical methodologies are inappropriate for analyzing such data. They typically assume exponential decorrelations (e.g. autoregressive or moving average processes) that have variability confined to narrow ranges of scale. In addition, they assume that the variability is quasi Gaussian or at least that it can be reduced to quasi Gaussian through a simple ~~transformations~~transformation of variables (e.g. by taking logarithms). In this paper, using standard spectral and probability distribution analysis, we show that both the spectral and the probability tails were power laws, not exponential, requiring nonstandard approaches.

The high resolution of the data allowed us to not only quantitatively compare glacial-interglacial cycles with each other, but also to subdivide each cycle into 8 successive phases that could also be compared to one another. One of the key findings was that there was a great deal of statistical similarity between the different cycles and that within each cycle there were systematic variations of the statistical properties with phase. These conclusions would not have been possible with the corresponding much lower resolution temperature proxy data.

Our variability analysis using real space (Haar) fluctuations confirmed that the majority of the variability was in the macroweather and climate scaling regime "backgrounds" with an average transition scale τ_c of about 300 years. In the climate regime (time scales above τ_c), dust variability is more affected by long-term hemispheric-wide climate changes affecting slow response subsystems like glaciers and vegetation~~.~~, which explains the high correlation of dust and temperature at these scales. In contrast, dust variability in the macroweather regime (time scales below τ_c) would have been more influenced by short-term atmospheric perturbations~~.~~ such as droughts and precipitation minima.

Using various techniques, τ_c was found to be systematically larger in the youngest two phases than in the middle and oldest phases; about 2 kyrs but with nearly a factor of 4 cycle to cycle spread and equal to 300 years (with a factor of 2 spread) for the six remaining phases. For the Holocene, τ_c was found to be 7.9 kyrs, which makes it an exceptionally stable interglacial, but not a statistical outlier compared to other interglacials. Similarly, the typical (RMS) variation in flux amplitude was smaller in the early phase increases by (on average) a factor of 4 from $\pm$0.13 mg/m^2/yr to about $\pm$0.5 mg/m^2/yr in the middle and later phases. The Holocene (with an amplitude of ±0.08 mg/m^2/yr) was again particularly stable with respect to the phase 1 of other cycles, but it was not an outlier.

We addressed the task of statistically characterizing the cycles ~~reduced~~by primarily ~~to the problem of~~ characterizing the phases' variability exponents H, C_1, q_D and amplitude A. We show that the atmosphere was relatively stable during glacial maxima and interglacials, but highly variable during glacial inception and mid-glacial. However, the low amplitude of dust variability during glacial inceptions indicates that ~~the Patagonian ice sheet was~~vegetation cover and dust production processes did not ~~very active~~significantly change until ~30 kyr after glacial inception.

We interpret the intermittency indicators as suggesting a higher frequency of drought events and more severe droughts during glacial inception, interglacials, and glacial maxima than during mid-glacial conditions. These short-term spikes in atmospheric dust could have helped trigger southern hemisphere deglaciation through albedo feedback of ice-sheet surfaces, or glacial inception through negative radiative forcing.

The results presented in this paper are largely empirical characterisations of a relatively less known source of climate data: dust fluxes. Dust flux statistics defy standard models: they require new analysis techniques and better physical models for their explanation. These reasons explain why our results may appear to be rough and approximate. Readers may nevertheless wonder why we did not provide standard uncertainty estimates. But meaningful uncertainties can only be made with respect to a theory and we have become used to theories that are deterministic, whose uncertainty is parametric, and that arises from measurement error. The present case is quite different: our basic theoretical framework is rather a stochastic one, it implicitly involves a stochastic "earth process" that produces an infinite number of statistically identical planet earths of which we only have access to a single ensemble member. From this single realization, we neglected measurement errors and estimated various exponents that characterized the statistical variability over wide ranges of time scale, realizing that the exponents themselves are statistically variable from one realization to the next. In place of an uncertainty analysis, we therefore quantified the spread of the exponents (which themselves quantify variability). In the absence of a precise stochastic model we cannot do much better.

This paper is an early attempt to understand this unique very high-resolution data set. In future work, we will extend our methodology to the EPICA paleo temperatures and to the scale by scale statistical relationship between the latter and the dust fluxes.

5 Acknowledgements

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Paleoclimate. MN Paleoclimate is supported by the Millennium Scientific Initiative of the Ministry of Economy, Development and Tourism (Chile). ~~Data~~The dust flux data is available here: https://doi.pangaea.de/10.1594/PANGAEA.779311

Figures

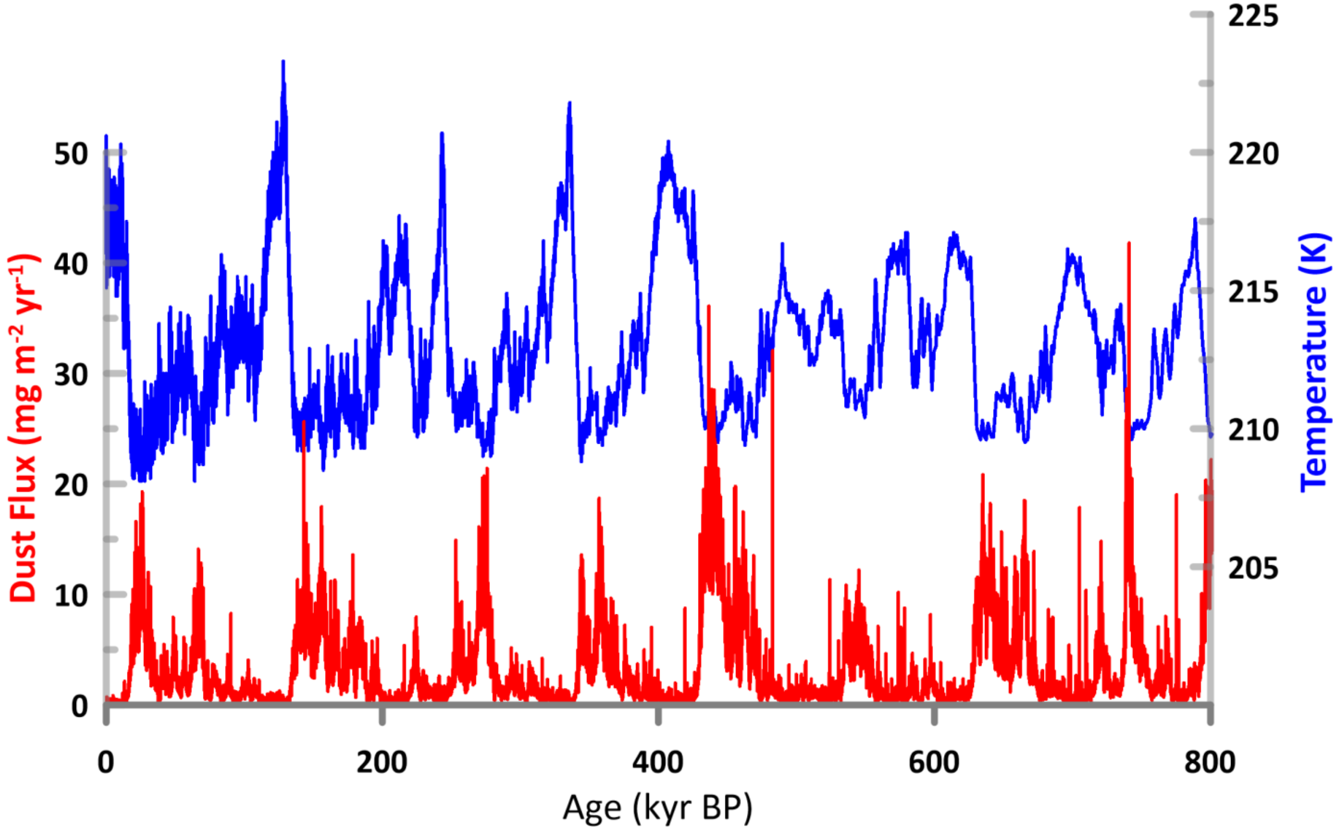
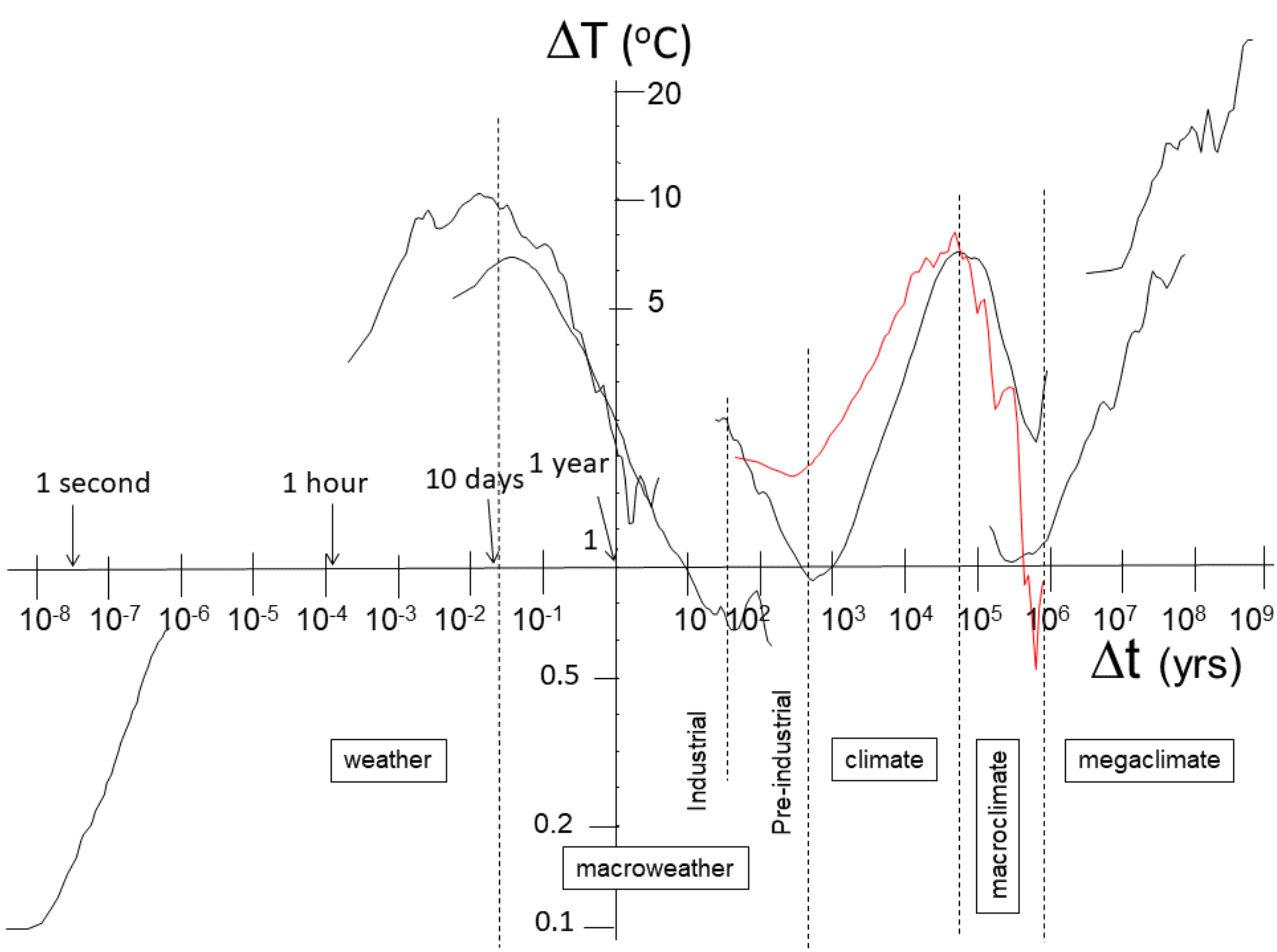


Figure 1: Temperature (blue) and dust flux (red) from the EPICA Dome C ice core (Jouzel et al., 2007; Lambert et al., 2012). The dust flux time series has 32,000 regularly spaced points (25-year resolution), the temperature series, has 5,752 points. The temperature data are irregularly spaced~~,~~ and lose resolution as we go back into the past (number of temperature data points in successive ice ages: 3022, 1117, 521, 267, 199, 331, 134, 146). In both cases we can make out the glacial cycles, but they are at best only quasi-periodic.



