Final answer to the Comments

Acevedo et al.,

March 24, 2017

Dear Prof. Goosse and Dear reviewer #2,

Thank you so much for your constructive comments. In a new version of the manuscript we tried to remove unnecessary technical details and describe our motivations and findings in a clear format. We provided a track changes version of the manuscript at the end of this answer (Red is deleted and blue is added). We reply to all your comments here:

1 Answer to Reviewer 2

We wish to thank you so much for your constructive review. It would be our pleasure to do all the modifications and make the improvements you have suggested, in the next version of the manuscript. We answer your comments (*italic*) point by point (**Bold**):

The authors have done a fine job addressing my comments and I feel the manuscript may be suitable for publication after addressing several large remaining issues with the text. I'm suggesting further revisions for additional problems with language and brevity. I would like to thank the authors for a much clearer and better-written manuscript and for addressing our concern about the time-varying soil moisture through further work, and for making a large effort to revise and add to the text for clarity and flow. However, the manuscript is still lacking transitions and sufficient detail, and is still too brief and in many places very confusing to follow. It is still a bit jarring to read and I think the abstract in particular could be more motivating and clear regarding the new science this work has added. The sectioning is over-done, and there are multiple places where you have a single sentence constituting an entire paragraph without any transition between them, an issue I highlighted in my first revision. For the new sections added, I couldn't take the time once again to heavily edit for language; please do go back over the additional sections to ensure that your sentence structure and word choices are sound. Guide the reader slowly through what you did and motivate it clearly. Some of the word choices and arguments are still awkward and hard to understand. A few that I caught are listed below. Please make appropriate edits throughout. Especially in the results and Discussion, I was really lost.

We agree. In the current version, the Discussion, abstract and introduction have been heavily edited. Now we open our main scientific questions in introduction and close them in Discussion. Regarding the introduction we give more citations on the previous studies around the forward modeling and its different applications as well as similar DA studies recently published. Regarding the discussion we merged the subsections together and formed a homogeneous story of our main results.

Page 1

8 rephrase "Our knowledge of the climate system...governing the evolution of the oceans and atmosphere."

9 "state of the flow" is too vague. Be specific!

12 delete comma after (forecast),

12 delete "Furthermore," and start sentence with The

21 'adjoint model' – unclear on what you mean here Could you add some more transitional sentences to guide the reader through the subsections in Section 2? Be careful about extra-short paragraphs that only have one sentence.... Combine where appropriate.

We merged many subsections and edited the whole section 2. We start with the Data Assimilation basics, KF, EnKF and time-averaged EnKF. Then we introduced the forward model representations along with the concept of Fuzzy Logic. Finally we presented our simulation design.

Page 5

12 as 'a' consequence, not 'the' , add 'any one observation may present ....with distant ones'

2.1.1 line 23: what is the 'sensor' – I don't think you have defined this yet...

Once again, the line numbering in this text just changes from 5-30 throughout which made it very difficult to give line-by-line comments.

23-25 the wording of this sentence is a bit confusing. You're trying to say that rain gauges and anemometers take hourly-scale measurements but paleoclimate data contain a time-averaged signal. It reads as if you're saying they're all the same. Revise for clarity.

Given that the "instantaneous" and "time-averaged" variables are frequently used by the climate community, we removed the sentence describing these two terms in the new version.

Page 6:

3 revise "comprising of a dynamical model" .... 'all which interact with each other'2.2.1 You need to define the V-S Lite acronym on first use and spell it out in this title. You do that later in the text at the moment and it's out of place. It needs to be here. 24 what is PLF ? Redefine, the reader has forgotten.

We have moved the text describing VSL into the introduction where we introduce TRW forward model as well as PLF.

Page 7

8: grown = growth

15: change definition of FL acronym to main text, not just in the title.

17: delete 'applied'

18: change to "FL has been applied in ecological ...

22: correspond = corresponds (add s)

Done.

Page 8, make line 8 into two sentences. Equation 15. Then, ...
2.4 Page 9
Change to "Experimental Design"
You define VSL here but it should be on the previous page.
16 change model to 'modeling'

Done.

Page 10: 10: boundaries THE model requires...
18 why is there a bullet here?
Page 11:
5 'lowest level of the state vector' – this is too much jargon. Do you just mean surface temperature?
12: what is meant by 'shifting of recorded variable?' unclear—revise
Delete Section 2.4.2 and move all of that paragraph to 3 Results.
19: change wording "We focus our analysis on temperature due to the larger error reduction in this field as compared to other variables...."
23: delete parentheses around near surface temperature and add "of near-surface temperatures" Page 12 typo line 26 "acenso"
Page 13 at the very top here you introduce a PRESCRIBED experiment for the first time. There must be some problem with your LateX file here—You meant to have two bullets above, one with SLAB and one with PRESCRIBED, but one got lost....correct above.

Yes there were some problems with the latexdiff. Now it is corrected. The bullets are for the two ocean configurations.

Page 14
Do not put a new paragraph between lines 10 and 11.... You're still discussing the same figure. You've got LaTeX error instances of "DIFdelbegin" and end throughout the text...
17 comma splice. This is also a good example of a long and convoluted sentence that appears throughout that is hard to follow and understand.
20-25 this is a bit of a weak statement regarding optimal network design. We already know that trees at higher latitudes are going to be more sensitive to changes in temperature, so saying that we should measure trees in Alaska instead of South Africa isn't very helpful. We're actually desperate for more terrestrial records from the Southern Hemisphere....I know what you're trying to say here, but you need to make a better argument.
23 contains comma splices

We edited the text. We added your comment as an extra sentence about the terrestrial records from the Southern Hemisphere.

25 change to "This study, where a DA method...." 32 I think you mean to say "constraint on the forecast motivates us to perform our DA experiments in an offline regime"

These sentences are already appearing in the discussion with a new format.

Page 15
10 delete comma after Prod
25-30 you say that a previous study supports Online .... But then you say "therefore we apply offline" – this doesn't follow or make sense. Review previous work and then compare it to what you did and say how it's different. Also, "performs a more realistic temporal varialbity" makes no sense—rephrase: 'simulates more realistic ..' and also, temporal variability of what?

Given that our DA attempt appeared to be in an "off-line regime", in the next step we have conducted an "off-line DA". There is a difference between "off-line regime" and "off-line DA method" which we made clearer in the new manuscript. The first one indicates that the forecast state of the DA has no skill (a DA with reinitialization after analysis) but the second is done using the forecast from free ensemble in an off-line strategy with no reinitialization. We have changed the sentence to "They concluded that in the off-line method temporal consistency of the model is lost."

Ever since the results section started I have felt lost trying to follow the writing. I also feel the sections are hyper-sectioned....out to four subsections is unnecessary. Just have two paragraphs and introduce them for 3.2.2.1, 3.2.2.2. Page 17?? 3 what is an elbow? Use mathematical language. Inflection point? What is Free run and why is it capitalized?
Your sections just jumped from 3.2.3 to 3.2.1. Re-label as 'Time-Varying Soil Moisture" 12 I don't know what the 'new set' is—you have to be specific. New set is the "time-varying soil moisture fields." Spell it out.
13 I think when you say 'time evolution....' Do you mean the time series of global mean temperature? As in, the skill over time vs. the spatial skill? You need to clearly differentiate between these two metrics throughout the text. They are calculated differently and have different implications.
19 "improvement in the error reduction" – don't you just mean "error reduction" ? 28 rephrase "this methodology can be applied to techniques in Optimal Sensor Placement..." 29 effectiveness, not effectivity (that is not a word)
Page 18
Line 2, "has yet to be investigated"
5: you're talking about structural biases and how they carry forward using PSMs with GCMs in a DA framework. This is discussed at length in Dee et al., 2016, and you might reference that here. 10: 'indiscriminant'17-20 single, long, confusing sentence, single paragraph.

As described, the subsubsections are removed and the Result section is modified.

21-35 this section reads as a regurgitation of previous work that is not cohesively linked to what you did and with the writing. I cannot follow this text at all. "enter an offline regime" – don't you just mean "use an offline regime?" I don't understand the argument that you can't get observational constraints for internal variability with annual resolution records. Please clarify this argument *dramatically.*

We agree that the previous version did not describe clearly the concept of "off-line regime" and "off-line DA". Now in the Results and Discussion section of the new version we described these two very different concepts more clearly. If the forecast state of the DA shows no skill but the analysis, then we are in the "off-line regime", although we are doing an "on-line DA". An "off-line DA" means that we do not reinitialize the model when the observation is available. We use the free ensemble forecast (the ensemble run without assimilation) for that time to produce the analysis state. Generally, we do "on-line DA" to have skill in forecast which is dynamically consistence not the analysis. However, we showed that in our experiment using SPEEDY we have no skill in forecast. We hope that the new version of the manuscript can clarify these two concepts.

