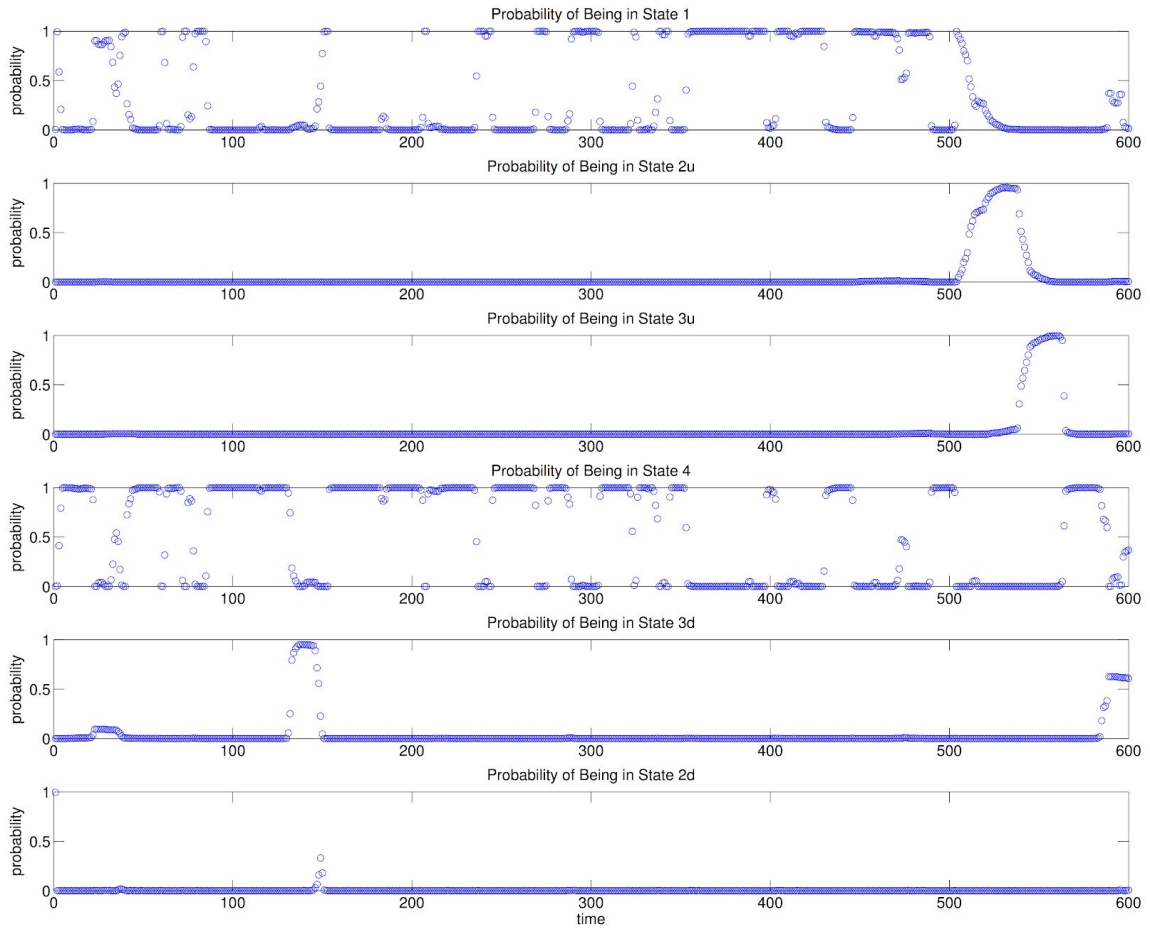


Reviewer #2's comments and our responses

The reviewer suggested improving the model descriptions:

1. The reviewer suggested adding a paragraph to describe the disadvantages of the classical correlation analysis. See the reply to the Reviewer #1's first comment. We wrote a paragraph that describes the advantages of the method we used in the study and the disadvantages of the correlation analysis (see above).
2. Reviewer's comment: *In the introduction, you should explicitly state that the four proxies come from the same core, as it is an important implicit hypothesis.*
In Section 2.1 Data Collection, we state that "... a sediment core retrieved from the central Peru margin". We can also modify the second sentence of Section 1.0 Introduction: "[The sediment record is retrieved from a single sediment core located at the central Peru margin and sampled as high-resolution to ...](#)".
3. The reviewer suggested describing the age model of the core.
The age model from [1] is used to estimate the age of the core top and base. We did not estimate the age for each sample because we assume a constant accumulation rate. The assumption does not affect the HMM and AR-HMM analysis significantly, because the sediment record is retrieved from a single sediment core so cross-correlations are not affected significantly by age. Modest variations in accumulation would only weakly affect the accuracy of the autocorrelations, as those are linkages assessed only between neighboring values of the data rather than long sections of the core. If the constant accumulation rate assumption is not valid, then we would consider either direct age assignments via C-14 dating ([2]) or indirect age assignments via synchronization based on a global climate variable, such as glacial ice volume using the benthic $\delta^{18}\text{O}$ proxy ([3], [4]). However, the age modeling efforts thus far do suggest that the site has a high and steady sedimentation rate across the Holocene. Therefore, we assumed a constant rate and used age estimates only for the top and bottom of the core.
4. The reviewer suggested describing the results when the number of states is assumed to be one or more than two. The goal of this study is to reveal the hidden regime changes under the sampled data, so we assumed that the number of states is two or more than two. We experimented during the early stages of the project by assuming that there might exist four states, which include two additional states as intermediate states, and found that the regime changes through the intermediate or transitional states was very fast, as shown the figure below. As this methodology does not exhibit persistence in the intermediate states, it is not meaningful to consider those intermediate states as separate regimes. This approach would also have indicated if there were three or four sustained regimes (i.e., nothing about the approach implied that these extra third and fourth states had to be transitional, but that was what was found). Therefore, we proceeded on to all later calculations assuming the number of states to be two, hence the two-state HMM and AR-HMM.



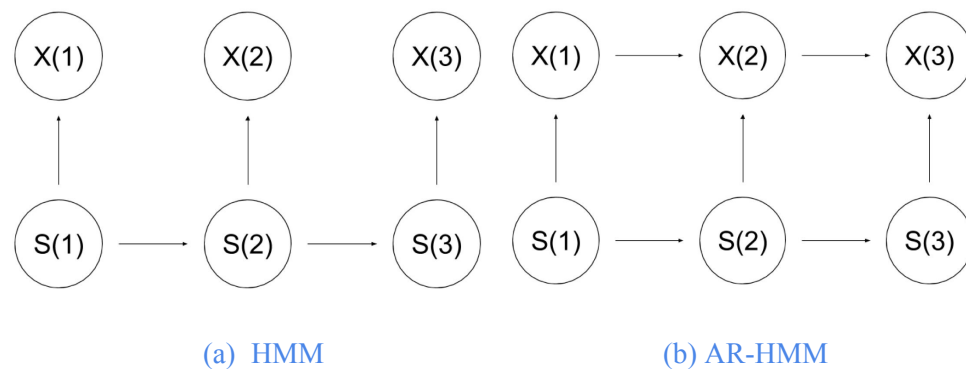
This figure shows the probability of being each state when we assumed four states, including 2 intermediate states. The state can change from 1 to 4 through 2u and 3u or directly from 1 to 4. Also, the state can change from 4 to 1 through 3d and 2d or directly from 4 to 1. Most state changes occur without intermediate states.

- The reviewer suggested adding more description of the HMM. See the reply to the Reviewer #1's first comment as well. The following paragraph can be added as an introduction of HMM and AR-HMM as well.

Both HMM and AR-HMM consist of observed data $X(t)$ and two kinds of hidden states $s(t)$. The measured data from the sediment core correspond to $X(t)$, and the unobserved state for each observed data corresponds to $s(t)$. The unobserved hidden states are analogous to the terms “regimes” that are described in climate studies. The states are hidden because they are to be determined from relationships within the data by the model, rather than indicated directly, e.g., if the value of one variable indicated which regime the data was in at any given time.

The figure below illustrates the dependencies among hidden states (S) and observed data (X) of the two models. State dependencies are the same in both models. Both models

have hidden states that have the Markov property, meaning that the future state does not depend on the past states given the present states. The difference between the two models is the dependency between observations (X) that are adjacent in time to each other ($X(1)$ to $X(2)$). In the HMM, a current observation is solely dependent on present observations and the current state. The HMM assumes that a current observation follows the normal distribution with means and variances determined by its state. Thus, a current set of observations is independent of other sets of observations at other times, although its state does depend on what state was determined at a previous time. In the AR-HMM, a current observation depends not only on a current and previous state but also on the previous observations. Therefore, the AR-HMM model allows for examination of causal relationships among the observed variables--inferring connections beyond just “regime” shifts and into relationships such as SST predicts productivity at a later time.



Dependencies among observations $X(t)$ and hidden states $S(t)$ for (a) HMM and (b) AR-HMM. Nodes are connected with an arrow if one node at the head of an arrow depends on another node at the origin of an arrow.

Also, after the equation (1), we can add the following statement to explicitly state that the equation (1) is for both states: “This equation represents the two-state AR-HMM, where the state value $s(t)$ can be either 1 (Noisy state) or 2 (Calm state).”