ΔT (°C)
20
10
5
1
0.5
0.2
0.1
1 second
1 hour
10 days
1 year
10-8
10-7
10-6
10-5
10-4
10-3
10-2
10-1
10
102
103
104
105
106
107
108
109
Δt (yrs)
weather
macroweather
Industrial
Pre-industrial
climate
macroclimate
megaclimate

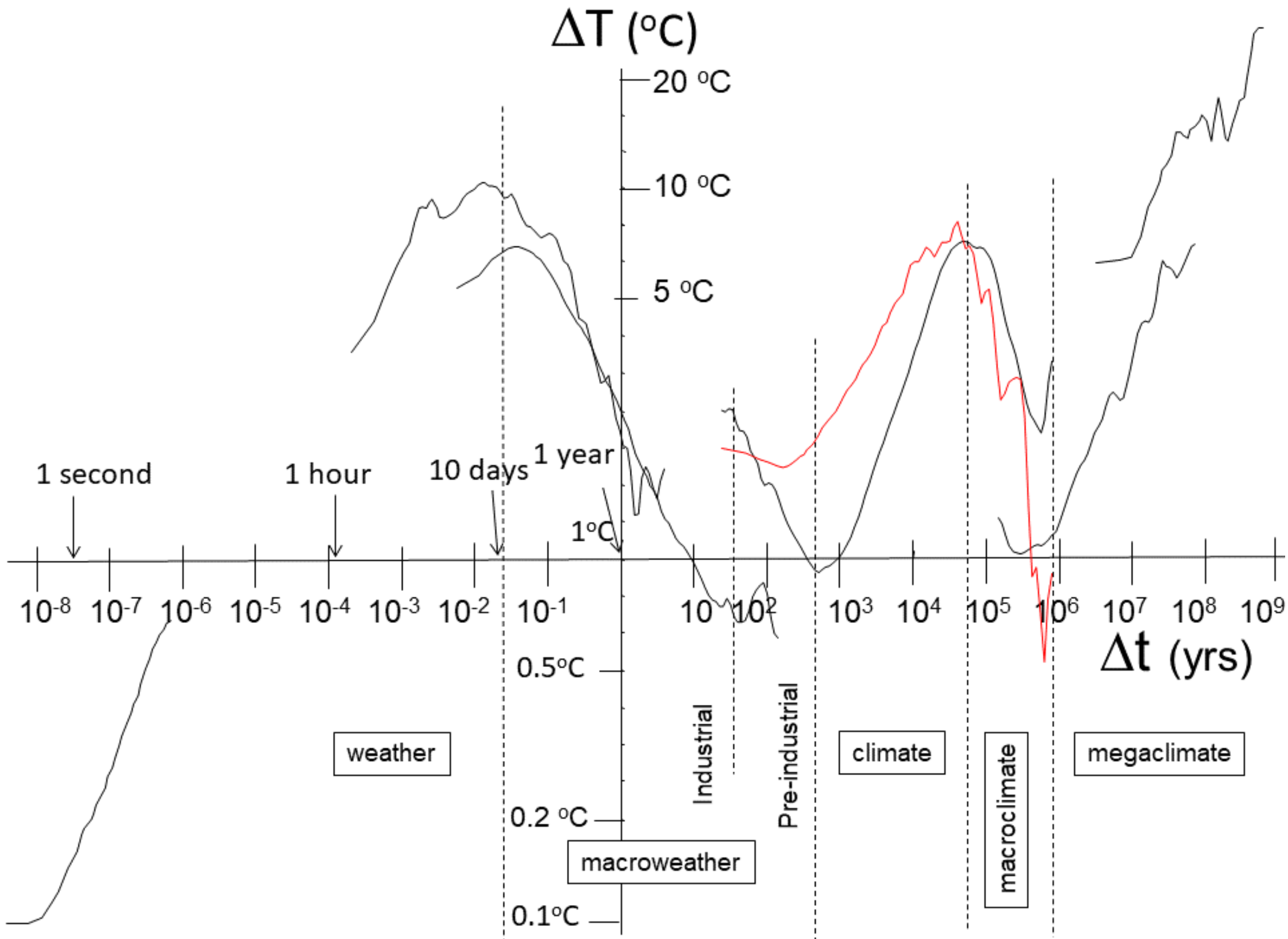


Figure 2: A composite showing root mean square (RMS) Haar fluctuations (ΔT in units of °C) black, and RMS dust fluctuations analysed in this paper (red, in units of mg/m^2/yr, (Lambert et al., 2012)). From left to right: thermistor temperatures at 0.0167s resolution (Lovejoy, 2018) , hourly temperatures from Landers Wyoming (Lovejoy, 2015) , daily temperatures from 75 °N (Lovejoy, 2015), EPICA Dome C temperatures (Jouzel et al., 2007), and two marine benthic stacks (Veizer et al., 1999; Zachos et al., 2001). The macroweather-climate transition is not in phase between the different records because the left ones (industrial side) are influenced by anthropogenic climate change, while the right data is pre-industrial natural variability. As elsewhere in this paper, the fluctuations were multiplied by the canonical calibration constant of 2 so that when the slopes are positive, the fluctuations are close to difference fluctuations. The various scaling regimes are indicated at the bottom. Adapted from (Lovejoy, 2017).

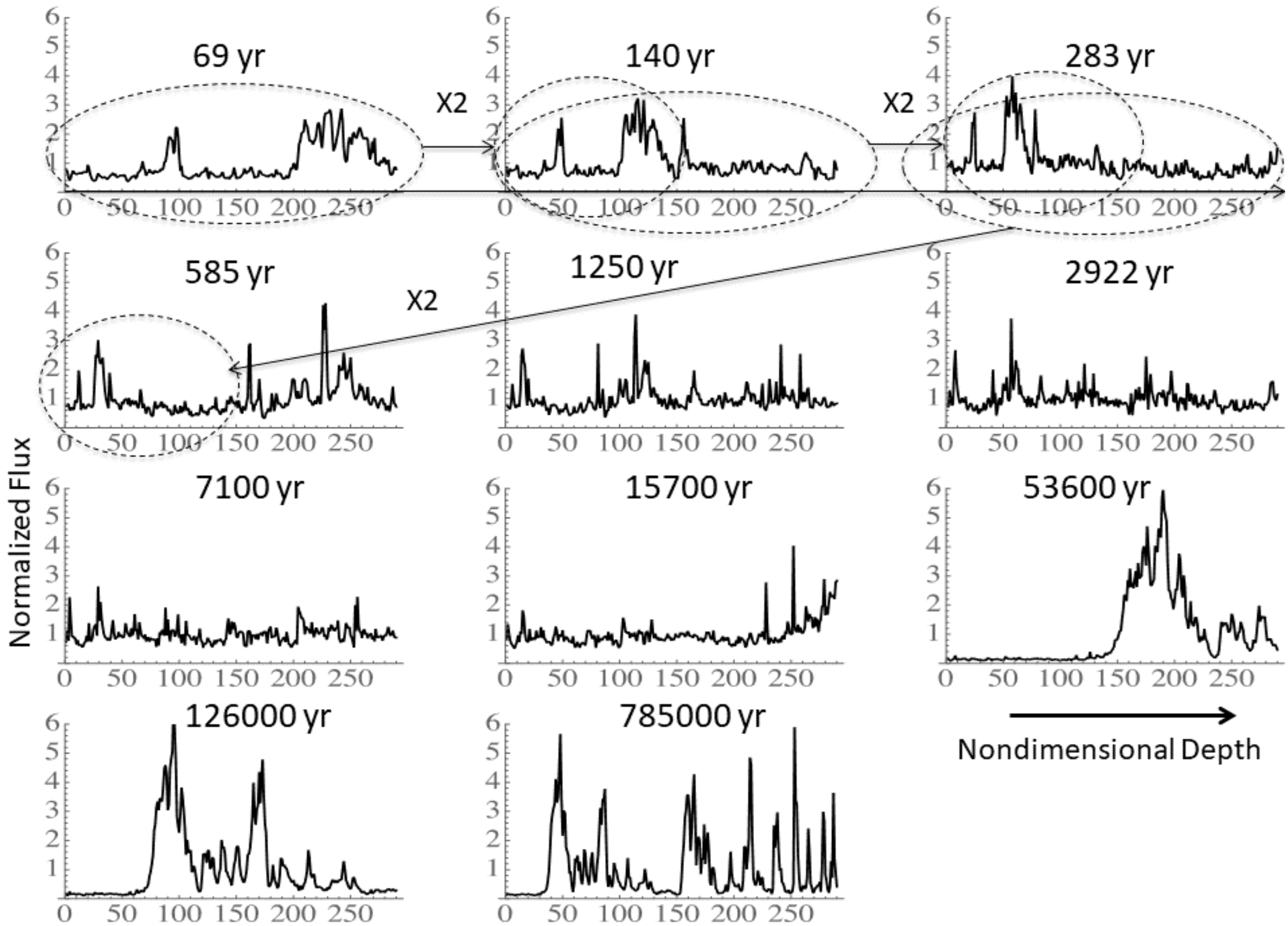


Figure 3a: Zooming out of the Holocene dust fluxes by octaves, by doubling the depth resolution from 1 cm (upper left) to 11m (lower right) resolution. Starting at the left and moving to the right and from top to bottom (see the ellipses on the first three in the sequence) we zoom out by factors of 2 in depth maintaining exactly 290 data points (effectively nondimensionalizing the depth; the small number of missing data points were not interpolated so that the final resolution is not exactly 2^{10}cm = 10.24m). The temporal resolution is not exactly doubled due to the squashing of the ice column, the total duration (in years) of each section is indicated in each plot, the average temporal resolution of plots are: 0.24, 0.48, 0.98, 2.02, 4.32, 10.1 24.5, 54.1, 184, 434, 2710 yr. In order to fit all the curves on the same vertical scale, the dust fluxes were normalized by their mean over each segment. The means (in mg/m^2/yr) are: 0.44, 0.38, 0.30, 0.36, 0.35, 0.33, 0.34, 0.39, 2.48, 2.18, 2.41 i.e. the first 8 plots have nearly the same vertical scales whereas the last three are about 6 times larger range. This means that all the plots except the last three are at nearly constant normalization.

