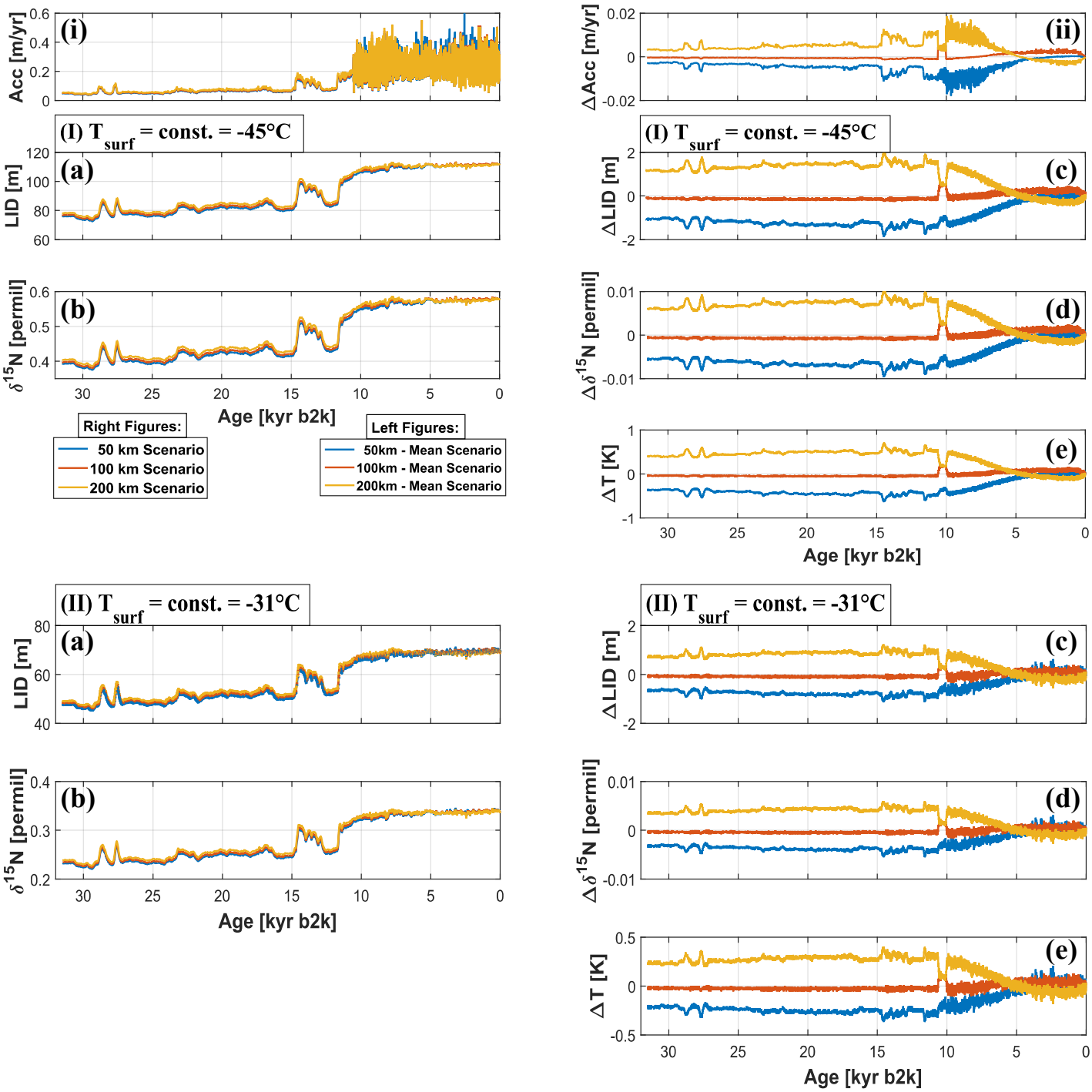

Figure S01: Possible accumulation rate scenarios (i), the deviation from the averaged scenario (ii) and resulting modelled lock-in-depth (LID) (a) and δ15N values (b) and deviations (c), (d), as well as, corresponding temperature uncertainties ΔT (e), for a constant temperature input (I) -45 °C and (II) -31 °C.

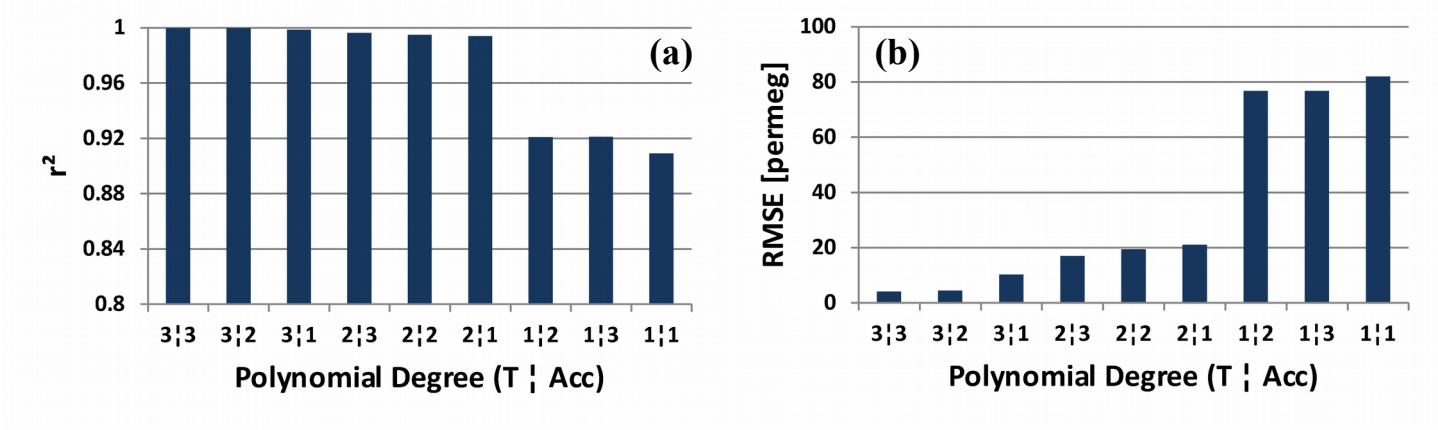

Figure S02: Static model behaviour: Two-dimensional (temperature T, accumulation rate Acc) polynomial surface fits. Correlation coefficients (a) and root mean squared error (b) versus polynomial degrees in temperature (T) and accumulation rate (Acc).