The reviewer also suggested investigating other methods that do not require the interpolation procedure. There exist some methods that can be applied to irregularly sampled data without the interpolation procedure, as the reviewer commented. However, those methods are not easily applicable when we consider multiple regimes and regime changes. In this study, we used the first order autoregressive model, meaning the model includes one immediately preceding value. To use this model, we need an estimated values for each missing observation value. Therefore, we estimated the expectation of the missing values by using the Kalman filter. To develop a method to analyze irregularly spaced samples with regime changes but without interpolations would be an interesting topic for future study, but is outside the scope of this paper and probably also the audience of this journal.

6. The reviewer suggested adding more statistical analysis for the conclusion that the four proxies are not causally related. If the four proxies are causally related, then the off-diagonal entry values

of the autocovariance matrix should be similar to the diagonal entry values. If the off-diagonal entry values and diagonal entry values are comparable, then further analysis would be required. However, the difference in absolute values between diagonal and off-diagonal values are extremely large; See the estimated autocovariance regression matrix (θ) values in Table 2. The smallest value of the diagonal entries is 13 times larger than the largest value of the off-diagonal entries, so we did not conduct a further statistical analysis. Also, it is not desirable to apply a frequentists approach when using a Bayesian model. Ideally, we would prefer a full Bayesian model comparison. However, that requires summing and integrating over all the unknowns which is not possible in this because there exist too many interconnections in the model parameters to complete all the sums and integrals simultaneously.

Or, to put it more simply, the AR-HMM results are not quantitatively different from the HMM, they are qualitatively different. The dominant relationships change entirely, not just slightly or within the range of statistical uncertainty, the whole nature of the AR-HMM and HMM models is different. We note again that this is not expected, unless the additional degrees of freedom of the AR-HMM system have systematically allowed for different behavior. This change is the nature of “Granger causality”, i.e., if the system changes qualitatively when past values are allowed as predictors, it implies that same-time correlations cannot be causations.

7. Reviewer’s comment: *In Fig. 2, 3 and 4, please draw in function of the time/age rather than the time step.*

The sediment record is retrieved from a single sediment core located at the central Peru margin. Also, the site of the sample has a high and steady sedimentation rate across the Holocene, and the samples are obtained from 2cm slices taken every 5cm. Therefore, we assumed a constant accumulation rate. Because we assumed a constant accumulation rate, we did not use additional age model to estimate the age for each sample and used the time step instead of the time or age. The accumulation rate does not affect the cross-correlations or autocorrelations directly.

8. Reviewer’s comment: *In Fig. 5, you should indicate state 1 for the top figures and state 2 for the bottom ones.*

The top and bottom rows represent the correlation matrices of the state 1 and state 2, respectively. We can add this information to the figure and also the figure description.

9. Reviewer’s comment: *In table 1, you should point out and explain why the 4-4 matrices are symmetric.*

The 4-by-4 matrix in Table 1 is the covariance matrix of the error term. The (i,j) entry of the covariance matrix is the covariance of the i th entry and j th entry. Because the covariance is commutative, meaning that $cov(\varepsilon_i, \varepsilon_j) = cov(\varepsilon_j, \varepsilon_i)$, the covariance matrix is symmetric. It can be seen from the definition of the covariance matrix as well. The (i,j) entry of the covariance matrix is defined as $\Sigma_{ij} = cov(\varepsilon_i, \varepsilon_j) = E[(\varepsilon_i - E[\varepsilon_i])(\varepsilon_j - E[\varepsilon_j])]$. Therefore, Σ_{ij} is equal to Σ_{ji} , and the matrix is symmetric ([5]).

10. Reviewer's comment: *In tables 3 and 4, please provide the explicit formulas for the computation of those matrices.*

The correlation matrix is defined as $corr(x_i, x_j) = \frac{cov(x_i, x_j)}{\sigma_i \sigma_j}$, where *cov* means covariance, and σ_i and σ_j are the standard deviations of x_i and x_j , respectively.

References

- [1] Chazen, C. R., M. A. Altabet, and T. D. Herbert (2009), Abrupt mid-Holocene onset of centennial-scale climate variability on the Peru-Chile Margin, *Geophys. Res. Lett.*, 36, L18704, doi:10.1029/2009GL039749.
- [2] Blaauw, M. and Christen, J.A., 2011. Flexible paleoclimate age-depth models using an autoregressive gamma process. *Bayesian analysis*, 6(3), pp.457-474.
- [3] Lin, L., Khider, D., Lisiecki, L.E. and Lawrence, C.E., 2014. Probabilistic sequence alignment of stratigraphic records. *Paleoceanography*, 29(10), pp.976-989.
- [4] Ahn, S., Khider, D., Lisiecki, L.E. and Lawrence, C.E., 2017. A probabilistic Pliocene–Pleistocene stack of benthic $\delta^{18}\text{O}$ using a profile hidden Markov model. *Dynamics and Statistics of the Climate System*, 2(1).
- [5] Wackerly, D., Mendenhall, W. and Scheaffer, R., 2007. *Mathematical statistics with applications*. Nelson Education.