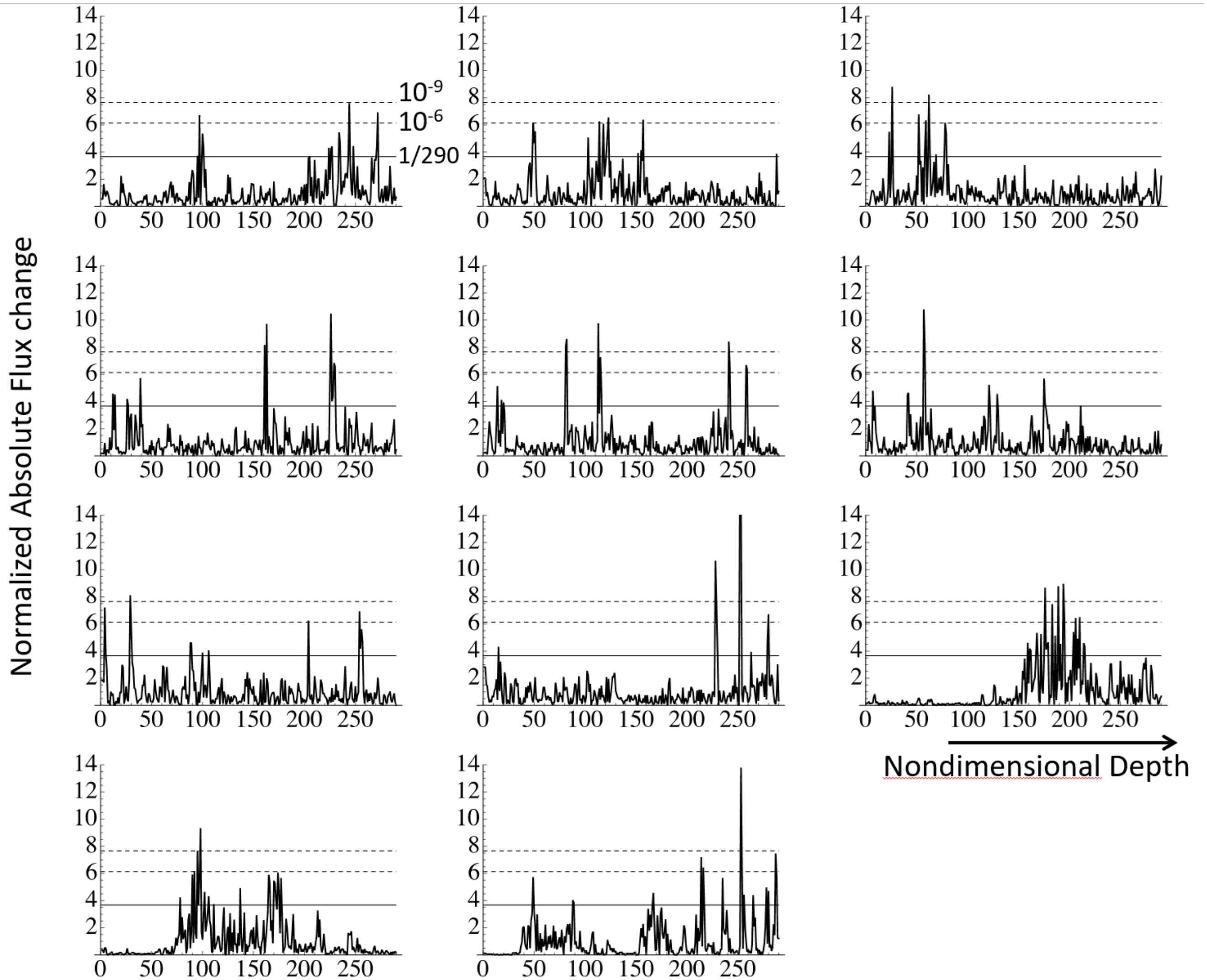


Figure 3b: Same as Fig. 3a but for the absolute changes between neighbouring values in dust flux normalized by the corresponding mean over the segment (290 points). The horizontal lines indicate the Gaussian probability levels for p = 1/290 (representing the mean extreme for a 290-point segment, full line), as well as $p = 10^{-6}$ (lower dashed) and $p = 10^{-9}$ (upper dashed).

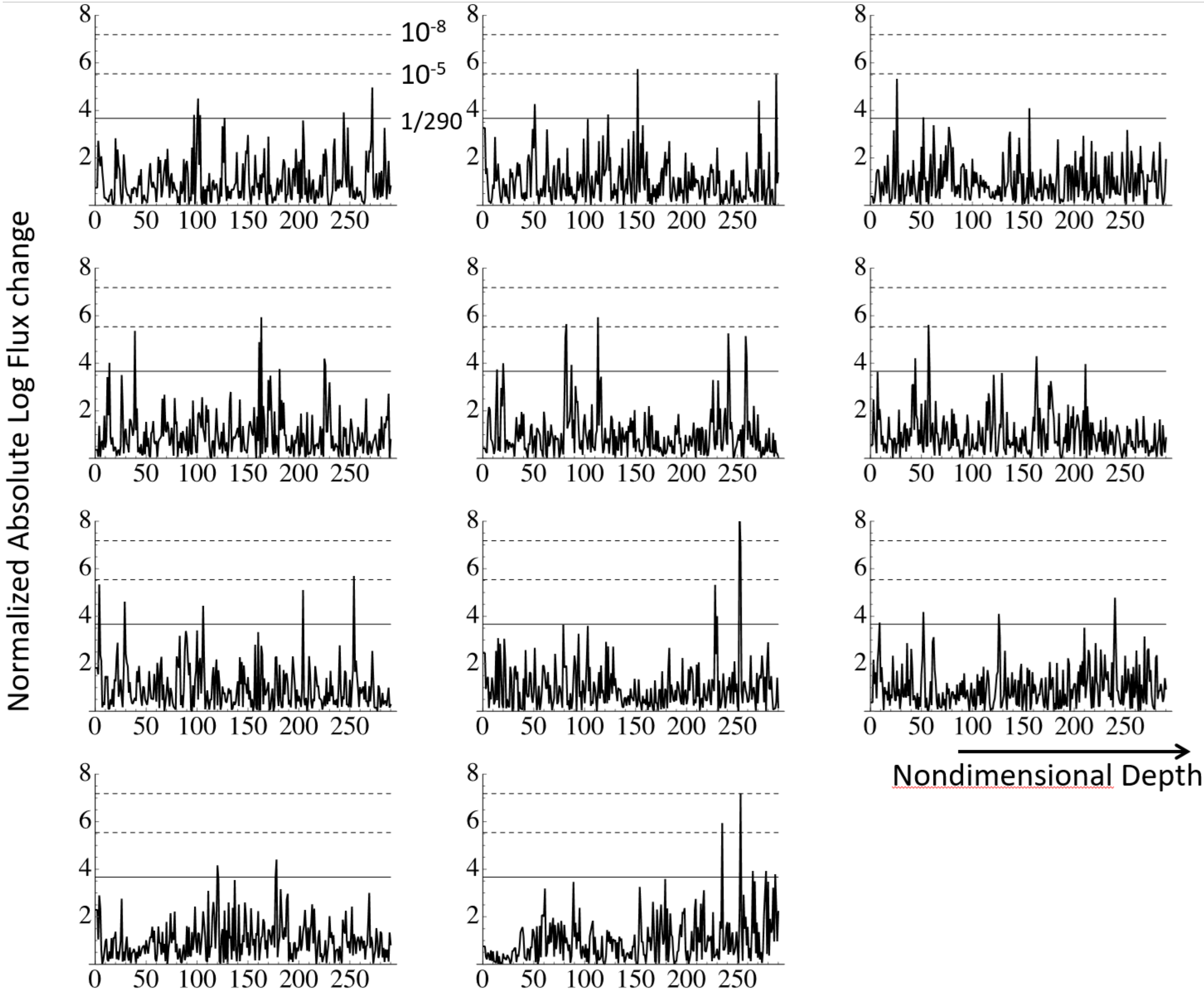
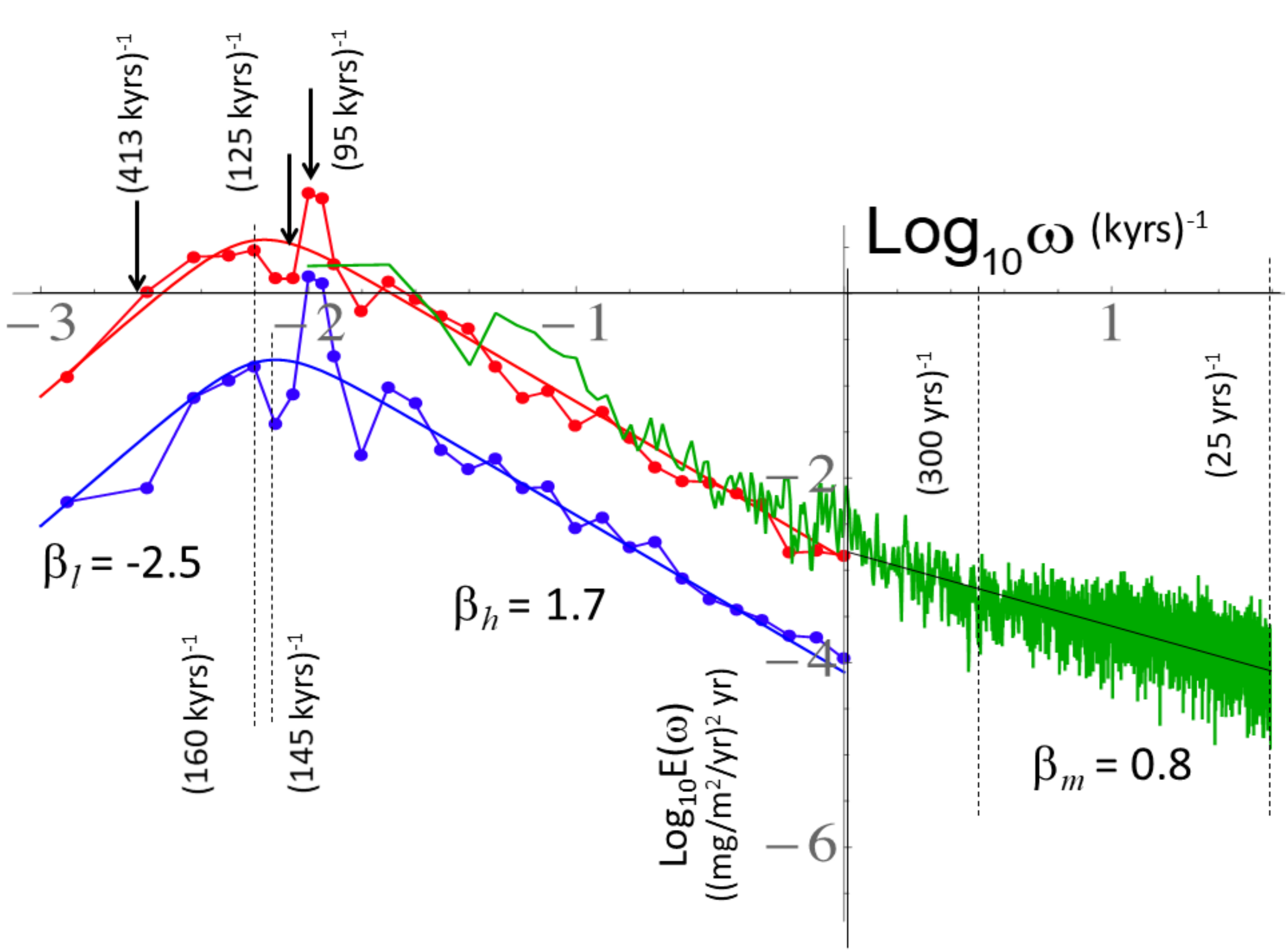


Figure 3c: Same as Fig. 3a but for the absolute changes between neighbouring values in the logarithms of dust flux normalized by the corresponding mean over the segment (290 points). The horizontal lines indicate the Gaussian probability levels for p = 1/290 (representing the mean extreme for a 290-point segment, full line), as well as p = 10^{-5} (lower dashed) and p = 10^{-8} (upper dashed, not the same as in fig. 3b).