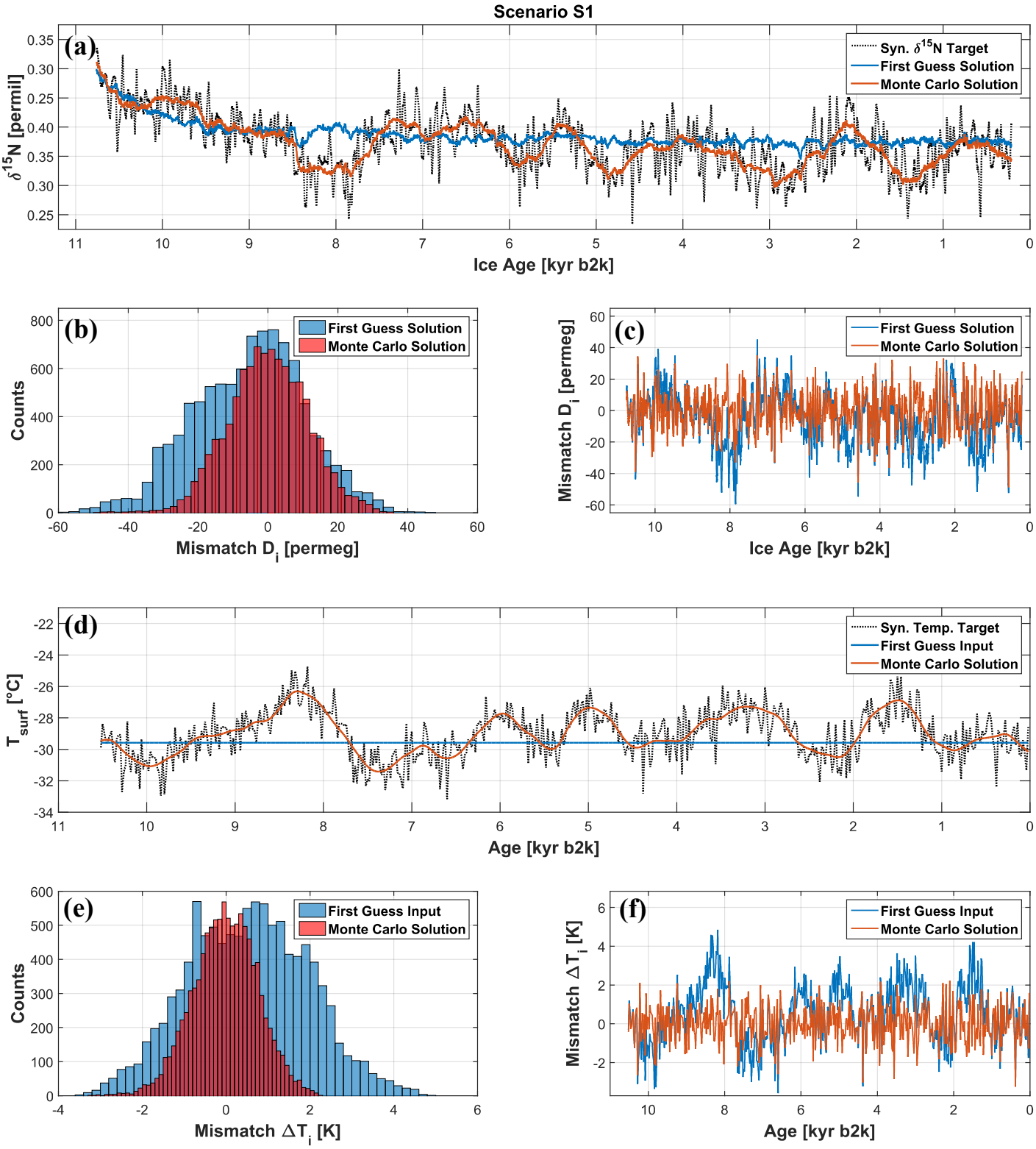

Figure S03: (a-c) First guess vs. Monte Carlo $\delta^{15}N$ solution for the scenario S1; (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series for the first guess input (blue line) and Monte Carlo solution (red line); (b) Histogram shows the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (c) Time series for the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (d-f) First guess vs. Monte Carlo surface temperature solution for the scenario S1; (d) Synthetic surface temperature target (black dotted line), first guess temperature input (blue line) and Monte Carlo solution (red line); (e) Histogram shows the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target; (f) Time series for the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target;

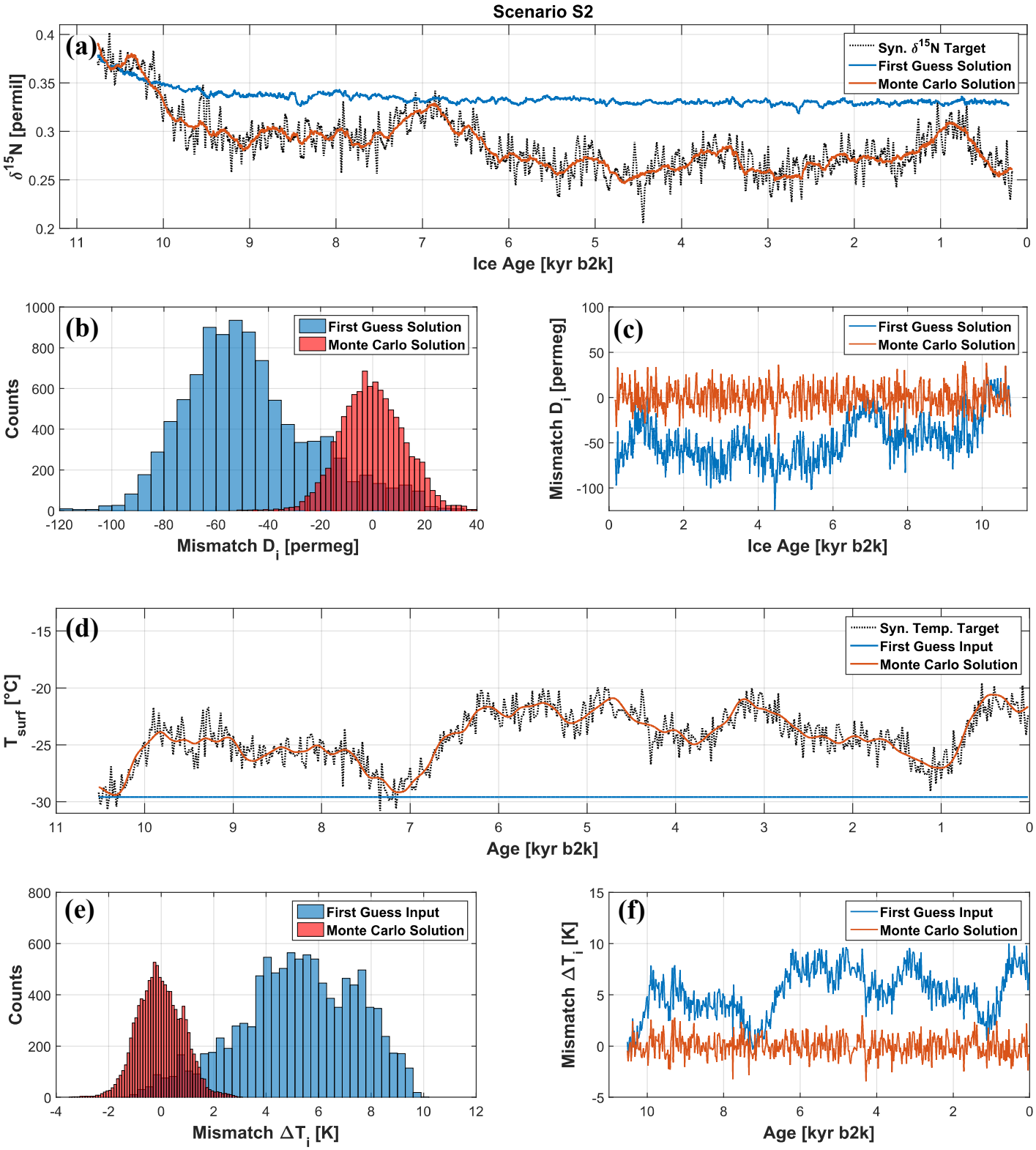

Figure S04: (a-c) First guess vs. Monte Carlo $\delta^{15}N$ solution for the scenario S2; (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series for the first guess input (blue line) and Monte Carlo solution (red line); (b) Histogram shows the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (c) Time series for the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (d-f) First guess vs. Monte Carlo surface temperature solution for the scenario S2; (d) Synthetic surface temperature target (black dotted line), first guess temperature input (blue line) and Monte Carlo solution (red line). (e) Histogram shows the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target; (f) Time series for the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target;

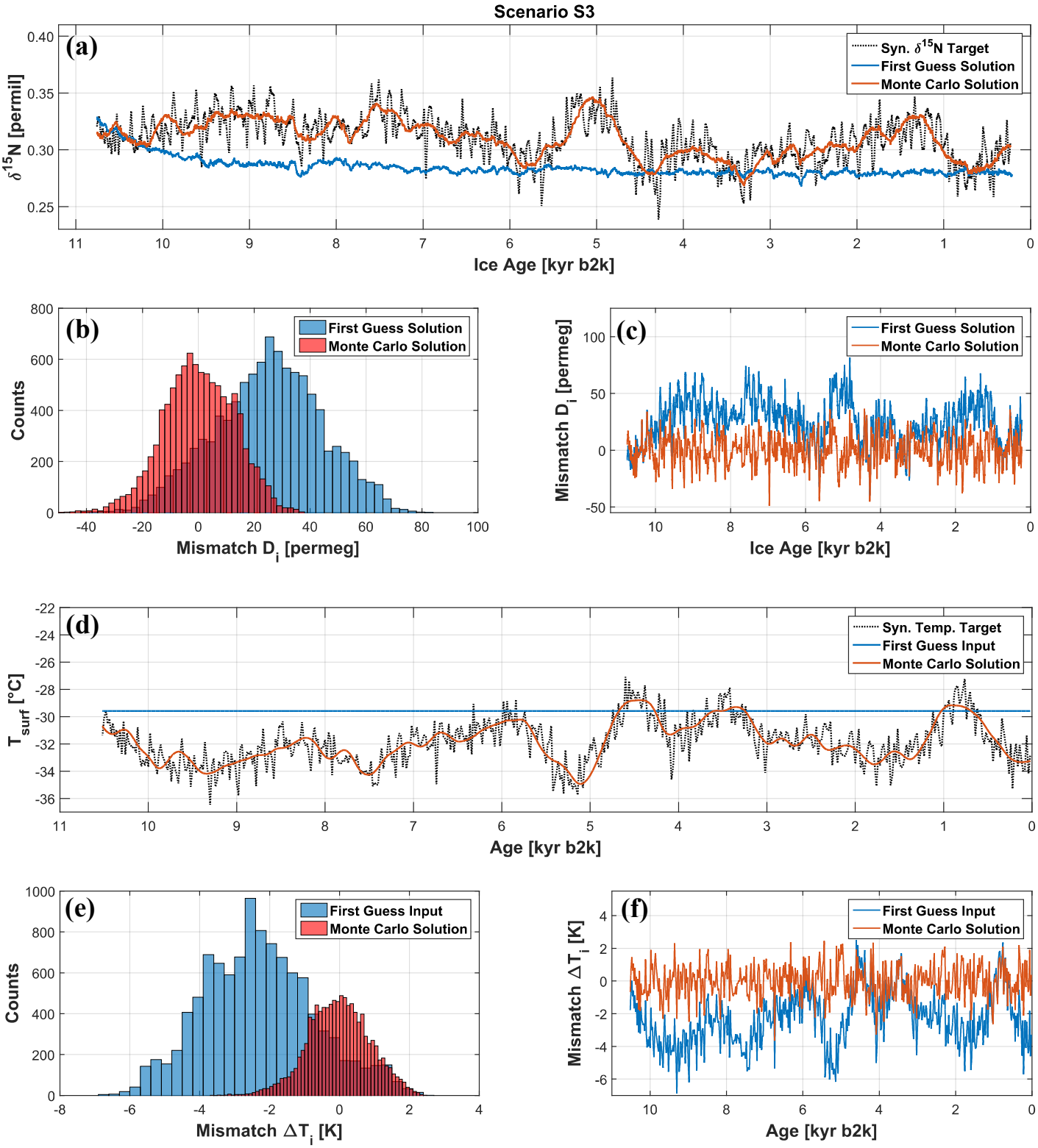

Figure S05: (a-c) First guess vs. Monte Carlo $\delta^{15}N$ solution for the scenario S3; (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series for the first guess input (blue line) and Monte Carlo solution (red line); (b) Histogram shows the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (c) Time series for the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (d-f) First guess vs. Monte Carlo surface temperature solution for the scenario S3; (d) Synthetic surface temperature target (black dotted line), first guess temperature input (blue line) and Monte Carlo solution (red line). (e) Histogram shows the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target; (f) Time series for the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target;