4.2 on page 21... you do not effectively review your new results or your new work here at all.... What is meant by pollutions? "in the face of" is too colloquial. In this discussion, and the outlook, you need to provide us with a concise summary of what you did, your new findings and how it compares to previous work, outline caveats, and then give suggestions for future work and the importance of yours. At present, it is too disjoint and I think the over-sectioning is a culprit. We don't need multiple sections here. Just "Discussion."

We agree and we have modified the manuscript according to your comment.

2 Answer to Editor

Dear Authors, Thanks for submitting the revised version of your manuscript. Both reviewers agreed that this revised version is significantly improved compared to the initial one but one reviewer considers that major changes are required, in particular because the text is still hard to follow. I agree with this evaluation and additional revisions following reviewers' suggestions are thus required, in particular in order to improve the clarity of the text, before a potential publication in Climate of the Past. Best regards, Hugues Goosse

Dear Prof. Goosse,
thank you very much for consideration of the manuscript. We have edited the text according to the suggestions of the reviewer and we hope that the new version of our manuscript is easier to follow and ready for publication in Climate of the Past.

Best regards,
on behalf of all the co-authors,
Bijan Fallah

Assimilation of Pseudo-Tree-Ring-Width observations into an Atmospheric General Circulation Model

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Abstract. ~~We~~ Paleoclimate Data Assimilation (DA) is a promising technique to systematically combine the information from the climate model simulations and the proxy records. Here we investigate the assimilation of Tree-Ring-Width (TRW) chronologies into an atmospheric global climate model using Ensemble Kalman Filter (EnKF) techniques and a process-based tree-growth forward model as observation operator. Our results, within a perfect-model experiment setting, indicate that the "on-line DA" approach did not outperform the "off-line" one, despite its considerable additional implementation complexity. On the other hand, it was observed that the nonlinear response of tree-growth to surface temperature and soil moisture does deteriorate the operation of the time-averaged EnKF methodology. Moreover, for the first time we show that this skill loss appears significantly sensitive to the structure of growth rate function, used to represent the Principle of Limiting Factor (PLF)s within the forward model. ~~On the other hand, it was observed~~ Additionally, our experiments showed that the error reduction achieved by assimilating a ~~particular~~ pseudo-TRW ~~chronology~~ chronologies is modulated by the strength of the yearly internal variability of the model~~at the chronology site~~. This result might help the dendrochronology community to optimize their sampling efforts. ~~In our experiments, the "online" (with cycling) paleao approach did not outperform the "offline" (no-cycling) one, despite its considerable additional implementation complexity.~~

1 Introduction

The low-frequency temporal variability of the climate system can not be estimated from the available time span of instrumental climate records. Accordingly, paleoclimate reconstruction must necessarily rely on the usage of the paleoclimate proxy records. ~~Nonetheless, these~~ These natural archives exhibit several problematic features, e.g., low time-resolution, sparse and irregular spatial distribution, complex nonlinear response to climate and high noise levels. Therefore the proper extraction of the climate signal therein contained can often remain opaque (Evans et al., 2013).

At present, many different paleoclimate modeling ideas have been proposed, e.g., data-driven statistical techniques, climate model hindcasts, and Bayesian probabilistic methods (see Crucifix (2012) as a recent review). Among this plethora of approaches, DA methodologies are today particularly appealing as they allow to systematically combine the information of paleoclimate records with the dynamical consistence of climate simulations ~~(Oke et al., 2002; Evensen, 2003; Hughes et al., 2010; Brönnimann,~~

~~Heretofore~~(Brönnimann, 2011; Hakim et al., 2016). So far, several very diverse paleo-DA schemes have been investigated~~providing very encouraging results (see (Hughes and Ammann, 2009; Widmann et al., 2010)~~ , including (i) pattern nudging (von Storch et al., 2000), forcing singular vectors (Barkmeijer et al., 2003; van der Schrier and Barkmeijer, 2005), 4D-Var (Paul and Schäfer-Neth, 2005; Kurahashi- particle filters (Annan and Hargreaves, 2012; Dubinkina et al., 2011; Dubinkina and Goosse, 2013; Mathiot et al., 2013; Matsikaris et al., EnKF (Huntley and Hakim, 2010; Bhend et al., 2012; Pendergrass et al., 2012; Steiger et al., 2014) (see (Hughes and Ammann, 2009; Wid reference)~~: Pattern Nudging (von Storch et al., 2000) and Forcing Singular Vectors (Barkmeijer et al., 2003; van der Schrier and Barkmeije were designed to curb the atmospheric circulation towards a target pattern by means of an artificial term added to the model dynamics. 4D-Var methodology has been used to assimilate pseudo-proxies into an ocean model (Paul and Schäfer-Neth, 2005; Kurahashi-N was adapted to time-averaged observations (Dirren and Hakim, 2005) and tested for a hierarchy of atmospheric models (Huntley and Hakim Finally, the use of a particle filter has been tested with an Earth system model of intermediate complexity (Annan and Hargreaves, 2012; Dub~~

An important difference between paleo-DA and traditional meteorological DA, is that the assimilation period might be very long compared to the time scales of the dynamical model. Under these conditions, the randomizing action of chaotic model dynamics becomes dominant and consequently the forecast appears completely decorrelated from the previous analysis state. This phenomenon, currently referred to as “off-line regime”, has been observed in several paleo-DA studies (Huntley and Hakim, 2010; Bhe Furthermore, some recent studies have assumed from the beginning the off-line condition and removed the reinitialization step after assimilation, giving raise to “off-line DA techniques” (Steiger et al., 2014; Dee et al., 2016; Hakim et al., 2016).

A typical assumption in most of the paleo-DA studies so far conducted is that the climate-proxy relation is linear. Nonetheless, currently it is widely recognized that climate proxies are the result of complex recording processes, which can have physical, chemical and biological nature. ~~Furthermore, several research groups have already developed and validated forward models for several proxy types (Evans et al., 2013; Dee et al., 2015). Hence, in order to increase~~ The dendrochronologists usually investigate the climate impact on tree-ring growth by empirical–statistical methods (Vaganov et al., 2006). More realistic methodologies have been recently sculpted by the paleoclimate community to investigate the climate-proxy relation that consider the distinct processes whereby the climate signal is recorded in proxy archives. Proxy forward modeling (Hughes et al., 2010; Evan to be one of the most promising methodologies in this area. In a proxy forward model the climate forcing is used as input data for producing the artificial proxy records which can be directly compared with the actual ones. One application of proxy forward models is to predict the evolution of proxy archives (Vaganov et al., 2006). They can also be applied as climate reconstruction strategies by using the probabilistic inversion methods like Bayesian hierarchical modeling (Tolwinski-Ward, 2012), Markov Chain Monte Carlo (MCMC) (Boucher et al., 2014) and DA (Hughes et al., 2010).

Several recent studies have investigated the applicability of process-based forward models into a paleo-DA setting, in the realism of DA-based climate reconstructions ~~, it is relevant and pertinent to connect the climate state space to the proxy space by way of forward models (Acevedo et al., 2015; Dee et al., 2016). Dee et al. (2016) applied~~ (Hughes et al., 2010; Acevedo et al., 2015; Dee e Acevedo et al. (2015) [AC15, hereafter] utilized the process-based TRW forward model Vaganov-Shashkin-Lite (VSL) (Tolwinski-Ward et an online EnKF scheme, to assimilate TRW records into a chaotic 2-scale dynamical system as a toy model. They found that the non-linearities of the forward model may deteriorate the performance of the EnKF. Furthermore, they observed that this loss of skill may be ameliorated by means of a Fuzzy Logic (FL)-based extension of VSL model. Matsikaris et al. (2015) compared an off-line and an on-line ensemble-based DA ("degenerat particle filter") and showed that the both methods outperform the model without DA. They concluded that in the off-line method temporal consistency of the model is lost. However, they encouraged to use a full particle filter strategy instead of a degenerate one. On the other hand, Dee et al. (2016) used three different nonlinear proxy ~~system forward models in a framework and investigated the utility of paleoclimate observations for constraining climate simulations~~forward models (including VSL) and an off-line EnKF scheme to assimilate TRW, coral and ice core records into two different isotope-enabled Atmospheric General Circulation Model (AGCM). They demonstrated that the linear-univariate models for tree ring width may not capture the ~~GCM~~AGCM's climate, especially for regions where the tree's growth is dominated by moisture. ~~The tree ring forward model used in our study is a modified version of the model used in Dee et al. (2016). Acevedo et al. (2015) AC15, hereafterevaluated the applicability of the process-based forward model (Tolwinski-Ward et al., 2011) as observation operator within a simplified setting. Using a chaotic 2-scale dynamical system as a toy model, AC15 generated pseudo-observations and assimilated them via the time-averaged -algorithm (Dirren and Hakim, 2005).~~

This paper follows ~~closely~~ the rationale of AC15, but within a more realistic scenario, where an AGCM is used as dynamical system and the observational network resembles the currently available TRW chronologies.