(413 kyrs)-1
(125 kyrs)-1
(95 kyrs)-1
Log10ω (kyrs)-1
−3
−2
−1
1
(300 yrs)-1
(25 yrs)-1
−2
−4
−6
β_l = -2.5
β_h = 1.7
β_m = 0.8
(160 kyrs)-1
(145 kyrs)-1
Log10E(ω)
((mg/m2/yr)2 yr)

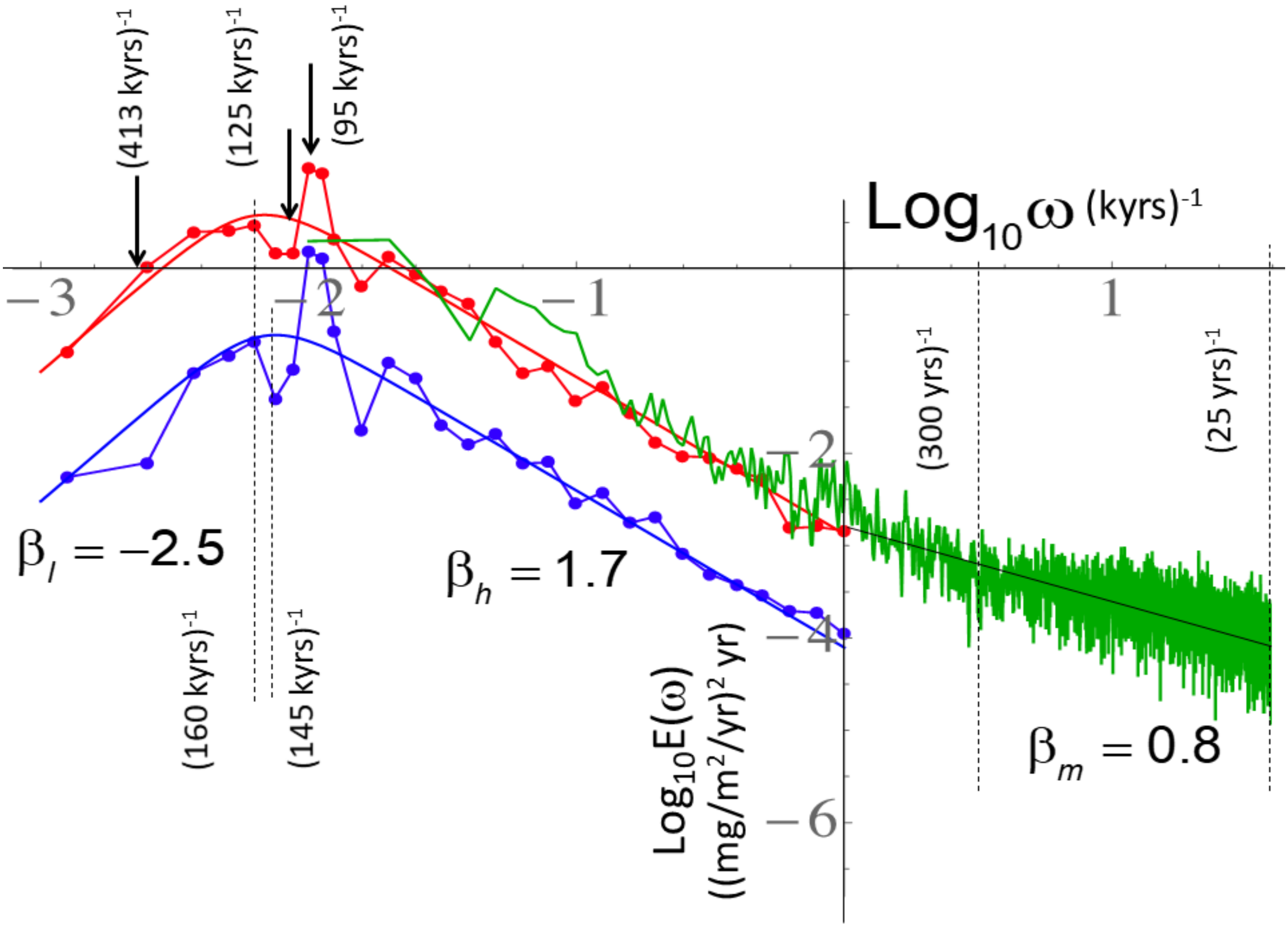
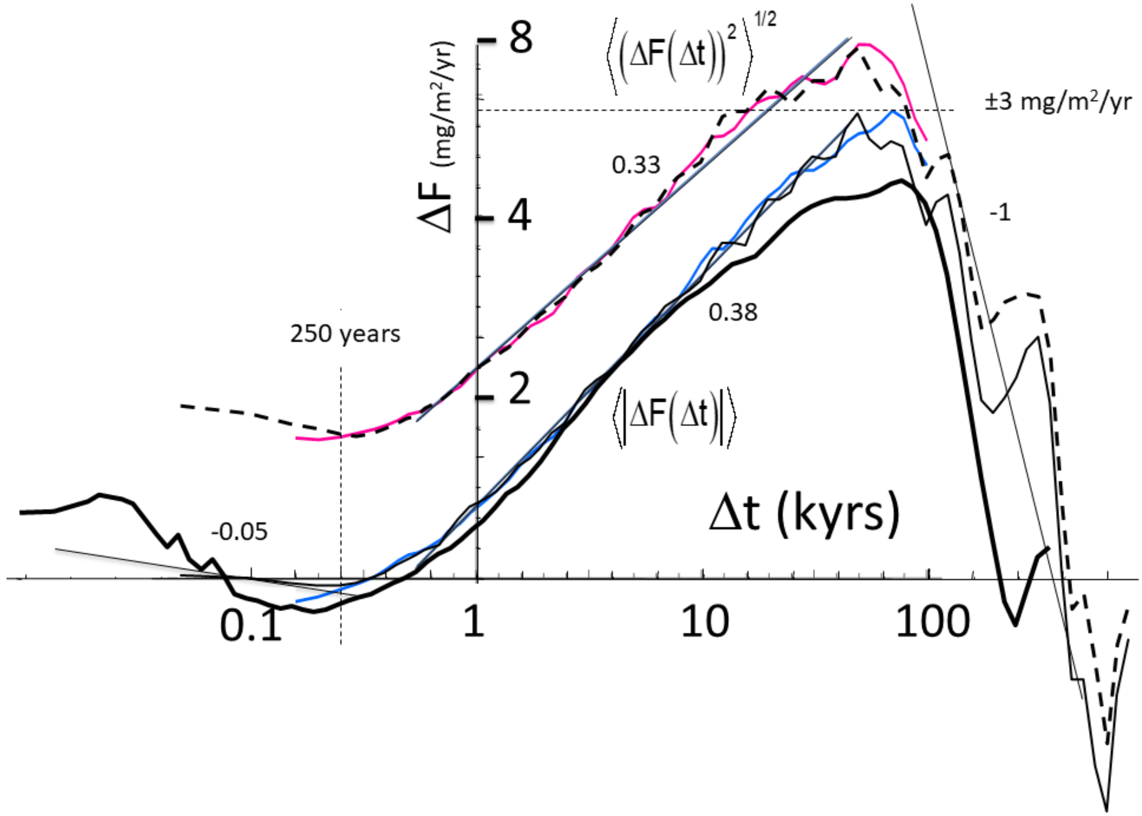


Figure 4: Log-log plot of the Fourier spectrum of the (25yr)$^{-1}$ resolution dust concentration in frequency units of kyrs^{-1} (red) and the same but of the logarithms of the flux (blue). Also shown is the average spectrum of the 5-year resolution data over the last 400 kyrs (green). For the latter, the periodograms of each the four most recent 100 kyr cycles were averaged, but the full spectral resolution (5yrs)$^{-1}$ was retained. The beta parameters are the exponents of the theoretical spectrum (see main text, the negative of the logarithmic slope) for the macroclimate (-2.5), climate (1.7), and macroweather (0.8) regimes. The spectra were analyzed using FFT with standard Hanning windows.



8
4
2
ΔF (mg/m²/yr)
$\left\langle \left(\Delta F(\Delta t)\right)^2 \right\rangle^{1/2}$
±3 mg/m²/yr
0.33
-1
0.38
250 years
$\left\langle \left|\Delta F(\Delta t)\right| \right\rangle$
Δt (kyrs)
-0.05
0.1
1
10
100