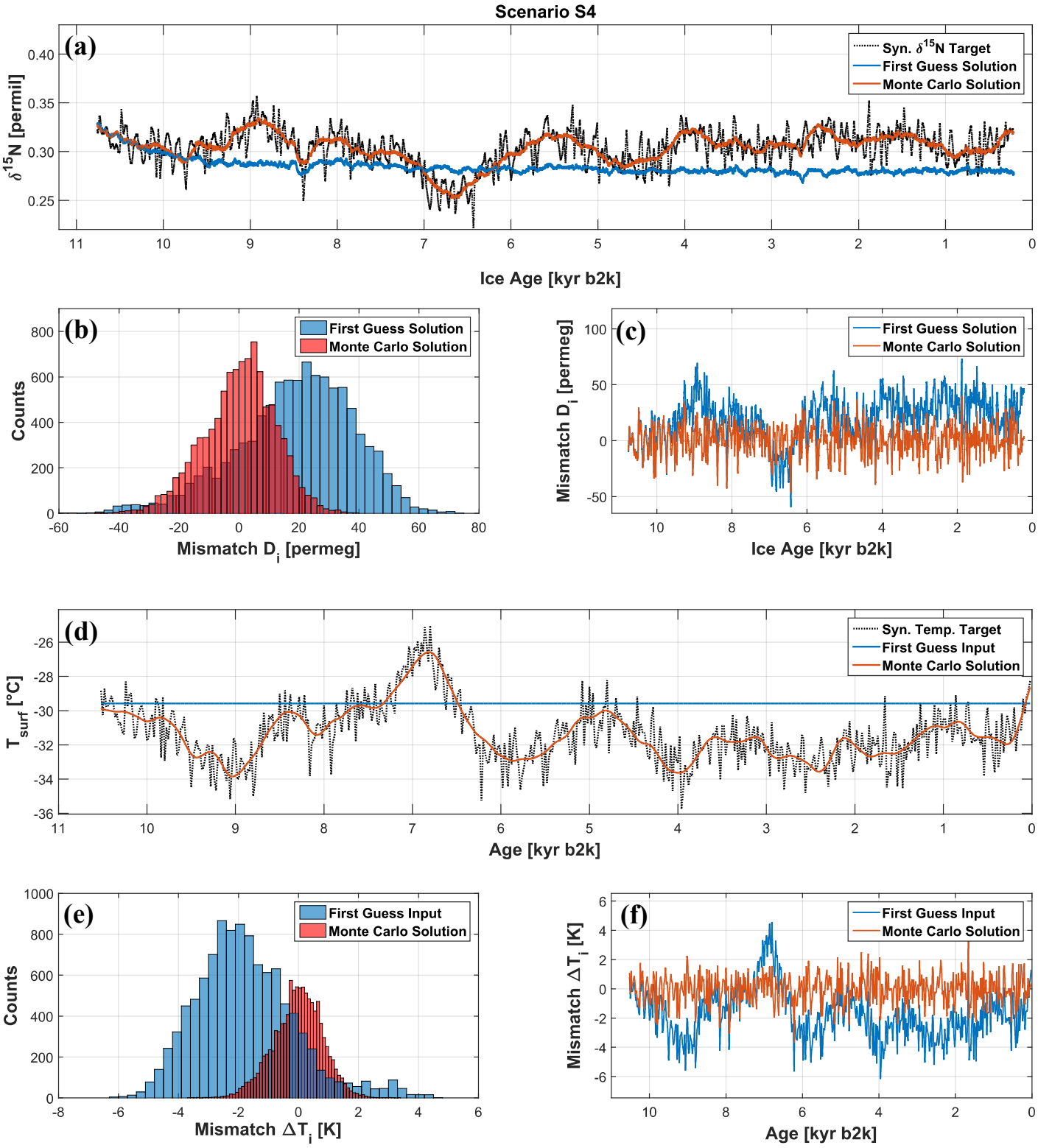

Figure S06: (a-c) First guess vs. Monte Carlo $\delta^{15}N$ solution for the scenario S4; (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series for the first guess input (blue line) and Monte Carlo solution (red line); (b) Histogram shows the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (c) Time series for the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (d-f) First guess vs. Monte Carlo surface temperature solution for the scenario S4; (d) Synthetic surface temperature target (black dotted line), first guess temperature input (blue line) and Monte Carlo solution (red line). (e) Histogram shows the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target; (f) Time series for the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target;

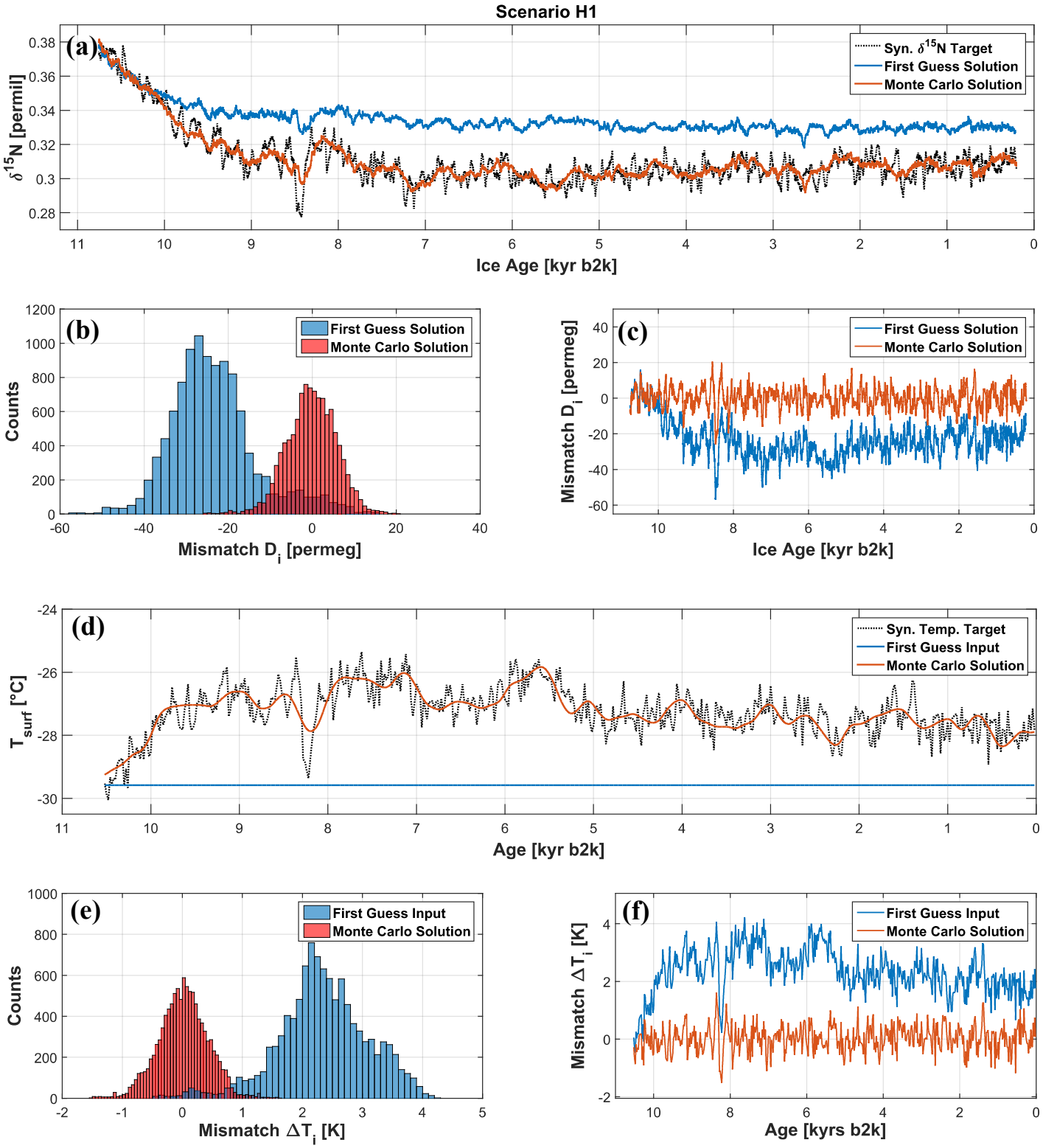

Figure S07: (a-c) First guess vs. Monte Carlo δ15N solution for the scenario H1; (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series for the first guess input (blue line) and Monte Carlo solution (red line); (b) Histogram shows the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (c) Time series for the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (d-f) First guess vs. Monte Carlo surface temperature solution for the scenario H1; (d) Synthetic surface temperature target (black dotted line), first guess temperature input (blue line) and Monte Carlo solution (red line). (e) Histogram shows the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target; (f) Time series for the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target;