~~In addition to the classical approaches used in paleoclimate studies, a so-called "off-line" -based climate reconstructions is presented by (Steiger et al., 2014; Dee et al., 2016; Hakim et al., 2016). In an off-line approach the climate model is not re-initialized at the observation time steps (no initialization cycle or "no-cycling").~~

~~The main objectives~~ The purpose of this study ~~are to shed light on the following four fundamental questions :~~

~~1) Can paleo-improve the skill of the model for the forecast (prior) state?~~

~~2) Can~~ is then to contribute to the present knowledge of paleo-DA ~~improve the skill of the model for the analysis (posterior) state?~~

~~3) Can an on-line ("with cycling") outperform an "off-line" ("nocycling") one (see Sec.4 for the definition of "~~techniques by addressing the following two questions: (i) Does the off-line ~~")?~~

~~4) How does the nonlinear response of tree-growth to surface temperature and soil moisture affect~~ regime naturally appear for the assimilation of TRW records into a AGCM? and (ii) is the Fuzzy Logic (FL)-based extension of VSL model still useful to improve the performance of ~~the time-averaged~~ EnKF ~~-method~~when a climate model is used?

~~The third question is one of the most important challenges in the paleo-field, given that the computational expenses of an on-line scheme with a realistic coupled GCM is far beyond the affordable limits of today's computers. On the other hand, state-of-the-art climate models have little or no predictive skill on the long timescale of proxy records (Hakim et al., 2016).~~

~~In~~ This study is structured as follows: in section 2 we describe the DA technique, the TRW forward model and the climate model as well as the experimental setting used. Our numerical results are shown in section 3, followed by a discussion in section 4.

2 Materials and Methods

2.1 Data Assimilation Basics

2.1.1 ~~Rationale~~

~~The knowledge about the climate is drawn from~~ The term DA designates in this paper the process of estimating the state of a system using observations and the physical laws governing the evolution of the ~~climate system . Numerical models apply the latter to estimate the state of the flow. is a process which applies both available information sources to estimate the state of the climate (Talagrand, 1997).~~

system as represented in a numerical model (Talagrand, 1997). In a typical ~~*sequential*~~ DA scheme, a ~~climate~~ dynamical model is integrated in time ~~steps over which observations are available. The predicted stateat an observed instant (forecast), is used as “background” for the scheme. Furthermore, the forecast~~until observations become available. Afterwards, the predicted state, also known as forecast, is “updated” ~~or “corrected” by observations to form~~ using the observational information in order to obtain a corrected state, also known as the analysis. ~~The model is then~~ Finally, the model is reinitialized from the analysis state and ~~propagates in time to reach the next observed instant. The analysis step is determined by availability of observations, their timescales and computational resources.~~ propagated in time until the next assimilation time, completing the so-called “analysis” cycle. DA methods have evolved from very empirical approaches, such as Newtonian relaxation, to probabilistic ones that ~~estimate the state~~ attempt to estimate the Probability Density Function (PDF) of the model state conditional to the observations (see ~~Kalnay (2003) and Lahoz et al. (2010) for review~~Kalnay (2003); Lahoz et al. (2010); Reich and Cotter (2015) as reference materials).

~~Among all the~~

Among the currently available DA techniques, EnKF (Burgers et al., 1998) occupies an outstanding position due to several reasons. It offers an appealing trade-off between accuracy ~~, relatively user-friendly implementation~~ and computational expenses. It works robustly for ~~very~~ sparse observation networks and moderate number of ensemble members (Whitaker et al., 2009). ~~Its~~ Furthermore, EnKF’s implementation does not require ~~adjoint model (calculations are outside the modelcode)~~ any modification of the model’s code and uncertainty estimates can be directly obtained from the ensemble spread (Hamill, 2006). The main disadvantage of EnKF, within a paleoclimate setting, is its inability to handle ~~strongly~~ non-Gaussian PDFs, which can ~~result~~ easily arise from the nonlinearities of climate models and observation operators. ~~Nonetheless, it is very difficult to remove this limitation, given that strictly~~ Recently, there have been several developments in the field of non-linear DA for high-dimensional systems (Van Leeuwen et al., 2015), however at the present fully non-Gaussian DA techniques ~~have historically been~~ are still prohibitively expensive to run for ~~high-dimensional systems~~general circulation climate models.

2.1.1 Kalman Filter

~~Given that the model's state is $\mathbf{x}(t)\in\mathbb{R}^n$,~~ Within the Kalman Filter (KF) (Kalman, 1960)~~assumes that~~ , the PDF of forecast state ~~is~~ $p(\mathbf{x})$ is assumed to be given by a Gaussian function ~~of~~ with mean $\mathbf{x}^f$ and covariance ~~$\mathbf{P}^f\in\mathbb{R}^{n\times n}$:~~ $\mathbf{P}^f$:

$$p(\mathbf{x}) \propto \exp(-\frac{1}{2}(\mathbf{x}-\mathbf{x}^f)^T(\mathbf{P}^f)^{-1}(\mathbf{x}-\mathbf{x}^f)). \tag{1}$$

The observations ~~$\mathbf{y}(t_j)\in\mathbb{R}^k$~~ $\mathbf{y}(t_j)$ are also assumed to have Gaussian errors and therefore the conditional probability of the observation vector $\mathbf{y}$ given the state $\mathbf{x}$ is:

$$p(\mathbf{y}\mid\mathbf{x}) \propto \exp(-\frac{1}{2}(\mathbf{y}-\widehat{\mathrm{H}}\mathbf{x}^f)^T\mathbf{R}^{-1}(\mathbf{y}-\widehat{\mathrm{H}}\mathbf{x}^f)), \tag{2}$$

where $\widehat{\mathrm{H}}$ ~~and $\mathbf{R}\in\mathbb{R}^{k\times k}$ are~~ is the observation operator and $\mathbf{R}$ is the observation covariance matrix, respectively. Following the Bayes theorem, the conditional probability of the state given the observations, i.e., the analysis PDF, is:

$$p(\mathbf{x}\mid\mathbf{y}) \propto \exp(-\frac{1}{2}(\mathbf{x}-\mathbf{x}^f)^T(\mathbf{P}^f)^{-1}(\mathbf{x}-\mathbf{x}^f)-\frac{1}{2}(\mathbf{y}-\widehat{\mathrm{H}}\mathbf{x}^f)^T\mathbf{R}^{-1}(\mathbf{y}-\widehat{\mathrm{H}}\mathbf{x}^f)). \tag{3}$$

~~Assuming the~~ Finally, assuming that $\widehat{\mathrm{H}}$ is a linear function, ~~equation 3 has~~ $p(\mathbf{x}\mid\mathbf{y})$ is also a Gaussian ~~(Eq.1 and Eq.2 are Gaussian). Therefore, its~~ function whose mean and covariance can be calculated by the ~~so called~~ so-called Kalman update equations (Lorenc, 1986):

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}(\mathbf{y}-\widehat{\mathrm{H}}\mathbf{x}^f), \tag{4}$$

$$\mathbf{P}^a = (\mathbf{I}-\mathbf{K}\widehat{\mathrm{H}})\mathbf{P}^f; \tag{5}$$

where the Kalman gain matrix $\mathbf{K}$ is given by:

$$\mathbf{K} = \mathbf{P}^f\widehat{\mathrm{H}}^\dagger(\widehat{\mathrm{H}}\mathbf{P}^f\widehat{\mathrm{H}}^\dagger+\mathbf{R})^{-1}. \tag{6}$$