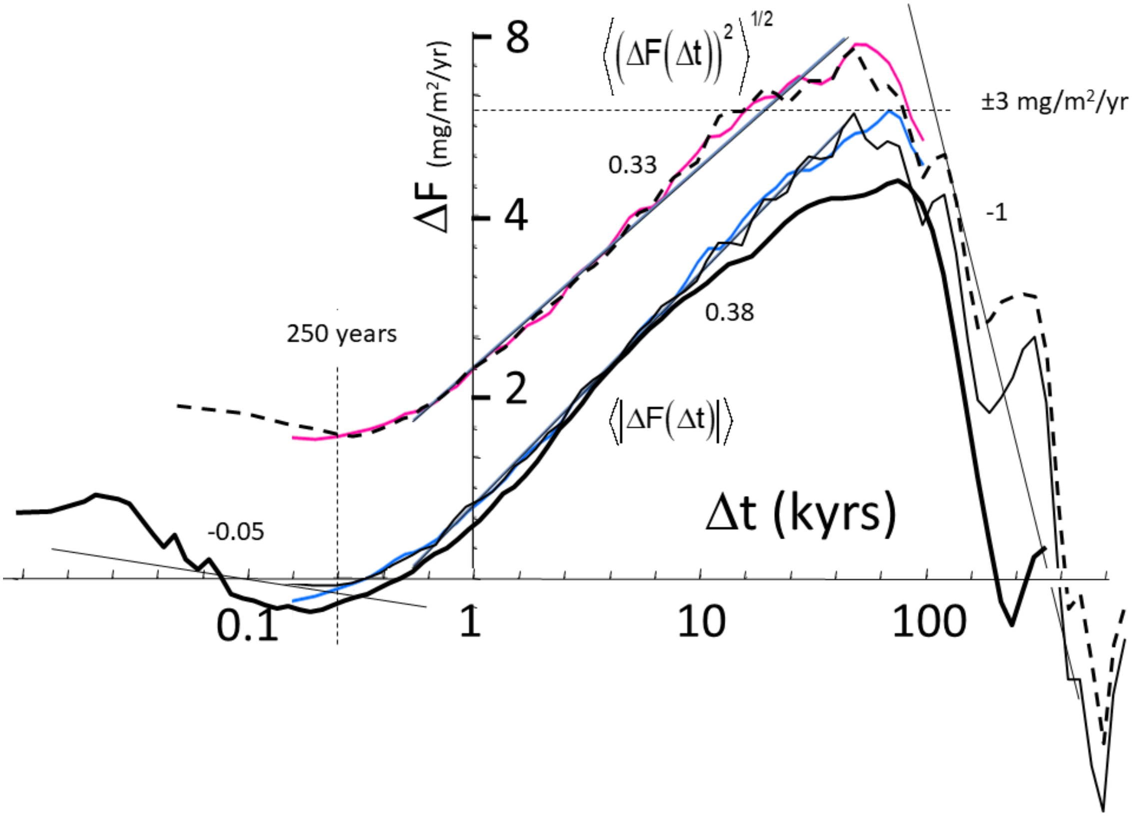
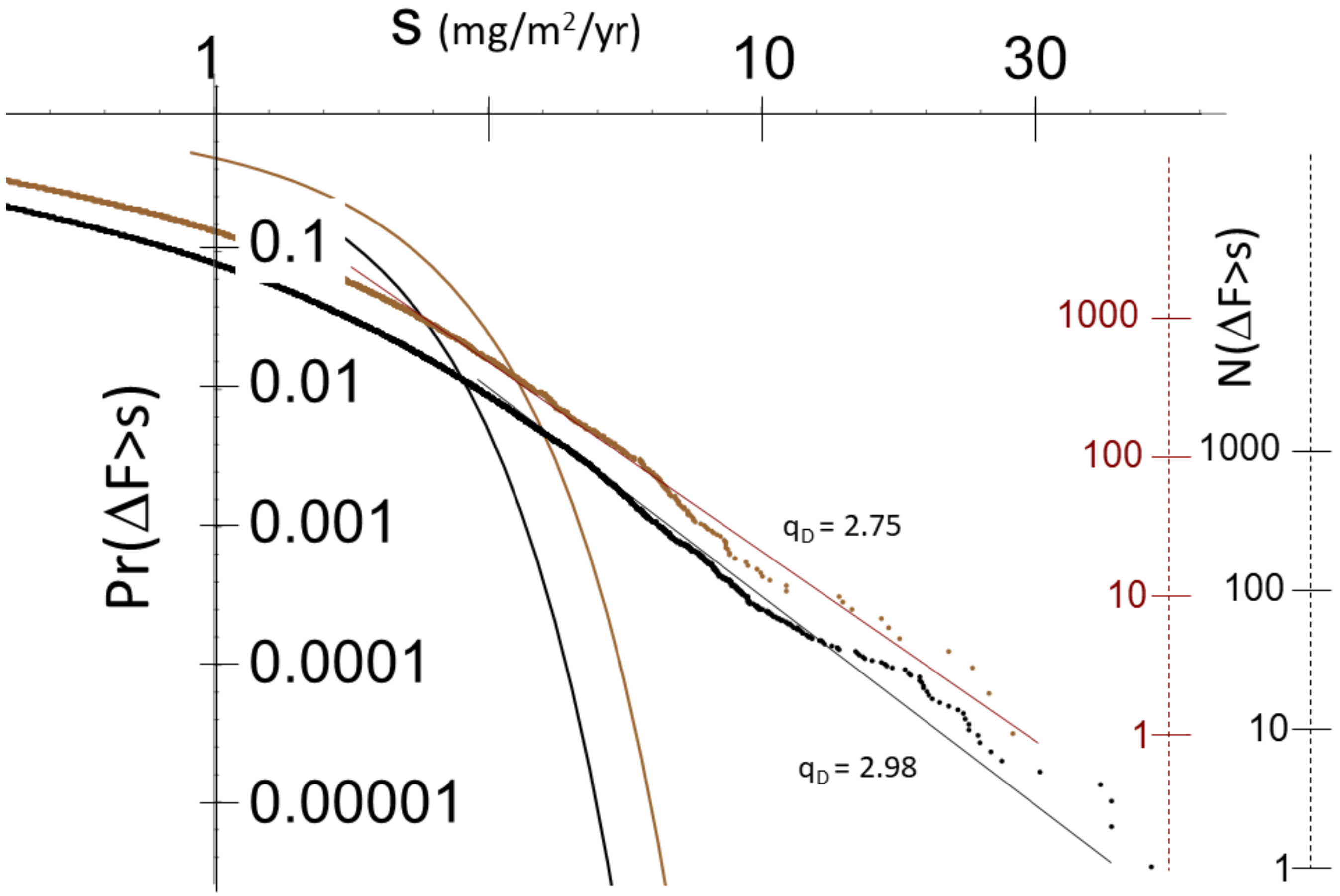


Figure 5: The Haar fluctuation analysis of the entire 800 kyr dust flux data set (thin lines). The dashed black and solid pink (top pair) represent RMS fluctuations for dimensional and non-dimensional time, respectively. The solid black and blue curves are the same but for the mean absolute (*q* =1) fluctuations. The curves with non-dimensional time lags have nominal (average) resolutions of 25 years and the fluctuation statistics are averaged over the 8 cycles. The thick black line shows the Haar fluctuations for the most recent 400 kyrs at 5-year resolution. Note that the peak in the curves occurs as expected at Δt ≈ 50kyrs i.e. at about a half cycle; and the horizontal dashed line shows that at this scale - corresponding to the largest difference in phases – the change in the mean absolute dust flux is about ± 3 mg/m^2/yr. Also shown (dashed vertical line) is the (average) time scale τ_c ≈ 250yrs at which the transition from macroweather to climate occurs. Several reference lines (with the slopes/exponents indicated) are shown showing approximate scaling behaviours.



S (mg/m2/yr)
1
10
30
0.1
0.01
0.001
0.0001
0.00001
Pr(ΔF>s)
N(ΔF>s)
1000
100
10
1
1000
100
10
1
qD = 2.75
qD = 2.98

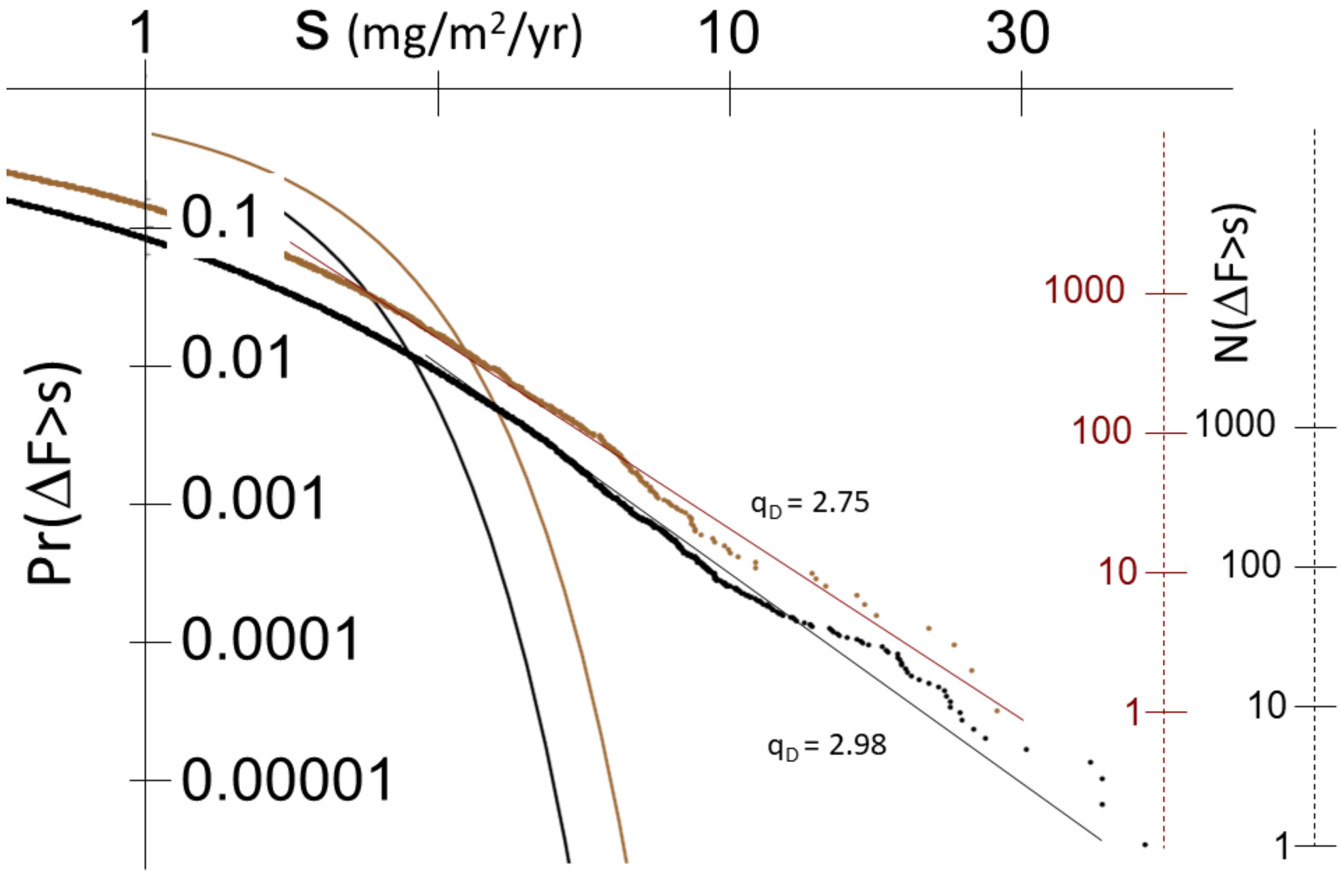
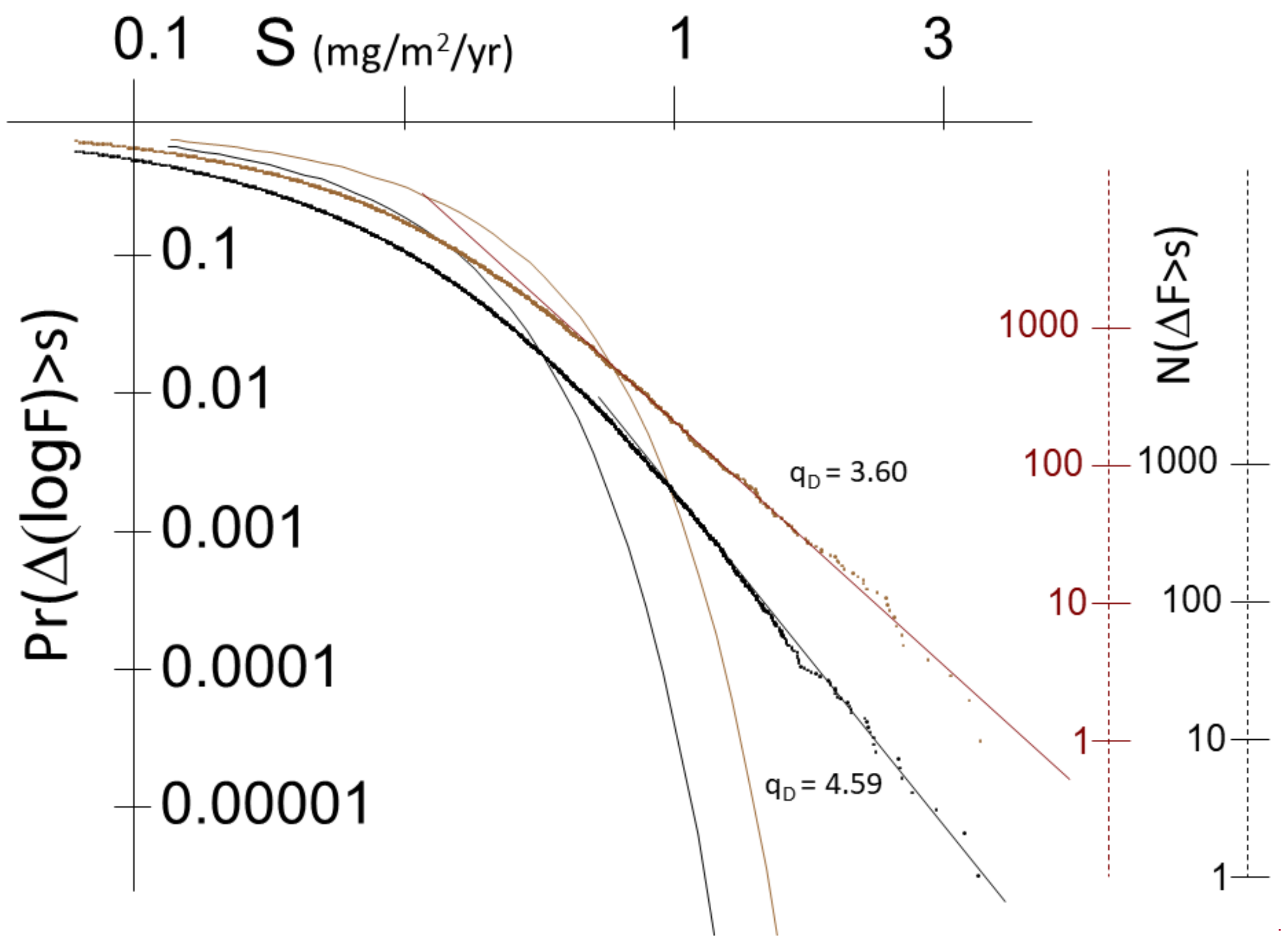