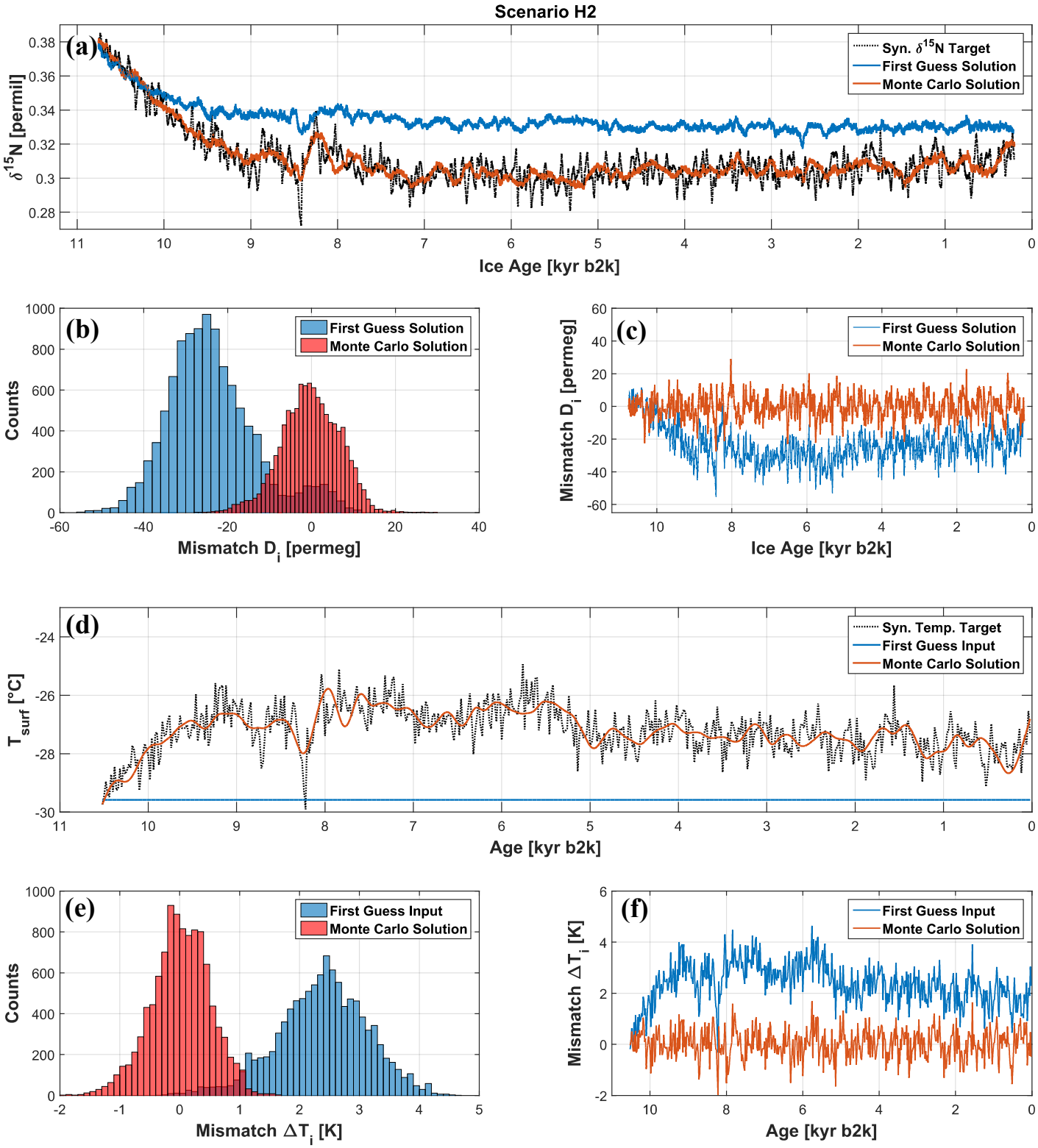

Figure S08: (a-c) First guess vs. Monte Carlo $\delta^{15}N$ solution for the scenario H2; (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series for the first guess input (blue line) and Monte Carlo solution (red line); (b) Histogram shows the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (c) Time series for the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (d-f) First guess vs. Monte Carlo surface temperature solution for the scenario H2; (d) Synthetic surface temperature target (black dotted line), first guess temperature input (blue line) and Monte Carlo solution (red line). (e) Histogram shows the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target; (f) Time series for the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target;

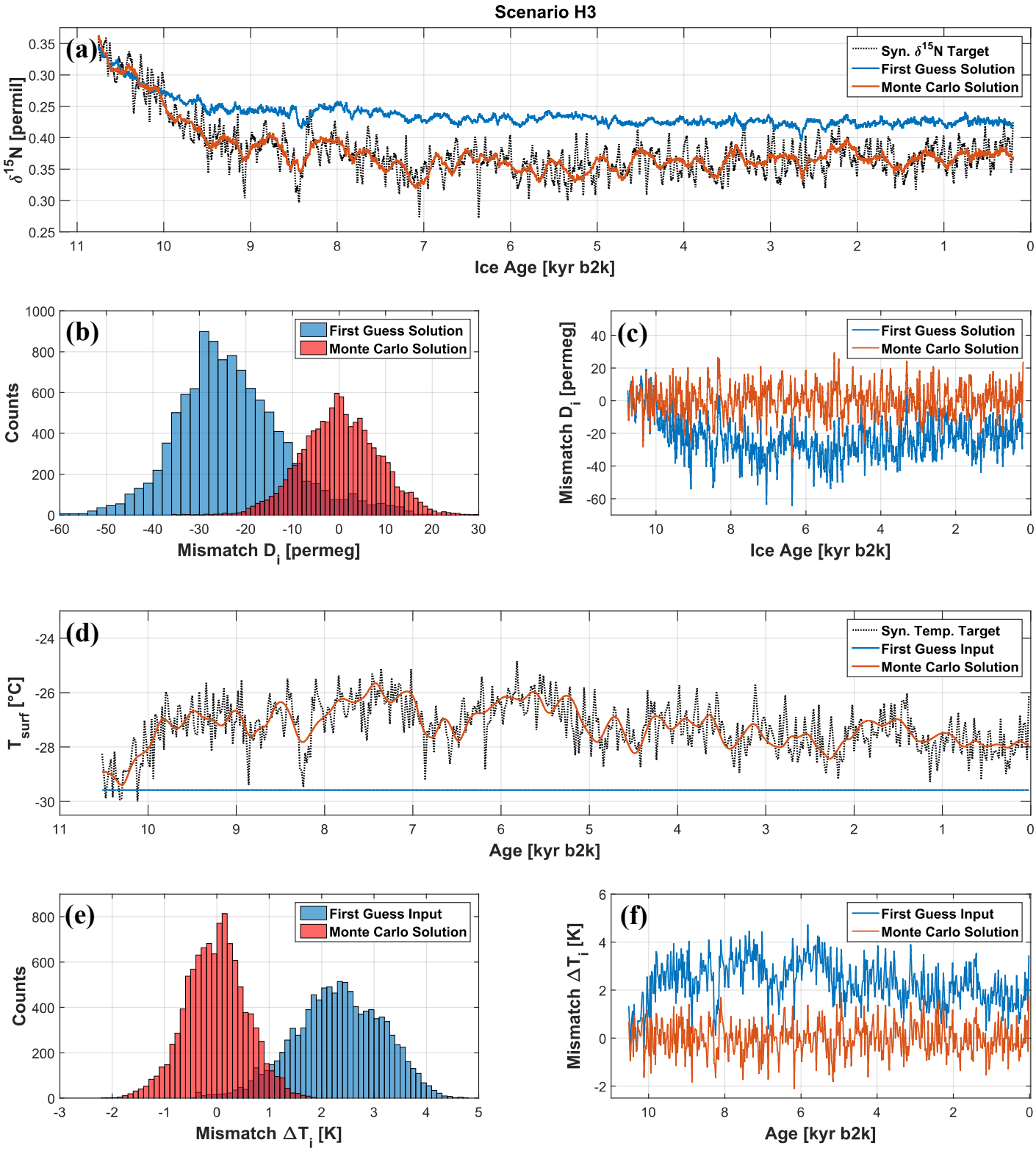

Figure S09: (a-c) First guess vs. Monte Carlo $\delta^{15}N$ solution for the scenario H3; (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series for the first guess input (blue line) and Monte Carlo solution (red line); (b) Histogram shows the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (c) Time series for the pointwise mismatches D_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic target; (d-f) First guess vs. Monte Carlo surface temperature solution for the scenario S5; (d) Synthetic surface temperature target (black dotted line), first guess temperature input (blue line) and Monte Carlo solution (red line). (e) Histogram shows the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target; (f) Time series for the pointwise mismatches ΔT_i for the first guess solution (blue) and the Monte Carlo solution (red) versus the synthetic surface temperature target;