2.1.2 Ensemble Kalman Filter (EnKF)

~~In a realistic model setting, the calculation~~ For realistic geophysical models, the dimensionality of the model state can be very high and then the calculation and storage of the covariance matrices ~~are numerically very expensive. Evensen (1994) have used~~ can be prohibitively expensive. A solution to this problem is provided by the EnKF (Evensen, 1994), which uses an ensemble of model states ($\mathbf{X}(t) = (\mathbf{x}_1,\ldots,\mathbf{x}_m)$) to approximate ~~the~~ KF equations. Following this approach~~the best state estimate and its uncertainty are presented by the ensemble mean and ensemble spread. The ensemble spread is given by the standard deviation of the ensemble around its mean. Thus, an cycle consists of an ensemble forecast step which provides the empirical~~ , the mean and covariance ~~for approximation of the equations:~~

of the forecast take the following form:

$$\langle\mathbf{X}_f\rangle = \frac{1}{m}\sum_{i=1}^{m}\mathbf{x}_i^f, \;\; \mathbf{P}^f = \frac{1}{m-1}\mathbf{X}_f{}'(\mathbf{X}_f{}')^T. \tag{7}$$

Here $\mathbf{X}_f{}' \in \mathbb{R}^{n\times m}$ denotes the forecast ensemble deviation matrix:

$$\mathbf{X}_f{}' = \mathbf{X}_f - \langle \mathbf{X}_f \rangle \mathbf{e}^T. \tag{8}$$

where $\mathbf{e} = (1,\ldots,1) \in \mathbb{R}^m$. ~~The~~ An analysis ensemble whose covariance satisfies equation 5 can be generated in different ways~~. Two main kinds of s are~~ , which can be classified into two main families: stochastic and deterministic filters (Hamill, 2006). In the stochastic approach an observational ensemble $\mathbf{Y}$ is generated by adding ~~a set of realization~~ realizations of the observational noise to the observation vector $\mathbf{y}$. The analysis ensemble is then created by the following updating equation:

$$\mathbf{X}_a = \mathbf{X}_f + \mathbf{K}\left(\mathbf{Y} - \widehat{\mathrm{H}}\mathbf{X}_f\right). \tag{9}$$

In the deterministic ~~updating scheme~~approach, instead of creating an ensemble of observations, the analysis mean ~~$(\overline{\mathbf{X}_a})$ and deviations $\mathbf{X}_a{}'$ are calculated by using different update formula (Tippett et al., 2003)~~and deviations are calculated using update formulae which do not involve random numbers (see Tippett et al. (2003) as reference).

~~Due~~ A practical problem of EnKF schemes, is that due to the limited ensemble size, the forecast uncertainty is usually underestimated~~in~~. This leads to an excessive confidence on the forecast and after several assimilation cycles the observations may ~~completely be~~ be completely ignored. This situation is normally avoided by means of an *ad hoc* procedure known as “~~filter divergence”and can treated by multiplying the ensemble spread~~ covariance inflation”, where the forecast covariance matrix is multiplied by a constant greater than one~~(covariance inflation).~~

. For the experiments presented in this paper, we employed ensembles of 24 members ~~(limited by the number of CPUs)~~ and constant multiplicative inflation of 1% after the ensemble update. ~~As the consequence of~~ Another undesired consequence of the limited ensemble size, ~~any observation may present~~ is that the ensemble state at any gridpoint will present non-negligible spurious correlations with ~~the distant ones and the filter performance may be affected. Therefore,~~ observations located far apart in space. This difficulty is solved using another *ad hoc* procedure known as “covariance localization”. Here we utilize the so-called R-localization (Hunt et al., 2007), where the elements of the observation error covariance matrix are multiplied by a function that increases exponentially with distance~~and an infinite error is assigned to the distant observations (R-localization (Hunt et al., 2007))~~. This is achieved using the following formula:

$$R_{loc} = R * \exp\left((r_h/2\lambda_h)^2 + (r_v/2\lambda_v)^2\right) \tag{10}$$

where r_h and r_v stand for the horizontal and vertical distances, respectively. Their corresponding scaling parameters were set to the values $\lambda_h = 500$ Km and $\lambda_v = 0.4\ln p$.

2.1.3 Time-Averaged Ensemble Kalman Filter (EnKF)

~~Usually the time scale of the measured system is sufficiently longer than the response time of the sensor and the measurements can be assumed to be instantaneous . However, this assumption can not be applied for precipitation gauges, wind meters~~

~~and proxy records. Proxies have averaged recording time spans ranging from months to decades. Time-averaged observations contain information of a segment of the model state trajectory instead of an instant of the model evolution.~~

The EnKF algorithm was initially designed to estimate the instantaneous state of a model given instantaneous observations. As a consequence, EnKF cannot be directly applied to paleoclimate data given that the observational information present in proxy records is typically the average of a function of the state over long time periods. A solution to this conflict is provided by the time-averaged ~~ensemble background fields are updated by the and the instantaneous deviations from the mean remain unchanged~~Ensemble Kalman Filter (EnKF) (Dirren and Hakim, 2005), where the instantaneous forecast is decomposed into its time-averaged part and the anomalies around it. Afterwards, the original EnKF update formula is used to assimilate the time-averaged observations into the time-averaged forecast, obtaining the time-averaged analysis. Finally, the instantaneous analysis is form by adding the unaltered time-averaged forecast anomalies to the time-averaged analysis. This approach is based on the ~~fact~~ assumption that the observations can only contain time-averaged information (Dirren and Hakim, 2005).

2.1.4 ~~Observational System Simulation Experiments~~

~~Given a prediction system comprising a dynamical model and a scheme, forecast and analysis errors arise from many different sources, e.g. model imperfections, inadequacy of the strategy and insufficiency of observational information, which interact with each other in practice. In order to disentangle the effects of these error sources, a scheme is typically tested under simplified conditions by means of numerical experiments, currently known as , whose realism level is gradually increased.~~

An ~~consists of (i) a single model trajectory $\mathbf{x}^{\text{NATURE}}$, typically referred to as "true" run or "nature" run, that is used as prediction target, (ii) pseudo-observations created by applying the observation operator to $\mathbf{x}^{\text{NATURE}}$ and adding simulated observational noise, and (iii) an observationally constrained run $\mathbf{X}^{\text{DA}}$, obtained by performing a sequence of analysis cycles where the pseudo-observations are assimilated (see Fig. 1).~~

~~The nature run is normally generated by running the dynamical model starting from a random sample of the model climatology. Notice that thanks to the availability of the truth model evolution for an , the forecast and analysis skill of the observationally constrained run can be directly assessed, using for example the of the ensemble mean:~~

$$\cancel{\mathbf{RMSE}(\langle \mathbf{X}^{\text{DA}} \rangle) = \left(\overline{\left(\mathbf{x}^{\text{NATURE}} - \langle \mathbf{X}^{\text{DA}} \rangle \right)^2} \right)^{\frac{1}{2}},}$$

~~where $\overline{}$ and $\langle\ \rangle$ denote the time and ensemble mean operators, respectively.~~

~~An additional run frequently performed for involving ensemble methods, is a free ensemble run $\mathbf{X}^{\text{FREE}}$, where no observations are assimilated and then the ensemble just freely evolve under the action of the model dynamics. $\mathbf{X}^{\text{FREE}}$ is intended to provide a benchmark of performance, against which it is possible to asses the the added value of~~ crucial aspect of time-averaged DA is the ~~scheme~~off-line regime, which manifest itself as a complete lack of estimation skill for the forecast quantities. This behavior was first observed for the time-averaged Ensemble Kalman Filter (EnKF) applied to a quasi-geostrophic atmospheric jet model (Huntley and Hakim, 2010; Pendergrass et al., 2012). Afterwards, several studies have used the simplified off-line time-averaged Ensemble Kalman Filter (EnKF) approach with global climate models (Bhend et al., 2012; Steiger et al., 2014; Dee et al., 20 the presence of the off-line regime. However, to our knowledge, there had not been numerical evidence of the onset of off-line conditions for a full time-averaged Ensemble Kalman Filter (EnKF) algorithm applied to a AGCM. As mentioned in the introduction, filling this knowledge gap is one the objectives of this paper.