Figure 6a: The probability distribution Pr(ΔF > s) of random changes in dust flux (ΔF) exceeding a fixed threshold s; in time at 25 year resolution (brown, 32,000 points), and in depth at 1cm resolution (black, 251,075 points corresponding to the last 400 kyrs). The frequency scales at the right give the number (N) of jumps in each of the series that exceeds the threshold s. The straight lines indicate power law probability tails with exponents q_D indicated. Also shown (parabolas) are the Gaussians with the same mean and standard deviations. In time, the maximum change in flux corresponds to about 28 standard deviations (i.e. to a Gauss probability ≈ 10^{-91}), in depth, to 51 standard deviations (i.e. to p ≈ 10^{-455}). On the right, we provide axes giving the actual number of flux increments that exceed *s*, brown for the fluctuations in time, black for those in depth.


0.1
S (mg/m2/yr)
1
3
0.1
0.01
0.001
0.0001
0.00001
Pr(Δ(logF)>s)
qD = 3.60
qD = 4.59
1000
100
10
1
N(ΔF>s)
1000
100
10
1

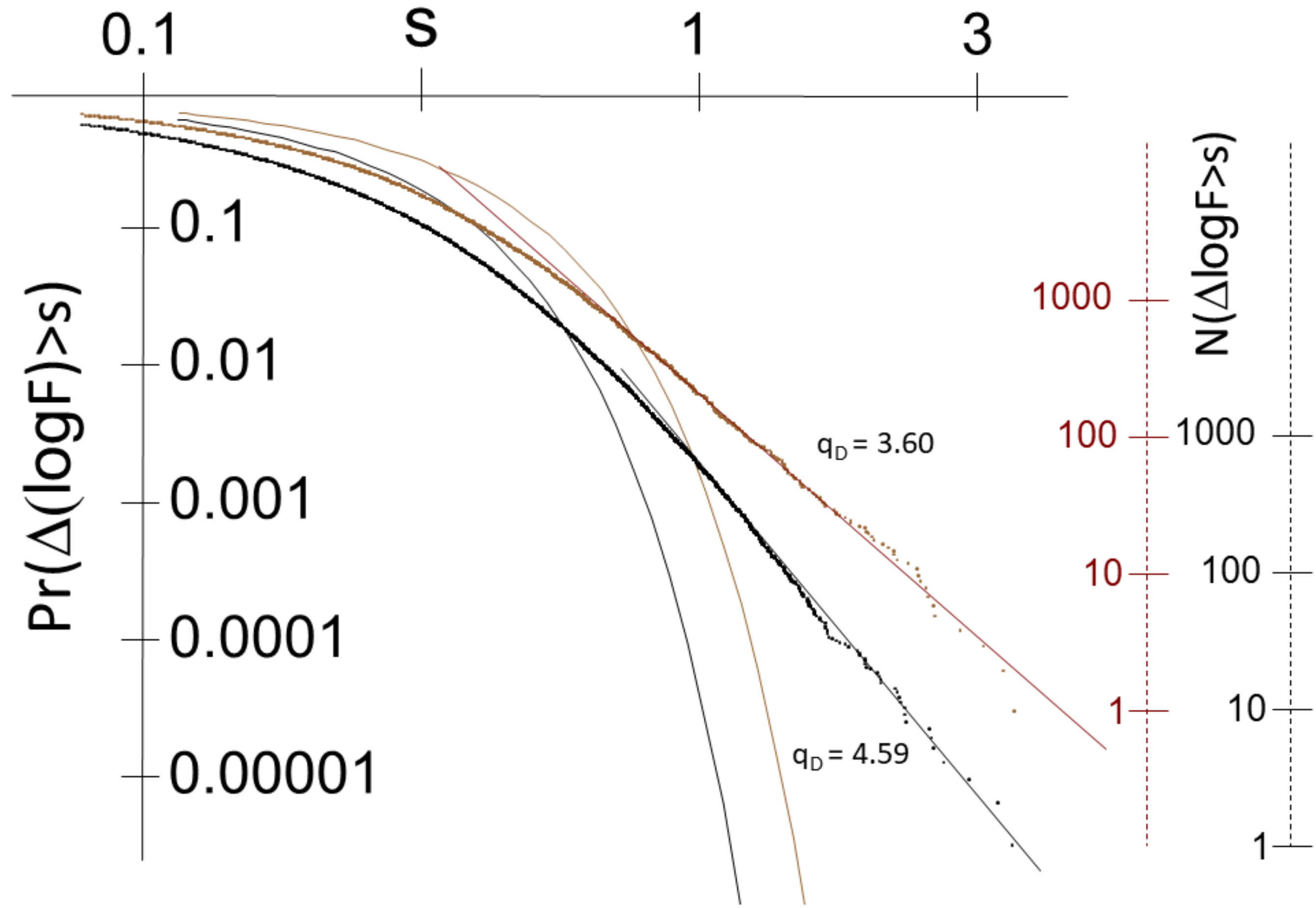


Figure 6b: Same as 6a except for the increments of the log of the dust flux (brown is in time, 25-year resolution, black is in depth, 1 cm resolution), the curves are the closest fitting (log) Gaussians. The threshold S is dimensionless, and the numerical values are correct if F is measured in units of mg/m^2/yr.

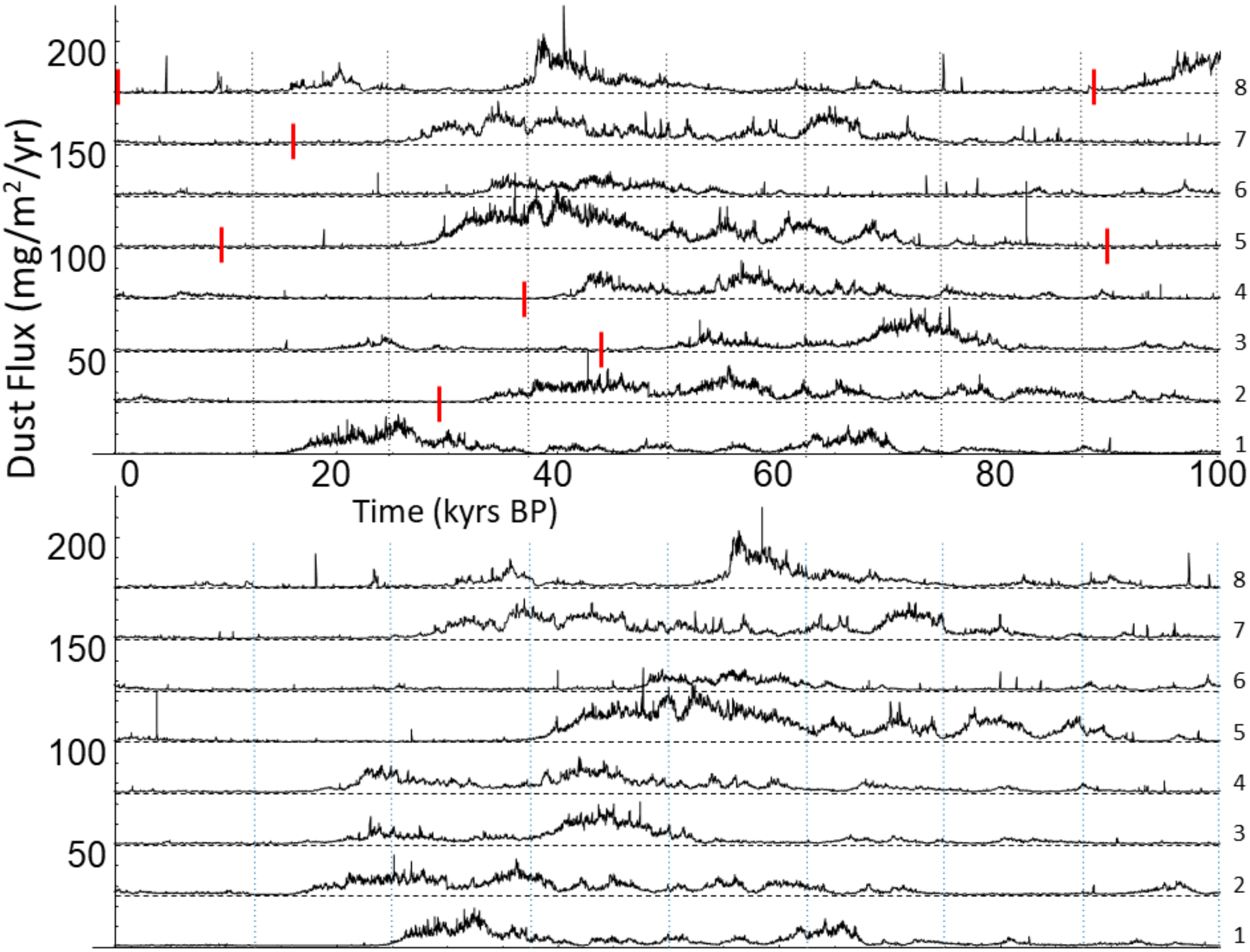


Figure 7: Top set: successive segments of theoretical 100 kyr–long glacial cycles using usual (dimensional) time (present to past: bottom to top, the segment number is at the far right) with the 12.5 kyr phases indicated by vertical dashed lines. The short red lines indicate the interglacial dust minima. Each glacial-interglacial cycle is shifted by 25 units in the vertical for clarity. The red markers in the upper plot get mapped to the first dashed blue line in the lower plot.

Bottom set: successive cycles using nondimensional time (interglacial to interglacial) and then shifted by one phase to better line up with the usual time segments (the left most phase of the bottom line of the lower plot is zeroed). The average (nominal) resolution is 25 years. The interglacial dust minima were taken as 128.5, 243.5, 336, 407.5, 490, 614, 700, 789 kyrs B.P. and the data start at 373 yrs B.P. Each cycle is shifted by 25 units in the vertical for clarity. The data older than 789 kyrs were not used in these nondimensional cycles.