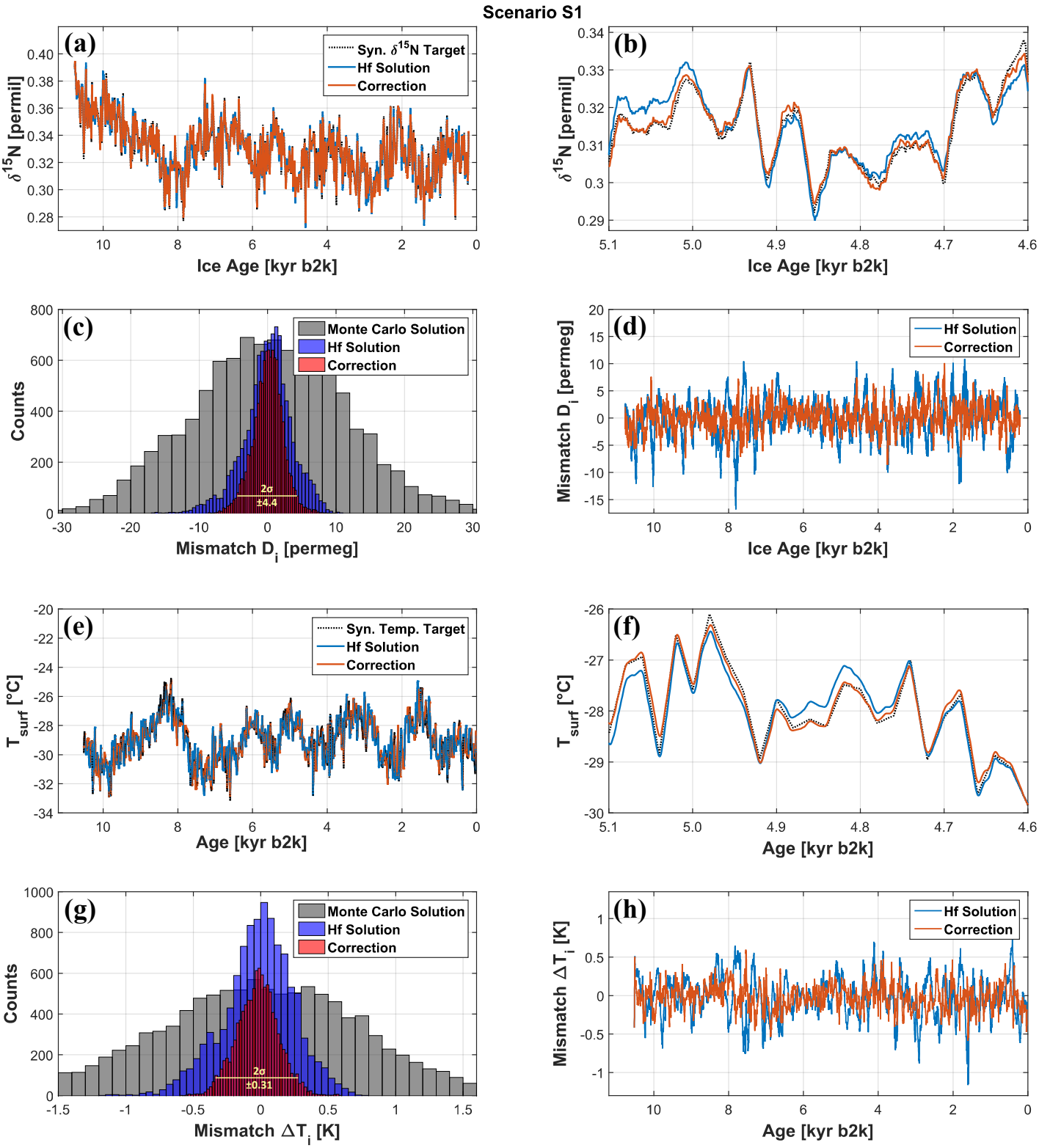

Figure S10: (a-d): (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series after adding high frequency information (blue line) and correction (red line) for the scenario S1; (b) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (c) Histogram shows the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 4.4 permeg (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (d) Time series for the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the high frequency solution (blue) and the correction (red); (e-h): (e) Synthetic temperature target (black dotted line), modelled temperature time series after adding high frequency information (blue line) and correction (red line); (f) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (g) Histogram shows the pointwise mismatches ΔT_i from the synthetic temperature target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 0.31 K (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (h) Time series for the pointwise mismatches ΔT_i from the synthetic temperature target for the high frequency solution (blue) and the correction (red);

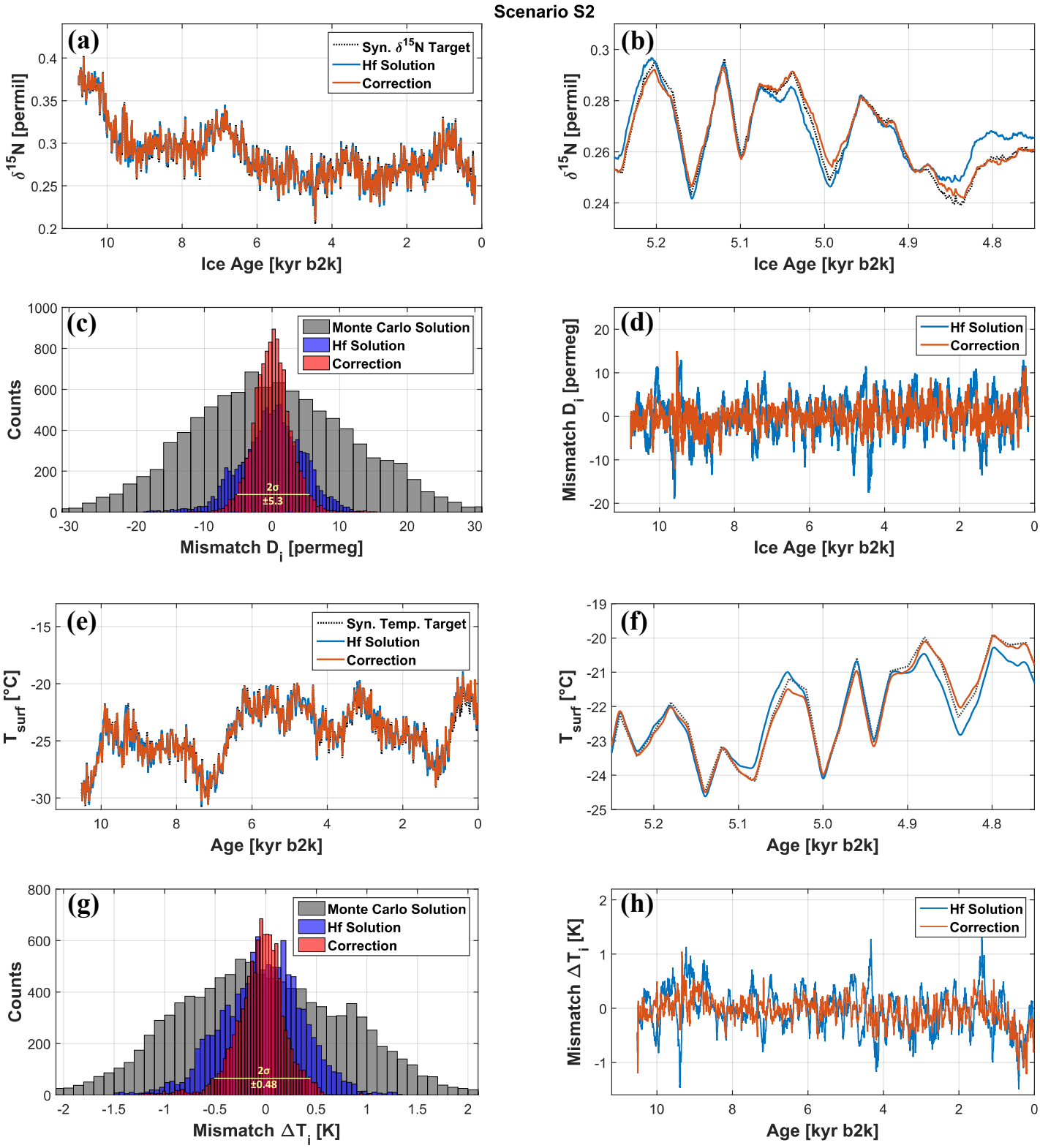

Figure S11: (a-d): (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series after adding high frequency information (blue line) and correction (red line) for the scenario S2; (b) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (c) Histogram shows the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 5.3 permeg (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (d) Time series for the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the high frequency solution (blue) and the correction (red); (e-h): (e) Synthetic temperature target (black dotted line), modelled temperature time series after adding high frequency information (blue line) and correction (red line); (f) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (g) Histogram shows the pointwise mismatches ΔT_i from the synthetic temperature target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 0.48 K (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (h) Time series for the pointwise mismatches ΔT_i from the synthetic temperature target for the high frequency solution (blue) and the correction (red);