2.2 TRW Forward ~~Modeling~~Model

2.2.1 ~~Model~~

~~The~~ The Vaganov-Shashkin-Lite (VSL) model for TRW chronologies offers an intermediate complexity approach between ecophysiological and completely data-driven models (Tolwinski-Ward et al., 2011; Tolwinski-Ward, 2012), where the climate-driven component of tree-ring growth is parametrized by way of a simple representation of the ~~(Fritts, 1976). This biological concept~~ Principle of Limiting Factor (PLF) (Fritts, 1976). The biological concept of PLF states that the pace at which a plant develops is controlled by the single basic growth resource, typically either energy or water, that is in shortest supply. Within VSL the limiting factors considered are near-surface air temperature (T) and soil moisture (M). These variables influence tree growth by means of "growth response" functions g_T and g_M using a piece-wise linear "standard ramp" function (Tolwinski-Ward et al., 2014):

$$\Psi(u) = \begin{cases} 0 & \text{if} \quad 0 \geqslant u \\ u & \text{if} \quad 0 < u \leqslant 1 \\ 1 & \text{if} \quad \phantom{0 <{}} u > 1, \end{cases}$$

VSL's growth responses at a particular time is expressed as:

$$g_T = \Psi\left(\frac{T - T^L}{T^U - T^L}\right) \tag{11}$$

and

$$g_M = \Psi\left(\frac{M - M^L}{M^U - M^L}\right). \tag{12}$$

Where T^L and M^L denote minimum thresholds for temperature and moisture below which there is no ~~grown~~growth, and T^U and M^U are upper thresholds above which tree growth is optimal. Afterwards, the growth rate G_{MIN} is determined by the smallest growth response, i.e.,

$$G_{MIN} = \min\{g_T, g_M\}, \tag{13}$$

The yearly TRW values W are obtained as following:

$$W_n = \int_{t_n - \tau}^{t_n} G_{MIN}(t) I(t)\, dt. \tag{14}$$

Where I is the relative local insolation.

2.2.1 VSL from the Fuzzy Logic Viewpoint

The term Fuzzy Logic (FL) was coined by Zadeh (1975) and refers to a mathematical theory which has been very successful at modeling complex systems involving imprecise data and vague knowledge of the underlying mechanisms. Since its introduction, FL has greatly influenced many ~~applied~~ disciplines, most notably control theory (Nguyen et al., 2002). Within the environmental sciences, FL has ~~also found numerous applications, including~~ been applied in ecological and hydrological modeling (Marchini, 2011; Salski, 2006; Se, 2009). Regarding climate proxy forward modeling, AC15 recently showed that VSL model can be completely embedded into the framework of FL. Within this reinterpretation, the growth response function g_T (g_M) ~~correspond~~ corresponds to the membership function to the set S_T (S_M) of optimal temperature (moisture) conditions for tree growth. Temperature (moisture) values lying below T^L (M_L) present null values for g_T (g_M) and accordingly do not belong to S_T (S_M). On the other hand, temperature (moisture) values lying above T^U (M_U) lead to g_T (g_M) values equal to 1, meaning they belong completely to S_T (S_M). All the other temperature (moisture) conditions present growth responses between 0 and 1 and consequently they are considered to belong partially to S_T (S_M). This idea of partial membership is the basis of fuzzy logic and the sets defined this way are called fuzzy sets. Furthermore, the intersection of the fuzzy sets S_T and S_M is again a fuzzy set $S_{T \wedge M}$, whose membership function can be calculated by evaluating the minimum between G_T and G_M:

$$g_{T \wedge M} = \min\{g_T, g_M\} \tag{15}$$

Equation 15 is completely equivalent to the equation 14~~and then~~. Then VSL's growth rate function can be interpreted as the membership function for the fuzzy intersection set $S_{T \wedge M}$. In FL theory, the minimum function (Eq. 15) is one of the most popular representations of the intersection operation, however it is not the only, existing actually a whole family of appropriate functions referred to as t-norms (see Nguyen et al. (2002)). In AC15 a number of t-norms was tested as replacement for VSL's growth rate function within a highly simplified paleo-DA setting. In particular it was found that the product t-norm $g_{T \wedge M} = g_T \cdot g_M$ might improve significantly the performance of the time-averaged EnKF technique. Accordingly, beside the minimum t-norm we consider also in this paper the product growth response VSL with Product t-norm (VSL-Prod):

$$G_{PROD} = g_T \cdot g_M. \tag{16}$$

2.3 ~~Atmospheric General Circulation Model~~Experimental Design

Following the rationale used in the experiments of AC15, a set of "perfect model" Observation System Simulation Experiments (OSSE)s (see fig. 1) was conducted using SPEEDY model (Molteni, 2003) as dynamical system and VSL forward model as observation operator. The time-averaged state of the atmosphere is estimated via the EnKF approach of Dirren and Hakim (2005). In the following, we describe in detail each of the components of our experimental setting.

2.3.1 Atmospheric General Circulation Model

The Simplified Parametrizations, primitivE-Equation Dynamics (SPEEDY) model (Molteni, 2003) is an intermediate complexity AGCM comprising a spectral dynamical core and a set of simplified physical parametrizations, based on the same principles as state-of-the-art AGCM but tailored to work with just a few vertical levels.

~~SPEEDY's dynamical core solves the hydrostatic primitive equations by means of the spectral transform developed by Bourke (1974), which uses absolute temperature, logarithm of the surface pressure, specific humidity, divergence and vorticity as basic prognostic variables. The time stepping is performed via a leapfrog scheme with an standard Robert–Asselin filter (Robert, 1966). The sub-grid scale processes parametrized in speedy are convection, large-scale condensation, clouds, short- and long-wave radiation, surface fluxes, and vertical diffusion.~~

~~In this paper we employ version 32 of SPEEDY, with seven vertical levels (L7) and standard Gaussian grid of 96 by 48 points in the horizontal (T30). The top and bottom layers represent the stratosphere and the planetary boundary layer , respectively~~Regarding the ocean, SPEEDY offers two possible configurations: (i) *PRESCRIBED* where the sea surface temperature is directly imposed as forcing and (ii) *SLAB* where the model is coupled to a slab ocean model ("q-flux adjusted mixed layer model") forced by climatological ocean dynamics. Despite of its low resolution and the relative low complexity of its ~~parametrizations~~parameterizations, SPEEDY still captures many observed global climate features in a realistic way, while its computational cost is at least one order of magnitude lower than the one of sophisticated state-of-the-art AGCM's at the same horizontal resolution (Molteni, 2003). The latter makes SPEEDY specially suitable for studies involving long ensemble runs, like the ones necessary for this study.

2.4 ~~Experimental Setting~~

~~Following the rationale used in the experiments of AC15, pseudo-observations are generated using Vaganov-Shashkin-Lite () (Tolwinski-Ward et al., 2011, 2013) as observation operator. Afterward, the time-averaged state of the atmosphere is estimated via approach Dirren and Hakim (2005). The impact of the representation of the on the filter performance is studied using the assimilation of time averaged linear observations as a reference.~~

2.3.1 ~~Filter Implementation~~

The SPEEDY model ~~is~~ was embedded by Miyoshi (2005) into the ~~ensemble framework using the (Hunt et al., 2007), the~~ so called SPEEDY-Local Ensemble Transform Kalman Filter (LETKF) framework~~. The parallel FORTRAN 90 implementation of the~~ , which offers a parallel implementation of LETKF (Hunt et al., 2007). Among the different flavors of EnKF, LETKF is particularly promising for high resolution ~~model~~ models given that the calculation of the analysis for a particular grid point requires only the information of the neighboring grid points. Therefore, LETKF offers outstanding scalability properties. SPEEDY-LETKF is an open-source software which have already been widely used for several DA studies (Li et al., 2009; Miyoshi, 2010; Lien et al., 2013; Ruiz et al., 2013; Amezcua et al., 2014). Here, SPEEDY-LETKF was extended ~~for~~ to allow the assimilation of time averaged linear observations and pseudo-TRW observations. ~~This was done by~~

2.3.1 Perfect Model Experiments

Given a dynamical climate model and a DA scheme, forecast and analysis errors arise from many different sources, e.g. model imperfections, inadequacy of the DA strategy and insufficiency of observational information, all which interact with each other in practice. In order to disentangle the effects of these error sources, a DA scheme is typically tested under simplified conditions by means of perfect numerical experiments, currently known as OSSE, whose realism level is gradually increased. An OSSE consists of (i) ~~modification of the model time cycling~~a single model trajectory $\mathbf{x}^{\text{NATURE}}$, typically referred to as "true" run or "nature" run, that is used as prediction target, (ii) ~~addition of the time-averaged updating approach of Dirren and Hakim (2005)~~ pseudo-observations created by applying the observation operator to $\mathbf{x}^{\text{NATURE}}$ and adding simulated observational noise, and (iii) ~~development of~~ an observationally constrained run $\mathbf{X}^{\text{DA}}$, obtained by performing a sequence of analysis cycles where the pseudo-observations are assimilated (see Fig. 1). The nature run is normally generated by running the dynamical model starting from a random sample of the model climatology. Notice that thanks to the ~~-like observation operator.~~

~~Additionally, in order to avoid catastrophic filter divergence (ref. sec. 2.1.2), observations with large divergence from their corresponding predicted values were neglected. Moreover, in order to avoid the crash of the model after assimilation steps, the following quality control criterium is applied: The observations whose corresponding innovation vector norm (absolute mismatch regarding the forecast observation) is bigger that 10 times its error standard deviation are discarded~~availability of the truth model evolution for an OSSE, the forecast and analysis skill of the observationally constrained run can be directly assessed, using for example the Root Mean Square Error (RMSE) of the ensemble mean:

$$\mathbf{RMSE}(\langle \mathbf{X}^{\text{DA}} \rangle) = \left(\overline{\left(\mathbf{x}^{\text{NATURE}} - \langle \mathbf{X}^{\text{DA}} \rangle\right)^2} \right)^{\frac{1}{2}}, \tag{17}$$

where $\overline{}$ and $\langle\ \rangle$ denote the time and ensemble mean operators, respectively. An additional run frequently performed for OSSE involving ensemble DA methods, is a free ensemble run $\mathbf{X}^{\text{FREE}}$, where no observations are assimilated and then the ensemble just freely evolve under the action of the model dynamics. $\mathbf{X}^{\text{FREE}}$ is intended to provide a benchmark of performance, against which it is possible to asses the the added value of the DA scheme.