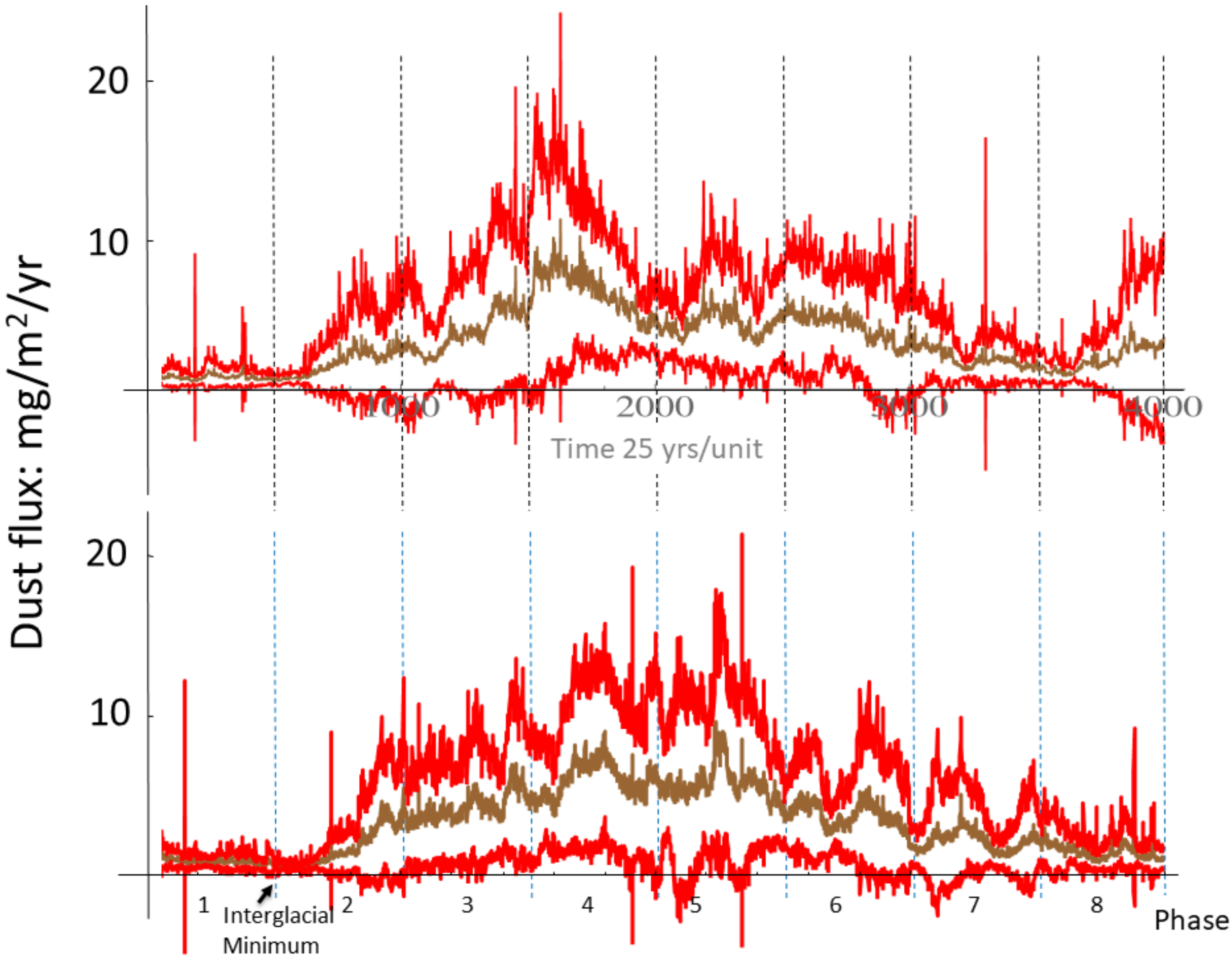


Figure 8: Top set: Averaging over the 8 cycles at 25-year resolution, we get the above picture: the mean is brown and the one standard deviation cycle to cycle variability is shown by the red. The dashed vertical lines give a further division into 8 x 12.5kyr segments, the 8 "phases" of the cycle.

Bottom set: the same but for the nondimensional time. The relative position of the interglacial minimum at the first dashed line is indicated.

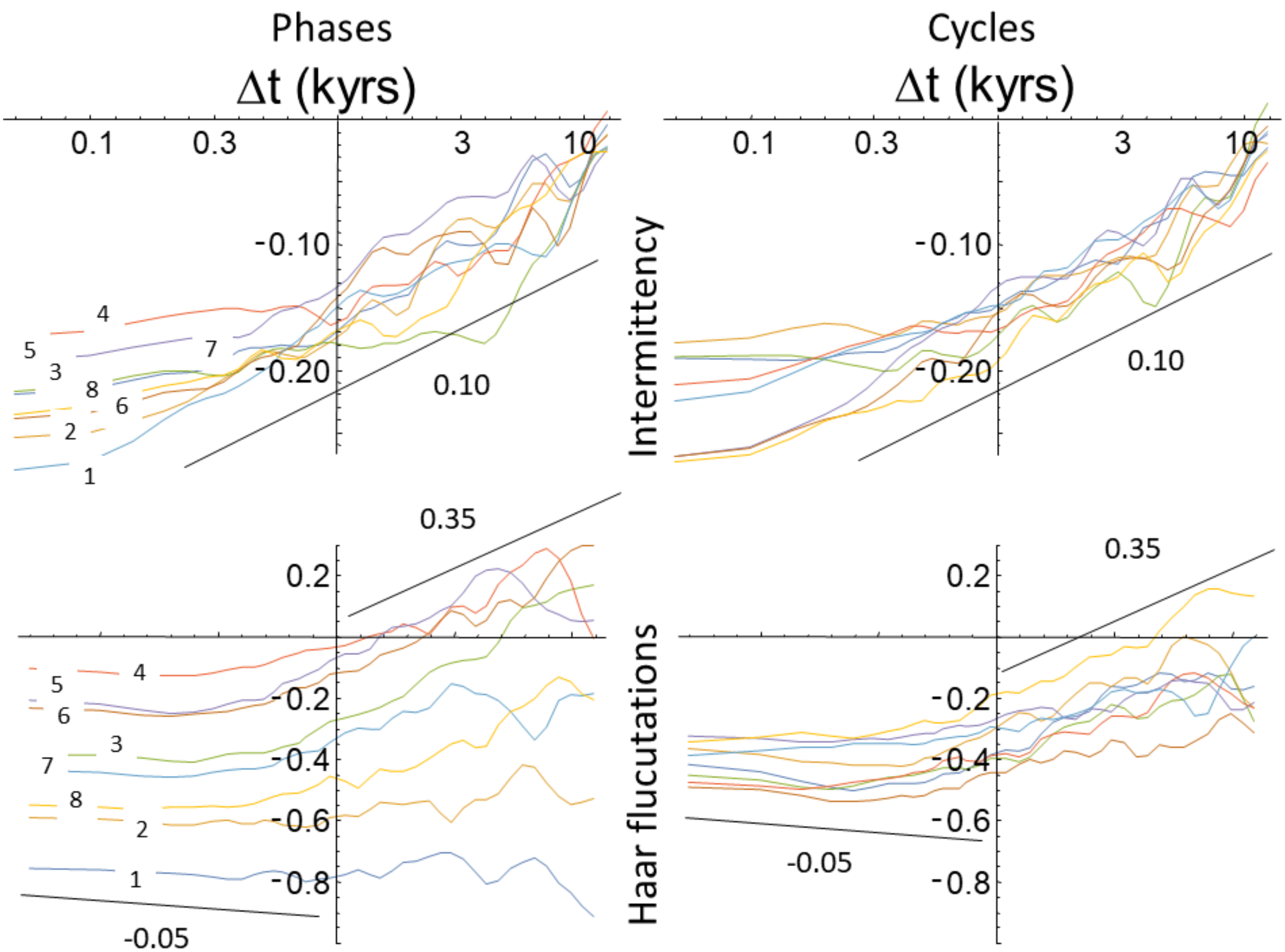


Figure 9: The top row shows the intermittency function $G(\Delta t)$ (whose slope on the log-log plot is C_1) and the bottom row, the mean absolute Haar fluctuation $S_1(\Delta t)$ (whose slope on the log-log plot is H), the left column shows the result for each phase after averaging over the 8 cycles with the numbers next to each line indicate the phase number~~;~~ (each colour corresponds to the ~~right hand~~same number); the righthand column shows the result for each cycle after averaging over the phases. Whereas each cycle is fairly similar to every other cycle (the right column), each phase is quite different (the left column). We see the most significant difference is the fluctuation amplitude as a function of phase (lower left).

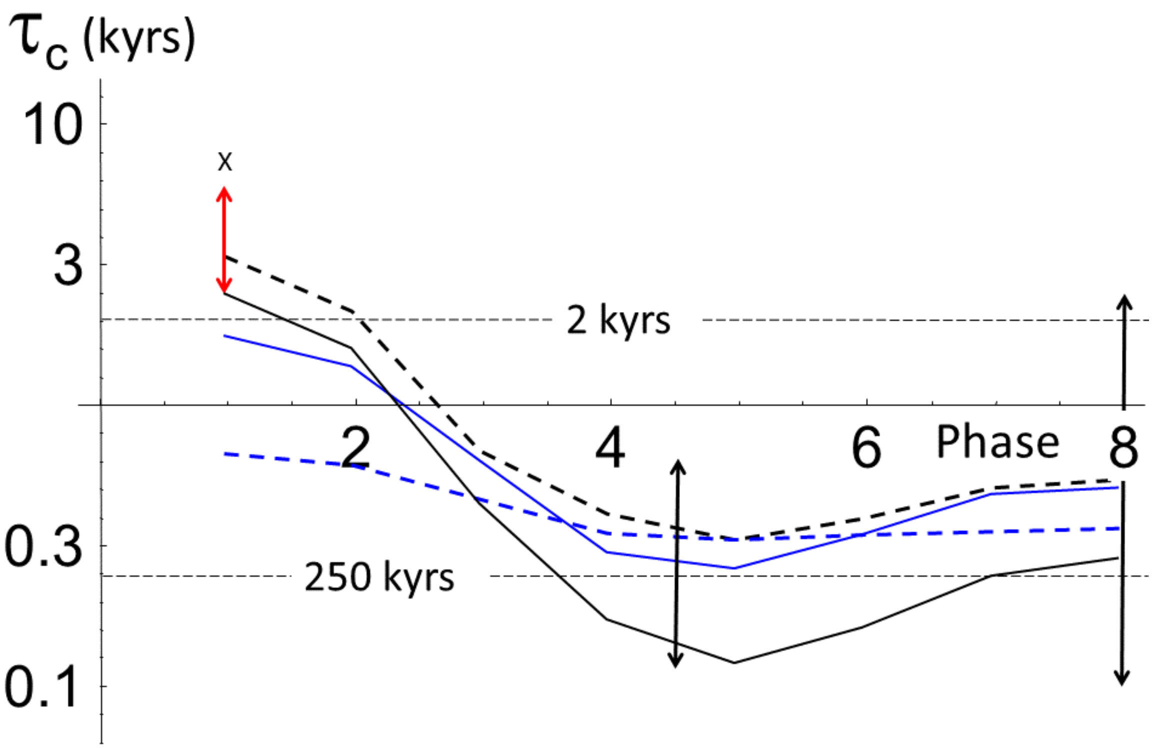


Figure 10: The transition scale τ_c estimated in two ways for each of the 8 phases and from two definitions of the phases. The first method (solid lines) used a bilinear fit to the (logarithm) of the Haar q=1 structure function (i.e. mean absolute fluctuation) as a function of log time lag Δt. To obtain robust results, a small Δ*t* region with the slope -0.05 and a large Δt slope +0.25 was imposed with the transition point (τ_c) determined by regression. This was done for each segment and cycle. For each phase there were thus 8 transition scales, which were used to calculate the mean of the logarithm of τ_c and its standard deviation. Results are shown for dimensional (segments, blue) and nondimensional time (cycles, black).