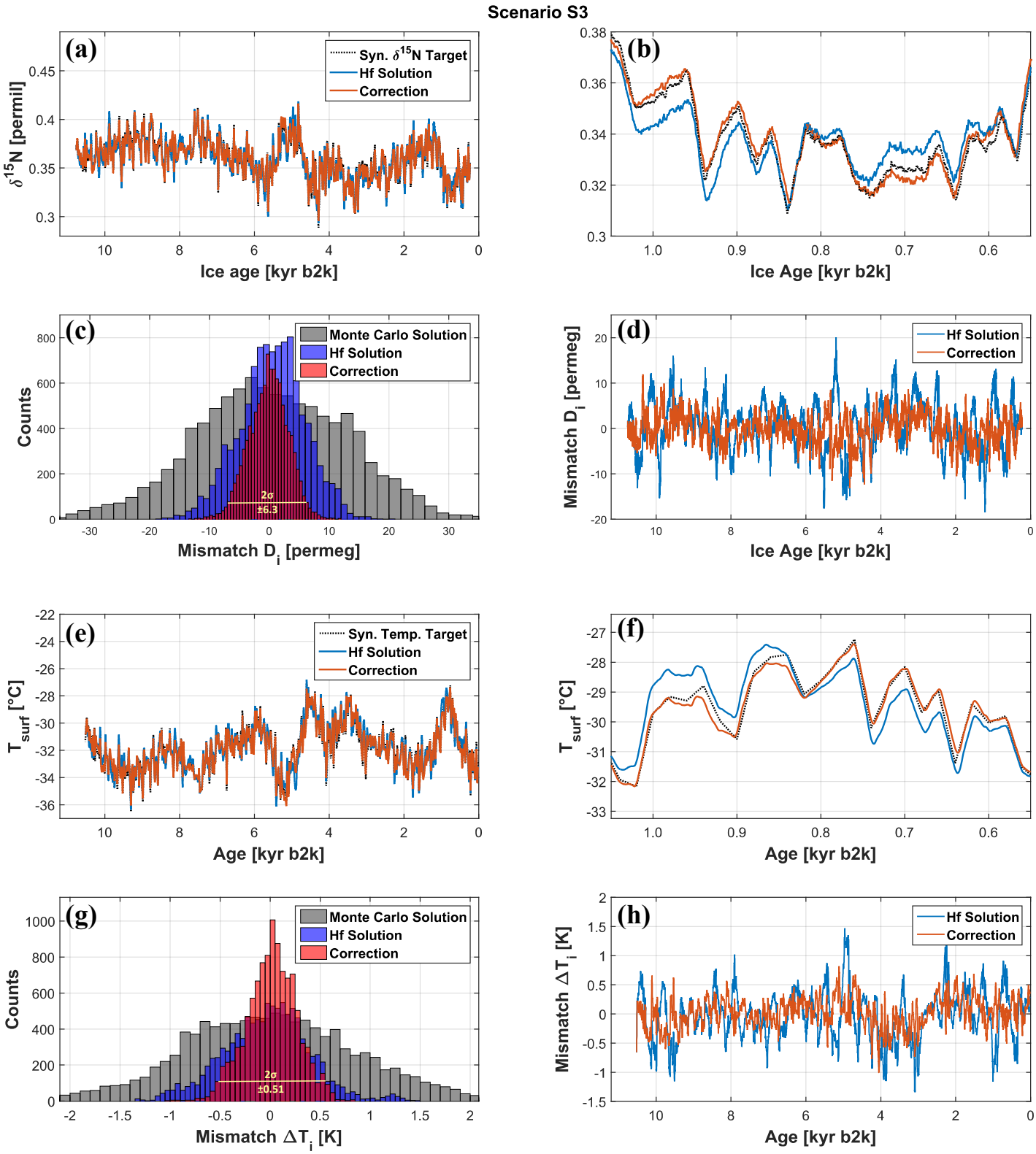

Figure S12: (a-d): (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series after adding high frequency information (blue line) and correction (red line) for the scenario S3; (b) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (c) Histogram shows the pointwise mismatches D_i from the synthetic δ15N target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 6.3 permeg (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (d) Time series for the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the high frequency solution (blue) and the correction (red); (e-h): (e) Synthetic temperature target (black dotted line), modelled temperature time series after adding high frequency information (blue line) and correction (red line); (f) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (g) Histogram shows the pointwise mismatches ΔT_i from the synthetic temperature target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 0.51 K (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (h) Time series for the pointwise mismatches ΔT_i from the synthetic temperature target for the high frequency solution (blue) and the correction (red);

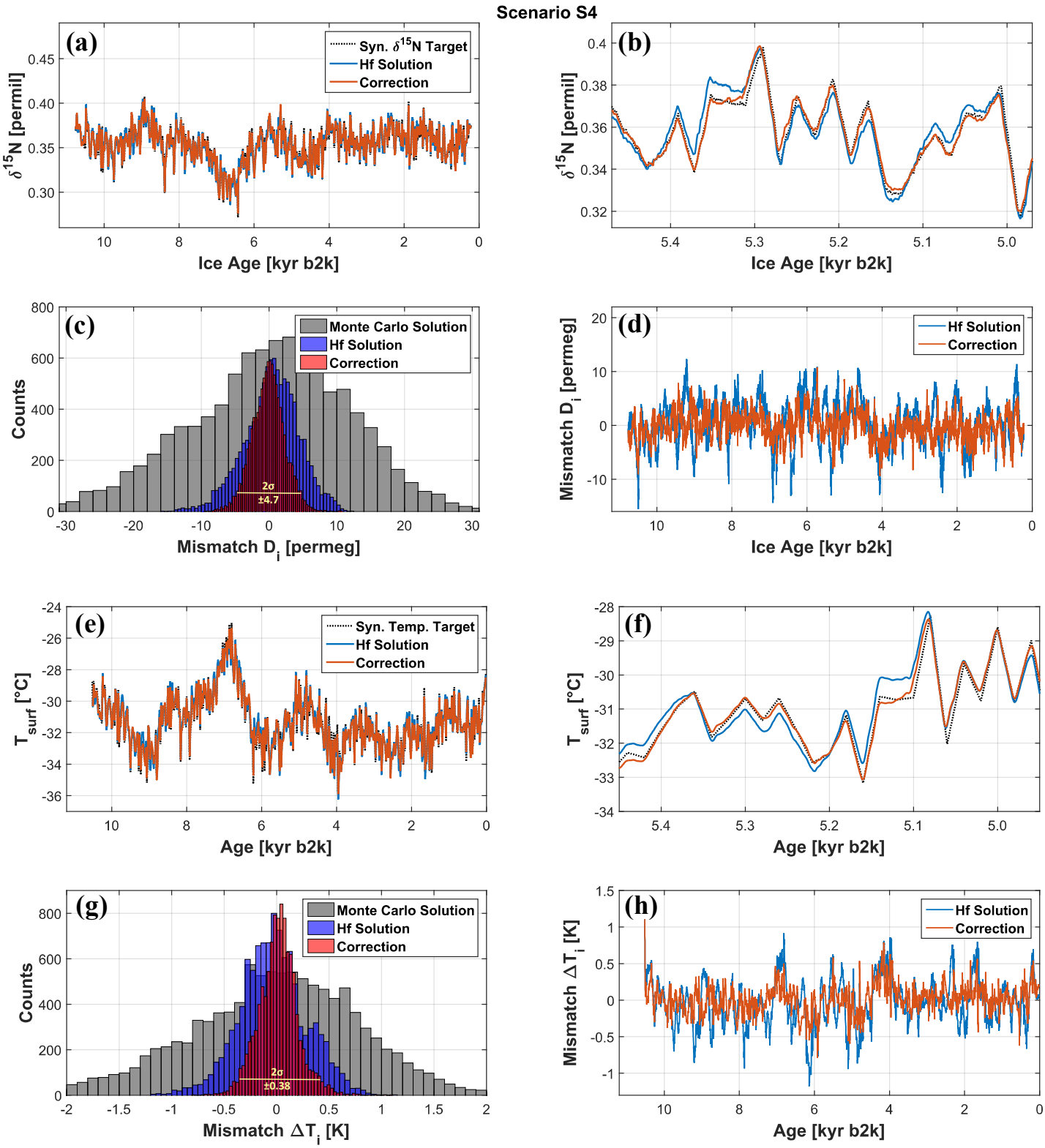

Figure S13: (a-d): (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series after adding high frequency information (blue line) and correction (red line) for the scenario S4; (b) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (c) Histogram shows the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 4.7 permeg (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (d) Time series for the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the high frequency solution (blue) and the correction (red); (e-h): (e) Synthetic temperature target (black dotted line), modelled temperature time series after adding high frequency information (blue line) and correction (red line); (f) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (g) Histogram shows the pointwise mismatches ΔT_i from the synthetic temperature target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 0.38 K (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (h) Time series for the pointwise mismatches ΔT_i from the synthetic temperature target for the high frequency solution (blue) and the correction (red);