2.3.2 ~~Simulations' Characteristics~~

The modified version of SPEEDY-LETKF is ~~utilized~~ used to carry out a set of standard ~~"perfect model"~~ OSSEs (Fig. 1). First ~~the~~ a simple representation of the PLF (VSL controlled only by Temperature (VSL-T)) is utilized for two sets of experiments under different ocean conditions:

- *PRESCRIBED experiment* is forced by the boundary conditions included in the version 41 of the code, which comprises the sea surface temperature (SST) anomalies from 1854 to 2010 with respect to the period 1979 to 2008 derived from NOAA_ERSST_V3 dataset (Smith et al., 2008; Xue et al., 2003), as well as climatological maps derived from input data of the European Centre for Medium-Range Weather Forecasts (ECMWF)'s reanalysis (Gibson et al., 1997). At the surface boundaries the model requires the climatological maps of sea surface temperature, sea ice fraction, surface temperature at the top of the soil, moisture in the top soil layer and the root-zone layer, snow depth, bare-surface albedo, fraction of land-surface vegetation. At the top of the atmosphere, the model calculates the flux of incoming solar radiation from astronomical formulae (Molteni, 2003). The solar radiation absorption by ozone in the stratosphere follows empirical functions with seasonal variability. The latitudinal variability of the optical depth depends on the daily averaged zenith angle (Molteni, 2003). The climatological fields are derived for the period 1981-1990 to have a better balance for warm and cold El Niño-Southern Oscillation (ENSO) events (Molteni, 2003). This procedure follows the AMIP-type experiments (Herceg Bulić and Kucharski, 2012).

- *SLAB experiment* is coupled with a slab ocean model ~~("q-flux adjusted mixed layer model")~~ forced by climatological ocean dynamics and no initialization is used. The model starts from a spun up state.

~~Two~~ We consider two representations of the PLF~~are considered~~: the "minimum" (G_{MIN}) and the "product" (G_{PROD}) t-norms. Initially, a one-year long spin-up run is performed for all experiments, starting from January 1st, 1860. The final state of this model trajectory is subsequently used as initial condition for a 150 year long nature ("true") run. The ensemble runs with and without DA are identically initialized from a set of states gathered daily from the last two months of the spin-up run (lagged 2 day initialization). Notice that the nature ("true") run and the different ensemble runs (~~priors~~forecasts) are generated with the same time varying forcing fields.

2.3.2 Observation Generation

Pseudo-TRW observations are produced following VSL's formulation, plus a final white noise addition step, where random draws from a Gaussian distribution are imposed on the time averaged observations. ~~The measurements' error is assumed not to be~~ In a perfect model experiment it is usually assumed that the measurements' errors are not correlated in time (no memory) ~~, therefore the white noise is used in this study~~ and the "white noise" is added to the observations (McShane and Wyner, 2011; Dee et al., 2016). Surface temperature data was extracted from the ~~lowest level of the state vector~~model, while soil moisture was taken from the surface boundary conditions. Notice that temperature is a prognostic variable of the model, whereas soil moisture is a prescribed variable with yearly periodicity. It is worthwhile to mention that although soil moisture is not a prognostic variable of SPEEDY, it does affect prognostic variables, such us humidity, through the parametrizations.

~~Regarding the geographical distribution of observations, we~~ We place a station at every grid box where at least one actual TRW chronology from the database of Breitenmoser et al. (2014) is present. This strategy yields ~~an~~ a realistic observational network comprising 257 stations (see figure 2). Concerning the configuration of the observation operator, for our experiment involving SPEEDY we focus on the effect of the first VSL's nonlinearity, i.e., the shifting of recorded variable (growth is limited by either temperature or moisture). Consequently, we configure VSL so that no thresholding takes place. This is done by setting the upper and lower response thresholds to the maximum and minimum values during the nature (true) run, respectively, so that the response functions reduce to linear rescaling operators (ref. AC15).

2.3.3 ~~Diagnostic Statistics~~

3 Results

~~Our results are presented in three sections~~Given the annual resolution of TRW chronologies, we study the filter performance for yearly averaged values of near surface temperatures. We focus our analysis on temperature due to the larger error reduction in this field as compared to other variables (*eg,* humidity, u-wind, v-wind) when DA is applied. The behavior of ensemble runs is monitored by means of RMSE for the near surface temperature. The results are shown as: 1) time-series of globally averaged temperature RMSE, 2) histograms of these time-series and 3) maps of time-averaged (150 years) temperature RMSEs. We ~~show the analysis of the temperature variable due to its larger error reduction compared to other variables (*eg,* humidity, u-wind, v-wind) when~~ begin with the investigation of the performance of the online and offline DA ~~is applied.~~

4 ~~Results~~

~~Given the annual resolution of chronologies, we study the filter performance for yearly averaged values (near surface temperatures). We monitor the behavior of ensemble runs by means of for the near surface temperature. SPEEDY presents spatially heterogeneous internal variability (Molteni, 2003). Due to this feature, for a particular time averaging length, there will typically be regions with very low internal variability (*eg.,* equatorial regions for temperature) for which shows very low values~~simulations. Then the performance of DA skill is tested for two different growth functions. Afterwards, we examine the performance of the off-line DA for different observational errors. Finally, the effect of using the time-varying soil moisture fields on the performance of DA approach is tested.

3.1 Free Ensemble ~~Run~~Spread and Error

An AGCM is an example of non-autonomous system and accordingly the evolution of its state is determined by both the atmospheric dynamics and the external forcing. The influences of these two distinct factors can be disentangled to some extent by considering atmospheric variability to be a superposition of an internal component, caused by the intrinsic dynamics, and an external one, resulting from the variations of the boundary conditions (Deza et al., 2014). Under this assumption, internal and external variability can be separated by way of a free ensemble run, using the ensemble mean as an estimate of the forced component. The magnitude of the internal variability can then be estimated from the ensemble spread. Note that using an ensemble DA method is only beneficial in the presence of internal variability, given that the forced variability can be well described by an unconstrained ensemble run (free ensemble run).

3.1.1 ~~Free Ensemble Spread and Error~~

The time averaging operator acts as a low pass filter that reduces the amplitude of fluctuations with time scales shorter than the averaging period. Subsequently, geographical areas dominated by fast processes, compared to averaging period, tend to present constant mean values, or equivalently no internal time averaged variability. ~~The climatology and the formulation of the SPEEDY model is fully described in Molteni (2003). Therefore, we focus only on the results of the approach, without considering the systematic errors of the model.~~ In the case of TRW chronologies, the characteristic one-year averaging period is long for atmospheric phenomena, and as consequence several areas show very low yearly internal variability for certain variables. A clear example of this is temperature around the equator (see figure 3a) where the temperature variability is dominated by the daily cycle and accordingly is strongly attenuated by the yearly averaging. On the other hand, planetary scale patterns are not completely stationary and fluctuate over longer time scales. These low-frequency processes introduce internal variability in the yearly means, as can be seen in figure 3a. Maximum temperature spread occurs near the surface at high latitudes around $\pm 70^{\circ}$. These yearly internal variability maxima can be related to leading modes of variability of the global circulation, such as the "annular modes" (e.g., ENSO) (Thompson and Wallace, 2000), migrations of the Inter Tropical Convergence Zone (ITCZ) (Schneider et al., 2014), as well as displacements of the jet stream (Woollings et al., 2011).