The second method used to estimate τ_c was graphical and relied on a somewhat subjective fitting of scaling regimes and transitions, but without imposing small and large Δ*t* slopes (exponents *H*). The results are shown in dashed lines, they are quite similar although we can note some differences for the first phase (dimensional, blue) and the middle phases (nondimensional, black). There is also considerable cycle to cycle spread that was quantified by the standard deviations. In order to avoid clutter, typical spreads are shown by the double headed black arrows. Dashed horizontal lines show the ensemble mean transition scale (about 250 years) as well as ensemble mean for phases 1 and 2 (around 2 kyrs), which stands out compared to the rest of the phases. The red arrow shows one standard deviation for the nondimensional first phases, while the X marks the value of the Holocene τ_c (7.9 kyr) just outside the 1-sigma limit.

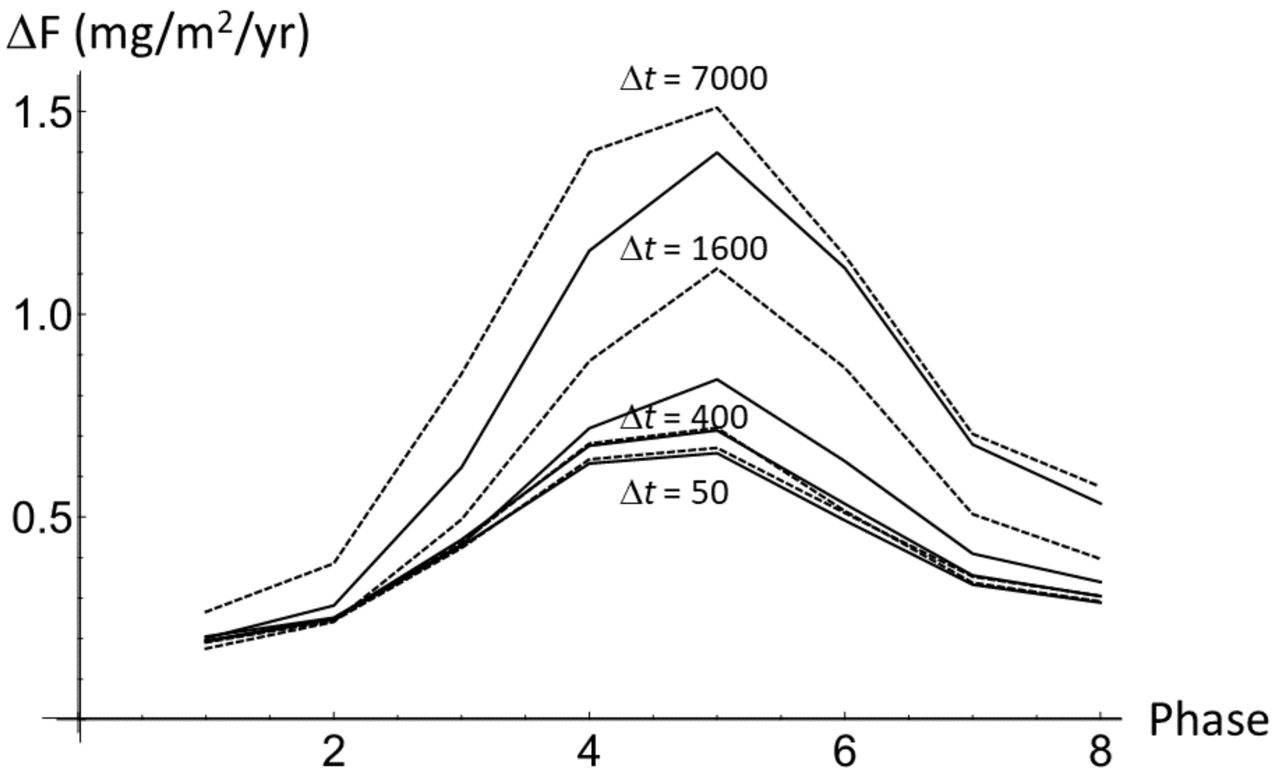
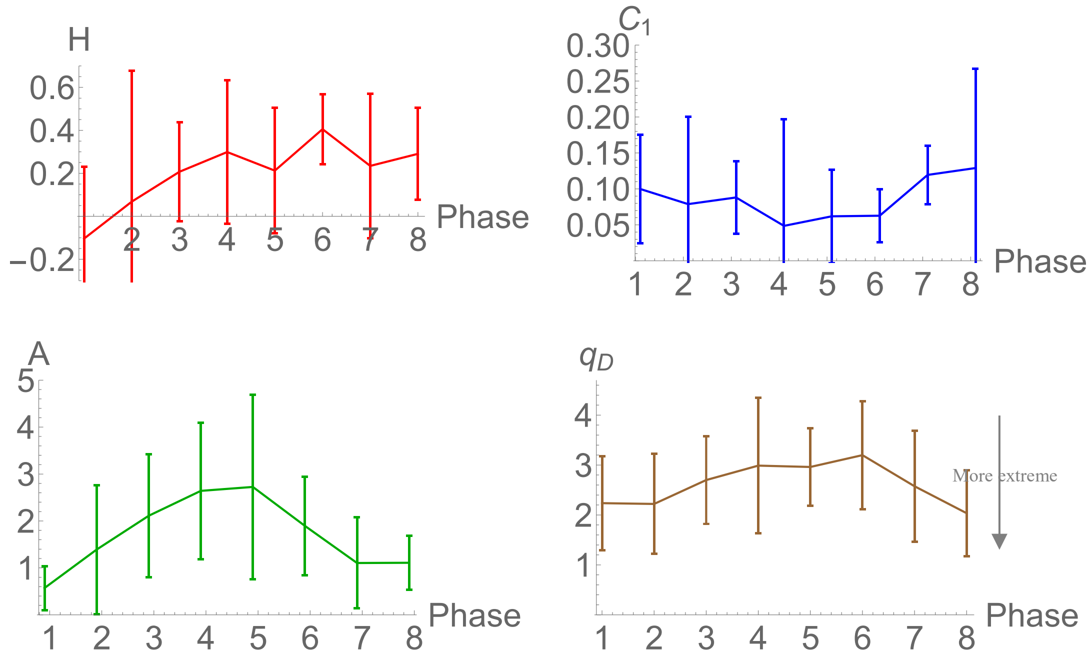


Figure 11: Using nondimensional time, the amplitude of the Haar fluctuations are averaged over all the cycles The curves from bottom to top are for time scales of Δt = 50, 100, 200, 400, 800, 1600, 3500, 7000 years, alternating solid and dashed (for clarity, only some of the Δt's are marked). The cycle to cycle variability (the dispersion around each line) is about a factor of 2 (it is not shown to avoid clutter).



H
0.6
0.4
0.2
−0.2
Phase
C_1
0.30
0.25
0.20
0.15
0.10
0.05
Phase
A
5
4
3
2
1
Phase
q_D
4
3
2
1
More extreme
Phase
1 2 3 4 5 6 7 8

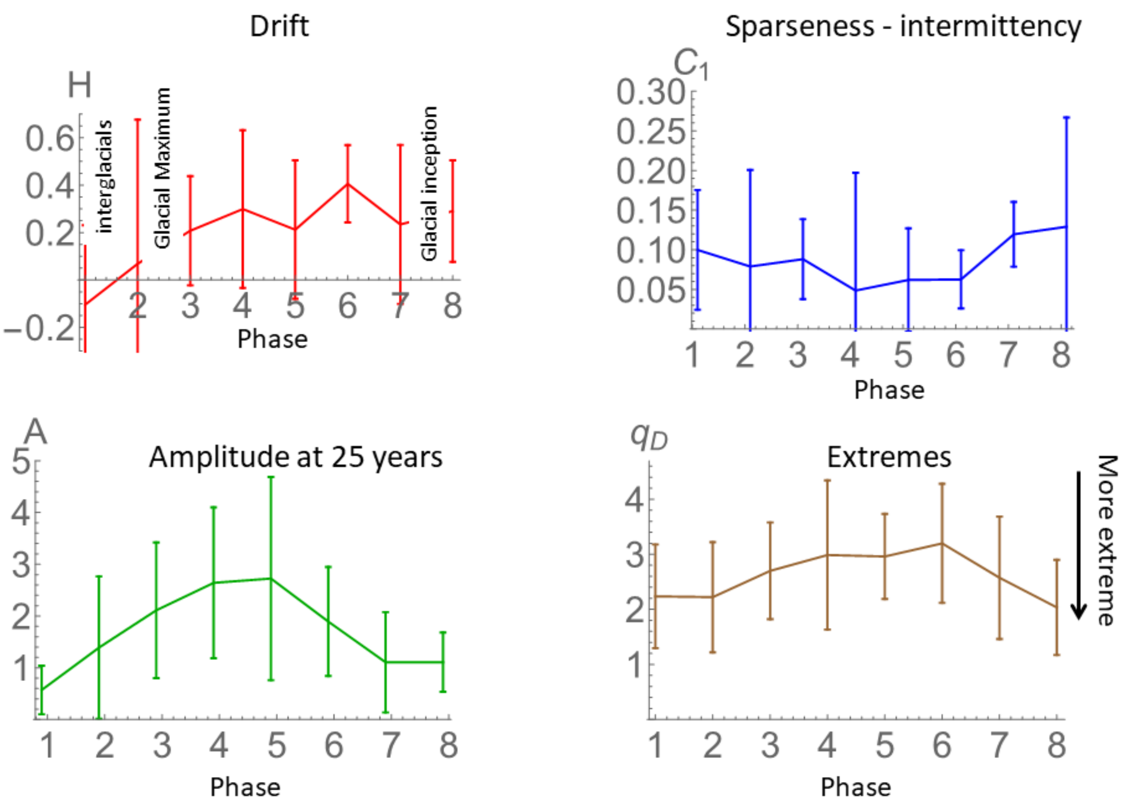


Fig.12: The fluctuation and intermittency exponents H and C_1 (top row) are estimated over the range 500 – 3000 years, as a function phase with the standard deviations from the cycle to cycle variability (all using nondimensional time). The upper left (H) plot shows low drift in phases 1 and 2 but become driftier in the middle and older phases. The intermittency (C_1, upper right) is moderate at the beginning and end of the cycles, and a little weaker in the middle. The lower left shows the amplitude of the fluctuations at 25 years determined by the standard deviation of the dust flux (units: mg/m^2/yr). We see that the flux has low amplitude fluctuations at the beginning and end of the cycles and 3-4 times higher amplitude fluctuations in the middle. The lower right shows the probability exponent q_D estimated from the 25-year resolution data for each phase; the extreme 5% of the flux changes were used to determine the exponent in each phase; the cycle to cycle spread is indicated by the error bars (overall average over the phases: q_D = 2.62±0.42).

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