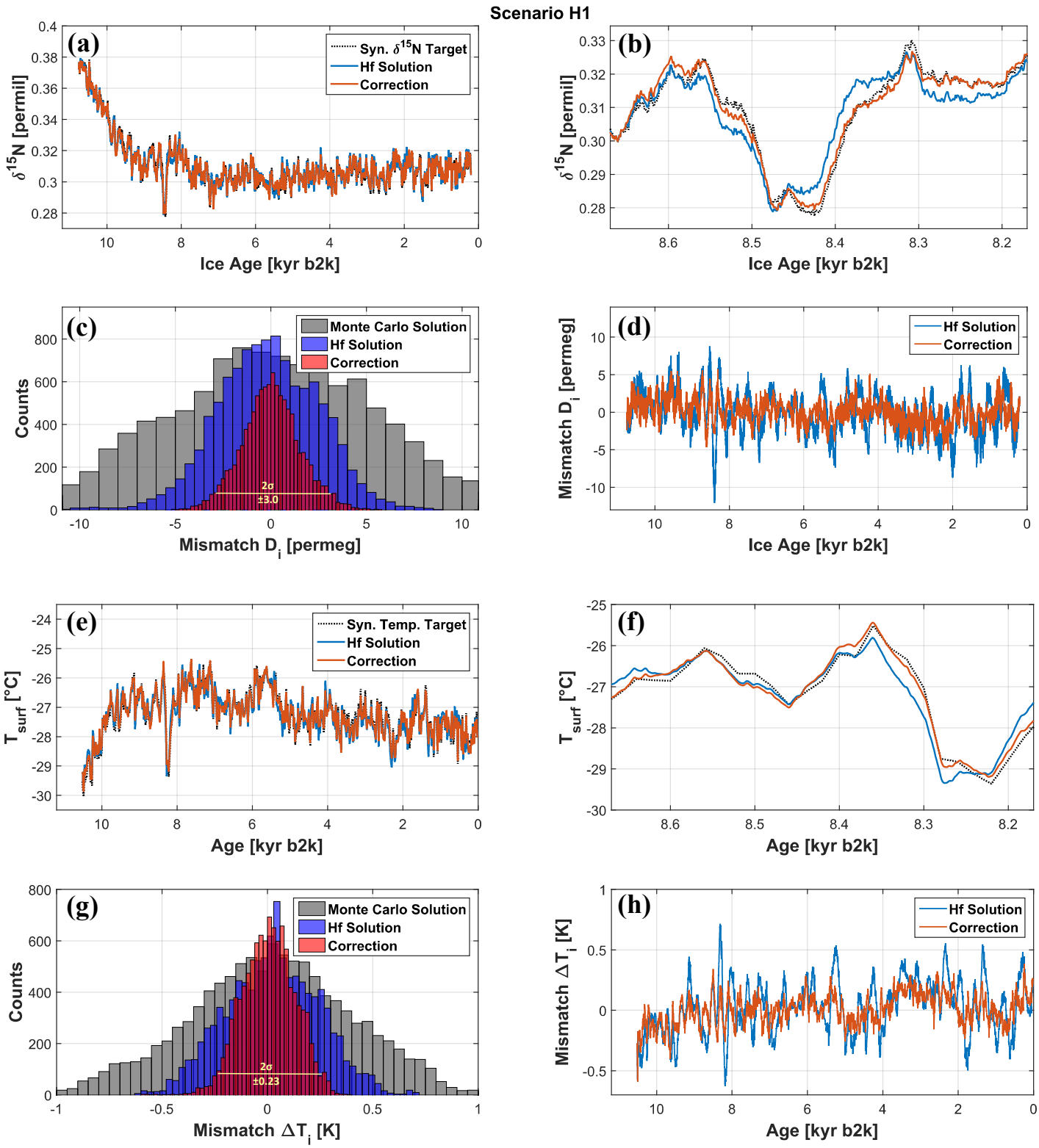

Figure S14: (a-d): (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series after adding high frequency information (blue line) and correction (red line) for the scenario H1; (b) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (c) Histogram shows the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 3.0 permeg (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (d) Time series for the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the high frequency solution (blue) and the correction (red); (e-h): (e) Synthetic temperature target (black dotted line), modelled temperature time series after adding high frequency information (blue line) and correction (red line); (f) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (g) Histogram shows the pointwise mismatches ΔT_i from the synthetic temperature target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 0.23 K (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (h) Time series for the pointwise mismatches ΔT_i from the synthetic temperature target for the high frequency solution (blue) and the correction (red);

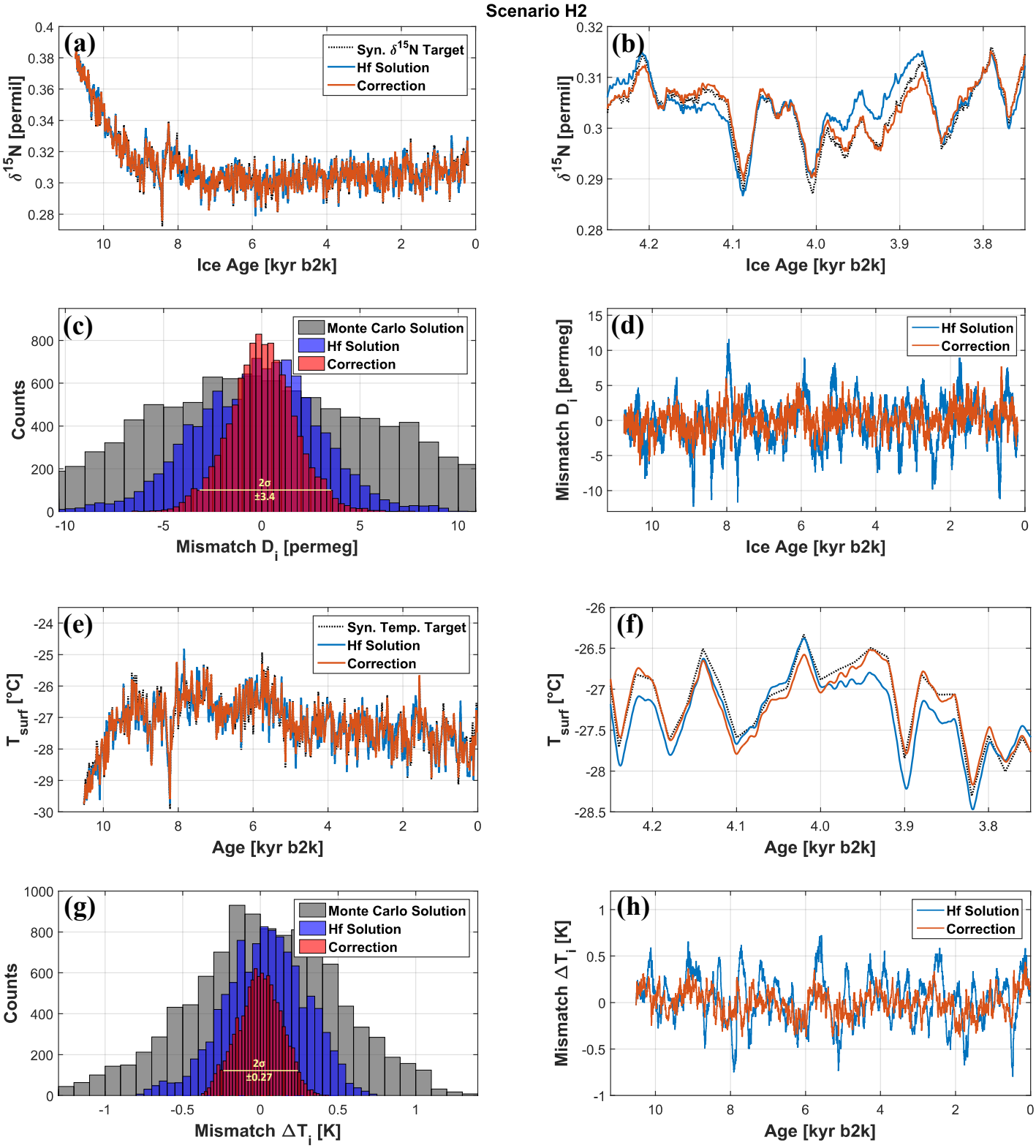

Figure S15: (a-d): (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series after adding high frequency information (blue line) and correction (red line) for the scenario H2; (b) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (c) Histogram shows the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 3.4 permeg (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (d) Time series for the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the high frequency solution (blue) and the correction (red); (e-h): (e) Synthetic temperature target (black dotted line), modelled temperature time series after adding high frequency information (blue line) and correction (red line); (f) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (g) Histogram shows the pointwise mismatches ΔT_i from the synthetic temperature target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 0.27 K (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (h) Time series for the pointwise mismatches ΔT_i from the synthetic temperature target for the high frequency solution (blue) and the correction (red);

