Review of

‘**Impact of different estimations of the background-error covariance matrix on climate reconstructions based on data assimilation’**

by V. Valler, J. Franke, and S. Brönnimann

Recommendation: minor revisions

This manuscript identifies the best choices from a number of different spatial and temporal localization approaches and from different inflation techniques for the background error covariance matrix in Ensemble Kalman Filters used in paleoclimatic applications. The optimization of these technical details in data assimilation is important for the growing paleoclimate data assimilation community. The results are systematically derived and the manuscript is in general well written. I support publication after the points listed below have been clarified or corrected.

Specific comments

Page 1, Line 2, replace ‘of the assimilation system’ with ‘of some assimilation systems’

Page 1, Lines 17/18, 24/25, ‘boundary conditions’ are specifications of state variables at the boundaries of a model domain, and thus not the same as ‘forcings’, which are external influences on the system. The two should be distinguished throughout the text. It seems that here the statement are about forcings. If so, reformulate avoiding the use of ‘boundary condition’.

Page 2, line 6, ‘linear models’ of what? I think it should by ‘linear dynamical systems’. A short comment on why KFs are used with non-linear systems, including GCMs, would be good. ‘Gaussian distributions’ of what? The state variables?

Page 2, line 13-16, I suggest using ‘stationary offline’ and ‘transient offline’ for the two approaches.

Page 2, lines 17/18, ‘The true climate state is not known, therefore it has to be estimated’. Does ‘it’ refer to ‘the true climate state’, as the sentence suggests or to ‘the uncertainty of the background state’, which would link better to the first sentence in this paragraph? This sentence should be clarified, or it could simply be deleted (which I think is the better option).

Page 2, line 20, What is a non-simplified KF? A KF with a ‘true’ background error covariance matrix? If so, how can this exist? The background error covariance has always to be estimated somehow. Please clarify the statement.

Page 2, line 23, It seems that the sampling error for the background error is not only a random error, but leads to a systematic underestimation of the background error, otherwise inflation would not be a suitable approach. Please explain better.

Page 2, lines 25/26. The statement on distribution of ensemble members refers to online approaches, but the approach used by the authors is an offline approach. This is confusing. Please briefly explain how the ensembles are generated in an online KF, and that in offline approaches the ensemble is given, but that the background error covariance still needs to be inflated.

Page 3, line 14, replace 'other method' with 'additive method'

Page 3, line 25, replace ''form' with 'from'

Page 3, line 28, Don't use 'forced by boundary conditions', as forcings and boundary conditions are different (see comment above). If I understand correctly for all ensemble members the same greenhouse gas, solar and volcanic forcings have been used, as well as the same SST boundary conditions. Please clarify.

Page 3, Lines 29-31, The SST reconstruction can be expected to strongly influence the results of this data assimilation approach with an atmosphere-only GCM. There should be some comments on how the SST reconstructions have been made, what is known about their uncertainties, and why this approach is taken rather than data assimilation with a coupled atmosphere-ocean GCM.

Page 3, line 30, I think 'till' should not be used in formal writing and should be replaced with 'until' (also later in the text).

Page 4, line 1, replace 'boundary conditions' with 'forcing', and make a separate statement on land-surface boundary conditions, including which variables are prescribed.

Page 4, line 16, 'CCC400' has not been introduced

Page 4, line 23, If I understand correctly the deviations of the ensemble members from the ensemble mean are updated in the online EnSRF according to equation 2, but not in the offline EKF, which uses an existing ensemble. Please clarify.

Page 4, lines 28-30, the statements on the 6 month periods are partly redundant

Page 5, lines 5-7, How have the error variances been chosen? Should sigma^2 be K^2? If so, why is the error for documentary data smaller than for instrumental data? Which multiple regression?

Page 5, lines 16 – 19. The notation is not clean. In line 16 it is said that R is a diagonal matrix, in line 19 that R is a scalar. The problem is that the same notation is used for an equation using the full set of observations (where R is a diagonal matrix) and for the equation when the individual observations are assimilated sequentially. Please reformulate.

Page 6, line 1, replace 'localization function' with 'the localization function'.

Page 6, line 19/20, replace 'additive inflation' with 'the additive inflation', and 'hybrid' with 'the hybrid'.

Page 6, line 23-25, The explanation is confusing. One can select ensemble members for the whole period or some or all ensemble members for some timesteps; how exactly are the climatological state vector and the associated error covariance matrix calculated? The simulations have already

been performed; why are there substantial computational costs for using a large number of ensemble members?

Page 6, line 29, H^T is at the end of all terms in the equation. Can it not simply be deleted?

Page 7, line 4-5. It is not clear how x^clim is calculated and updated, what n is, and what 'propagated' means in an offline assimilation scheme.

Page 8, line 7, replace 'isotropic localization' with 'an isotropic localization'.

Page 8, line 15, ENH is not defined

Page 9, line 3, Explain why P^b does not have full rank, why this is a problem, and what this has to do with inflation.

Page 9, lines 5-6. This is not well phrased; it is not the covered model space that is multiplied with the inflation factor.

Page 9, line 16, It has not yet been mentioned in the main text in section 3 that 250 members have been chosen and how they have been selected (see also earlier comment on this).

Page 12, line 2, replace 'verification' with 'validation' (this is used elsewhere in the text) or 'performance'

Page 12, line 6, missing full stop after '2'

Page 12 and also in main text, Add a comment on whether the skill differences found are substantial and practically relevant.

Page 14, line 11, 'kalman' should be upper case

Page 14, line 16/17, typos and missing spaces

Figs. 1, 3, 8, 10 should be bigger