An important consequence of the spatially heterogeneous yearly internal variability of SPEEDY is that the nature (true) run variables at geographical areas with low internal variability can be well predicted by the ensemble mean of the free ensemble run, as it can be seen in figure 3b for the tropical surface temperature. On the other hand, RMSE extremes take place in areas of maximal internal variability (compare figures 3a and 3b). Generally, the error of the free ensemble run, used as a predictor of the nature (true) run, is essentially the projection of the nature (true) run trajectory on the internal variability component (see schematic in Fig. 1). Figure 3 exhibits the results for the SLAB experiment. The PRESCRIBED experiment presents a very similar behavior.

3.2 Assimilating Pseudo-TRW Observations

3.2.1 ~~Temperature-based Representation ()~~SLAB and PRESCRIBED ocean

~~As described in Sec. ??~~ Here, we investigate two different experiments using SLAB and PRESCRIBED ocean conditions (see Table 1). ~~For the sake of simplicity, we set up the sensitivity experiments~~ Sensitivity experiments are conducted using the simple observation operator VSL-T to investigate the effect of ~~the SLAB oceanmodel. The use of SLAB ocean~~ using a SLAB ocean. This is motivated by the fact that the coupled ocean may lend predictability to the atmosphere as a slow component of the climate system. On the other hand, the climate of the PRESCRIBED experiment may follow the trends of the forced ocean conditions instead of the terrestrial proxy records. Therefore, the PRESCRIBED experiment's spread and error are expected to be smaller than the SLAB experiment. Figure 4 supports this hypothesis, showing a reduction in globally averaged free ensemble error in PRESCRIBED compared with the SLAB.

Figure 4a illustrates that no error reduction is obtained for the ~~forecasted temperatures~~forecast state. The expected value of the RMSE is slightly larger than the free ensemble simulation for both SLAB and PRESCRIBED. However, the analysis state has skill (Fig. 4b), especially prior to 1950s. The existence of the trend in the RMSE time-series may arise from cycling (reinitialization) of the ensemble or the choice of observation operator~~(more details in Sec. ??)~~. The distribution density of the proxy record locations are biased to the Northern hemisphere; therefore, the error reduction of the analysis is more evident in the RMSE maps (Fig. 5).

~~An important aspect of our results concerning the skill when yearly averaged linear observations are assimilated, is that the error reduction regarding~~ The error reduction of the free ensemble run ~~appears~~ is modulated by the magnitude of the yearly internal variability of the particular variable at a specific site (compare figures ~~4~~ 3 and 5). As a consequence, stations located in areas of strong yearly internal variability (i.e., Alaska) are more efficient than the others at reducing the error of the time-averaged state estimate.~~An example of this are the stations located in Alaska which constrain temperature with considerably larger skill than sites in South Africa. This finding may prove useful for the design of optimal chronology networks, in particular, and proxy networks in general~~ Additionally, more terrestrial records from the Southern Hemisphere may improve the state estimate (see Comboul et al. (2015) and the discussion at the end of this paper).

~~This situation, where a method presents time averaged analysis skill for averaging periods where the time averaged forecast skill is completely lost, has been previously observed in studies applying techniques on time averaged quantities (Huntley and Hakim, 2010; performed under these circumstances is generally termed "offline Data Assimilation". This term is used to indicate that, under the randomizing action of chaotic model dynamics, the prior is completely decorrelated from the previous analysis state. As a consequence, the observational information cannot accumulate over time, as opposed to the typical application of for short-range prediction. This complete absence of observational constraint on the forecast implies that our experiments are performed in an "off-line" regime.~~

3.2.2 Minimum and Product Growth Rate Functions~~(and)~~

~~Here~~ Here, the performance of ~~the~~ two different growth functions within the VSL's formulation, the product growth response (G_{Prod}) and the minimum growth response (G_{Min}), are investigated. ~~These formulations are tested for both "online" (with cycling) and "offline" (no-cycling) data assimilation set-ups.~~ In a simple DA experiment, AC15 have shown that the G_{Prod} performs slightly better than G_{Min}.

~~**with cycling**~~

~~Considering the SLAB experiment, we compare the two nonlinear representations in our setting (and).~~ As illustrated in figure 6a, the DA forecast presents no skill in the globally averaged temperature for both of the representations (VSL with Minimum t-norm (VSL-Min) and VSL-Prod). However, the use of VSL-Prod ~~,~~ instead of VSL-Min appears beneficial to the filter performance for the analysis, as demonstrated in Figure 6b. The expected value of the RMSE shifts significantly toward lower values for VSL-Prod compared to the free ensemble run. ~~Similar to the case of , the~~ The RMSE time-series shows an increasing trend for both VSL-Min and VSL-Prod.

The RMSEs of DA forecasts using different VSL representations (figures ~~5a,~~ 7a and 8a) do not indicate any improvement over the free ensemble run (Fig. 3b). The analysis of VSL-Prod performs ~~with~~ slightly better skill than VSL-Min over Europe, the United States and Central Asia. Due to the strong nonlinear features of VSL-Min and VSL-Prod, the performance of these filters is expected to be degraded with respect to the ensemble runs constrained with VSL-T linear observation (see AC15). This behavior can be readily seen by comparing the figures 5b, 7b and 8b.

~~with no-cycling~~

Our experiments show that the DA forecast has no skill over the ~~model climatology. Several recent studies have applied a similar methodology (Steiger et al., 2014; Dee et al., 2016; Hakim et al., 2016). Dee et al. (2016) have performed paleoclimate reconstructions by using a physically based for three kinds of proxies (tree ring, coral $\delta^{18}O$ and ice core $\delta^{18}O$) and two isotope-enabled atmospheric general circulation models. Matsikaris et al. (2015) compared an off-line and an on-line ensemble-based and showed that the both methods outperform the model without . They concluded that the on-line method performs a more realistic temporal variability~~free ensemble run. Therefore, ~~we investigate the idea of purely "~~using the forecast state of the free ensemble simulations we conduct an off-line ~~"~~ DA ~~. The~~ to reconstruct the analysis state. The forecast of the free ensemble simulation at any individual year is used as the ~~prior~~ forecast state vector for that year~~instead of the forecast~~. Following this methodology, the cycling step (reinitialization) of the ensemble is neglected~~and the time averaged technique is applied in parallel~~. A very interesting feature of figure 9 is that the increasing trends in the RMSE time-series of the analysis have vanished. ~~This indicates that the previously existing trends in the forecast and consequently in the analysis originated from the reinitialization step of the system but not the proxy records.~~ Figure 10 also confirms that the performance of ~~no-cycling~~ off-line DA can compete with the performance of the online DA.

3.2.3 Signal to Noise Ratio

The Signal to Noise Ratio (SNR) is expressed as the ratio of the standard deviation of the nature (true) run to that of the additive white noise. We examined the performance of the off-line DA with different SNRs~~(Fig. 11)~~. Figure 11 exhibits that the time-averaged global RMSE shows an ~~elbow~~ inflection point at values around $SNR = 1$ and reaches the error levels of ~~Free run~~a simulation without DA (free ensemble run) at $SNR = 0.03$, where all the pseudo-observations are ignored in the DA.

3.2.4 ~~Time Variable~~ Time-Varying Soil Moisture

To investigate the effect of using the time-varying soil moisture fields instead of climatological average in DA approach, we implemented the Climate Prediction Center (CPC) Leaky Bucket Model (LBM) (Huang et al., 1996) in our DA code. The LBM code was extracted from VSL v2_3 (ftp://ftp.ncdc.noaa.gov/pub/data/paleo/softlib/vs-lite/). ~~Instead of using climatological soil moisture for , the~~ The precipitation and temperature output from SPEEDY is used as input for LBM to produce the ~~new set of soil moisture with interannual~~ time-varying soil moisture fields with inter-annual variations. In the next step we repeated the off-line data assimilation runs for two VSL presentations (VSL-Prod and VSL-Min) ~~.~~using the new soil moisture data. The results show that using the ~~new set of soil moisture~~ time-varying soil moisture fields has improved the error reduction of VSL-Min with minor improvement for VSL-Prod ~~in both time evolution and maps of~~ (Figures 12 and 13). The RMSE of VSL-Min reaches the one of VSL-Prod when using the soil moisture calculated from LBM. Figure 14 shows the ~~histograms~~ boxplots of the RMSE time-series. The results show that ~~the is more sensitive to the choice of soil moisture and~~ while using the soil moisture calculated by the LBM improves the performance of the model ~~. However~~for VSL-Min, the improvement in error reduction for VSL-Prod is not significant~~when using the calculated soil moisture with the~~.