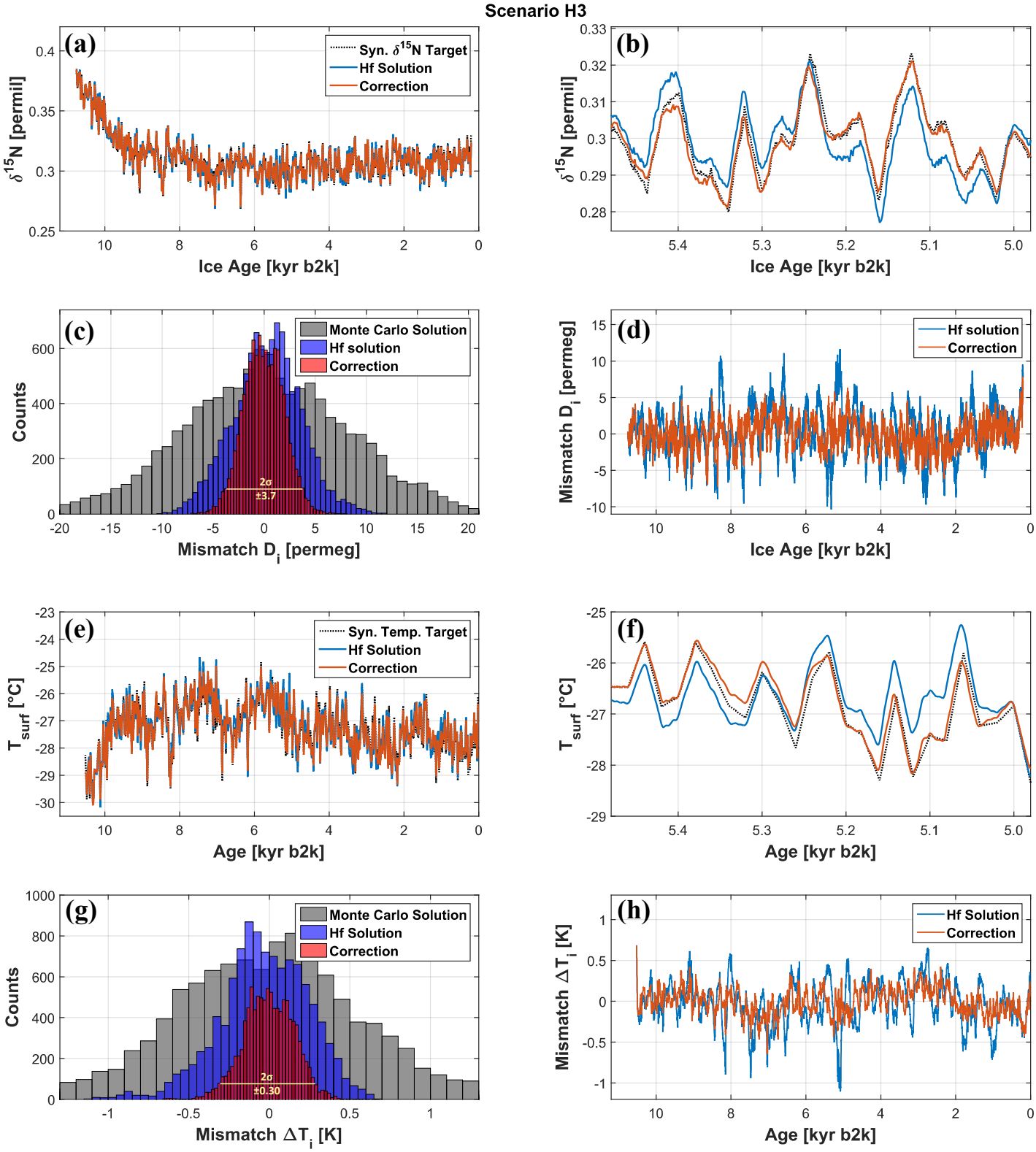

Figure S16: (a-d): (a) Synthetic $\delta^{15}N$ target (black dotted line), modelled $\delta^{15}N$ time series after adding high frequency information (blue line) and correction (red line) for the scenario H3; (b) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (c) Histogram shows the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 3.7 permeg (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (d) Time series for the pointwise mismatches D_i from the synthetic $\delta^{15}N$ target for the high frequency solution (blue) and the correction (red); (e-h): (e) Synthetic temperature target (black dotted line), modelled temperature time series after adding high frequency information (blue line) and correction (red line); (f) Zoom in for a randomly chosen 500 yr interval shows the decrease of the mismatch after the correction compared to the high frequency solution; (g) Histogram shows the pointwise mismatches ΔT_i from the synthetic temperature target for the Monte Carlo solution (grey), the high frequency solution (blue) and the correction (red); The 95 % quantile is 0.30 K (yellow line) and used as an estimate for 2σ uncertainty of the final solution; (h) Time series for the pointwise mismatches ΔT_i from the synthetic temperature target for the high frequency solution (blue) and the correction (red);

Tables (supplementary):

Polynomial: (T, Acc)	Coefficients (95% confidence bounds)		Unit	r^2	RMSE [permeg]
Static behaviour:					
(3,2) $\delta^{15}N(T,Acc) = p_{00} + p_{10}\cdot T + p_{01}\cdot Acc + p_{20}\cdot T^2 + p_{02}\cdot Acc^2 + p_{30}\cdot T^3 + p_{11}\cdot T\cdot Acc + p_{21}\cdot T^2\cdot Acc + p_{12}\cdot T\cdot Acc^2$	p_{00}:	106.4 (104, 108.7)	[‰]	0.9997	4.5
	p_{10}:	-1.253 (-1.283, -1.224)	$[‰/K]$		
	p_{01}:	29.16 (27.99, 30.32)	$[yr\cdot ‰/m]$		
	p_{20}:	$4.949\cdot 10^{-3}$ $(4.824, 5.074)\cdot 10^{-3}$	$[‰/K^2]$		
	p_{02}:	-6.887 (-7.505, -6.268)	$\left[\frac{yr^2\cdot ‰}{m^2}\right]$		
	p_{30}:	$-6.536\cdot 10^{-6}$ $(-6.711, -6.361)\cdot 10^{-6}$	$[‰/K^3]$		
	p_{11}:	-0.2075 (-0.217, -0.198)	$\left[\frac{yr\cdot ‰}{m\cdot K}\right]$		
	p_{21}:	$3.707\cdot 10^{-4}$ $(3.51, 3.904)\cdot 10^{-4}$	$\left[\frac{yr\cdot ‰}{m\cdot K^2}\right]$		
	p_{12}:	0.02594 (0.02335, 0.02853)	$\left[\frac{yr^2\cdot ‰}{m^2\cdot K}\right]$		
Dynamic behaviour:					
(3,2) $\delta^{15}N(T,Acc) = p_{00} + p_{10}\cdot T + p_{01}\cdot Acc + p_{20}\cdot T^2 + p_{02}\cdot Acc^2 + p_{30}\cdot T^3 + p_{11}\cdot T\cdot Acc + p_{21}\cdot T^2\cdot Acc + p_{12}\cdot T\cdot Acc^2$	p_{00}:	-1697 (-1821, -1574)	[‰]	0.6995	34.2
	p_{10}:	23.07 (21.44, 24.71)	$[‰/K]$		
	p_{01}:	-2221 (-2332, -2110)	$[yr\cdot ‰/m]$		
	p_{20}:	-0.1044 (-0.1116, -0.09714)	$[‰/K^2]$		
	p_{02}:	-392.3 (-422.8, -361.8)	$\left[\frac{yr^2\cdot ‰}{m^2}\right]$		
	p_{30}:	$1.571\cdot 10^{-4}$ $(1.465, 1.677)\cdot 10^{-4}$	$[‰/K^3]$		
	p_{11}:	19.47 (18.5, 20.45)	$\left[\frac{yr\cdot ‰}{m\cdot K}\right]$		
	p_{21}:	-0.04268 (-0.04481, -0.04055)	$\left[\frac{yr\cdot ‰}{m\cdot K^2}\right]$		
	p_{12}:	1.699 (1.571, 1.827)	$\left[\frac{yr^2\cdot ‰}{m^2\cdot K}\right]$		

Table S01: Two dimensional polynomial surface fit for the static and dynamic model behaviour: coefficients with units, correlationcoefficient r^2 and root-mean-squared error RMSE for the polynomial of degree 3 in temperature (T) and degree 2 in accumulation rate (Acc). Note: temperature is in Kelvin.

Scenario:	cop [yr]	s
S1	1135	0.2065
S2	1007	0.3967
S3	1177	0.4002
S4	1315	0.2952
S5	1244	0.2388

Table S02: Cut off periods and s values for creating the smooth temperature scenarios according to the Monte Carlo approach.