4 Discussion~~and Conclusions~~

4.1 ~~Error Reduction Efficacy of Chronologies~~

~~For the s studied here, it was found that the ability of a particular pseudo-chronology to reduce the error of the~~ Using the time-averaged EnKF ~~-based estimate of the time averaged state appears modulated by the strength of the yearly internal variability of the model at the chronology site. This methodology is termed and can in principle be employed to help the dendrochronology community to increase the effectivity of their sampling efforts by focusing on the sites with more potential to decrease reconstruction uncertainty (Ancell and Hakim, 2007; Hakim and Torn, 2008; Mauger et al., 2013). Furthermore, this approach can be directly applied to any proxy type with sufficiently stable time resolution (e.g., annual resolution)(Comboul et al., 2015). However, the application of this method for lower frequency climate data like sediment cores or speleothems has to be investigated. These results are likely to depend on the climate model, the proxy system model, the proxy network and their resolution (Comboul et al., 2015).~~

~~An evident caveat of the above mentioned rationale is that every model-based estimate of the climate internal variability strength for a particular time scale will necessarily exhibit the biases of the particular climate model used. We consider that this modeling subjectivity/imperfection issue can be ameliorated by means of multi-model~~ methodology and ~~multi-physics approaches, which in principle should increase the robustness of the results and provide uncertainty estimates. In any case, we believe that provided results are analyzed cautiously taking into account the weaknesses of current climate models. The climate dynamics knowledge condensed into an Earth system model can certainly be used profitably to reduce the cost of a indiscriminated proxy sampling strategy.~~

4.1 ~~Off-line Regime~~

~~Within our simplified perfect model, the observed situation of simultaneously having significant~~ a proxy forward model (VSL), we assimilated the pseudo-observations in an AGCM (SPEEDY). Using a set of perfect model experiments we studied two different aspects of DA~~skill for analysis quantities and none for forecast quantities, currently referred to as~~ , namely "on-line/off-line ~~regime ,~~ regime" and "on-line/off-line DA scheme". We concluded that the DA conducted here appears to be in the "off-line regime": while the analysis quantities have a significant skill, there is no skill for the forecast state. This result supports the studies of Huntley and Hakim (2010) and Pendergrass et al. (2012) who applied the time-averaged EnKF to a quasi-geostrophic atmospheric jet model. The appearance of the off-line regime can arise either from the dynamical model or from the DA scheme~~(answers to the questions 1 and 2 raised in Introduction).~~

. Regarding the dynamical model, the ~~most obvious reason to enter into the off-line regime~~ main reason for losing skill in the forecast is that the period between consecutive observations exceeds the predictability horizon of the model. Under these conditions, as already discussed in AC15, the DA ensemble spread reaches ~~climatological levels (~~the spread of the ~~Free ensemble run)~~ free ensemble run before new observations are assimilated~~and the accumulation of observational information is essentially lost~~. For SPEEDY, due to its purely atmospheric nature, it is likely to ~~enter the off-line regime~~ have no forecast skill for a 1-year inter-observation period. ~~This might be also the case for current operational (coupled) climate prediction systems, given their lack of useful lead times longer than one year.~~ Thus, it seems unlikely to ~~achieve effective observational constraints on the forecast~~ improve the forecast skill using proxy records with yearly time resolution. However, there is already evidence for the existence of potential sources of climate internal variability with time scales longer than ~~1~~ one year (Smith et al., 2012). The so called "annular modes" (Thompson and Wallace, 2000) may present internal variability in the high latitude areas. The latitudinal oscillation of the cell structure imposes variability at the fringes of the jet streams and oscillations of the ITCZ impacts the humidity (Holton and Hakim, 2013). ENSO affects a large portion of tropical and subtropical climate in time-scales larger than one year. Accordingly, we expect that it should be possible to obtain actual inter-annual predictability skill in the foreseeable future.

Regarding the DA scheme, a possible ~~culprit for the onset of the~~ reason for the appearance of off-line regime is the time-averaged update strategy (Dirren and Hakim, 2005). It is not clear ~~if~~ whether we can employ this technique with SPEEDY to properly estimate instantaneous quantities out of time averaged observations. In particular, ~~complete decorrelation between~~ it is not guaranteed that the time averaged and instantaneous variables ~~is not guaranteed~~are de-correlated.

~~In any case, despite~~ Given that our DA simulation is in the off-line regime (no forecast skill), we have conducted an "off-line DA" (no-cycling) for two different representations of VSL forward model. Despite its lack of accumulation of observational information over time, off-line DA has already been shown to be more robust than traditional Climate Field Reconstruction (CFR) techniques based on orthogonal empirical functions and stationarity assumptions (Steiger et al., 2014; Hakim et al., 2016). ~~Moreover, the implementation and running of~~ Our off-line DA ~~schemes is remarkably cheaper than on-line approaches.~~

~~Following the idea of (Steiger et al., 2014; Matsikaris et al., 2015; Hakim et al., 2016) for purely "offline" (no-cycling), our perfect model experiments indicate that the "online" scheme may not outperform the "offline" one in either the temporal or the spatial error reduction (answer to the question 3 raised in Introduction). It should also be emphasized that our model set-up (with slab ocean) can not capture the full atmosphere-ocean interactions. Therefore, using a more realistic coupled atmosphere-ocean model may improve the skill of the "online" .~~

4.1 ~~Filter Operation Sensitivity to the Growth Rate Function~~

~~The results of the experiments conducted with SPEEDY support results obtained~~ experiments support the results obtained by the two-scale Lorenz (1996) model (AC15) regarding the influence of the PLF representation on the filter performance. The efficacy of the EnKF-based time averaged state estimation strategy appeared to be significantly sensitive to the selection of the t-norm used to calculate the growth rate~~, with~~ . For the off-line DA presented here, the product t-norm (VSL-Prod) ~~outperforming~~ outperforms the minimum t-norm (VSL-Min) which is used in the original formulation of VSL forward model.

Tolwinski-Ward et al. (2014) described trees as fundamentally lossy[1] recorders of climate, due to the integrated nature of the information contained in them and the standardization process used to minimize the non-climatic effects on growth. Growth

[1]This adjective is currently used in the information technology area to designate data encoding methods that lead to information loss from the original version for the sake of reducing the amount of data needed to store the content.

is influenced by temperature and/or moisture and the transitions between limitation regimes may happen suddenly ("abrupt shifting") ~~Acevedo et al. (2015)~~(Acevedo et al., 2015). In the same vein, we argue that the "abrupt shifting" of recorded variable (temperature or moisture)–implied by the minimum function used in VSL's original formulation– might constitute an additional source of lossyness~~(at least within a EnKF-based setting used), which~~ . We conclude that this can be substantially reduced by resorting to alternative Fuzzy Logic-based representations of the PLF. ~~Our experiment indicates a higher skill performance with the for both "offline" and "online" regimes compared to the (answer to the question 4 raised in Introduction).~~

4.1 ~~Challenges to be Addressed~~

As a cautionary remark, we want to highlight the several important limitations of the experiments described in this paper. The generated pseudo-TRW observations lack a threshold for temperature or moisture after which the growth response does not change and their contamination with noise was performed assuming optimistically high SNR levels. Furthermore, the response thresholds were set in a completely homogeneous fashion for all the observational stations, whereas actual TRW networks are strongly heterogeneous in that sense, comprising chronologies generated under highly dissimilar growth limitation regimes. Additionally, the efficiency of EnKF technique used relies on the Gaussianity of all the variables of the model. Nevertheless, in a climate model some variables can present strongly non-Gaussian properties –specially definite positive quantities such as humidity– and their estimation should in principle be performed with more sophisticated strategies such a Gaussian anamorphosis (Bocquet et al., 2010; Lien et al., 2013). It is worth mentioning the necessity of explicitly addressing model errors by conducting imperfect model OSSE. ~~Finally, we note that our findings are based on a slab coupled ocean model and we encourage using a proper coupled atmosphere-ocean model in future studies.~~

4.1 ~~Outlook~~

~~Our results appear useful for chronologies in the sense that techniques are robust in the face of two strong nonlinearities, i.e., "switching recording" (Acevedo et al., 2015). Thus, it~~ It is important to emphasize that the OSSE presented in this manuscript represents the first step of the long hierarchy of DA experiments to achieve an effective assimilation of proxy records into climate models using forward proxy models. We encourage further experiments using comprehensive earth system models with longer time scale processes to bring the proxy DA into an online regime. However, assimilation of proxies in an earth system model with different components may lead to inter-component DA pollutions and is computationally very expensive.

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Table 1. Runs' characteristics.

No.	1	2	3	4
Forward Model	VSL-T	VSL-T	VSL-Min	VSL-Prod
Ocean	*SLAB*	*PRESCRIBED*	*SLAB*	*SLAB*

Simulations are 150 years long.

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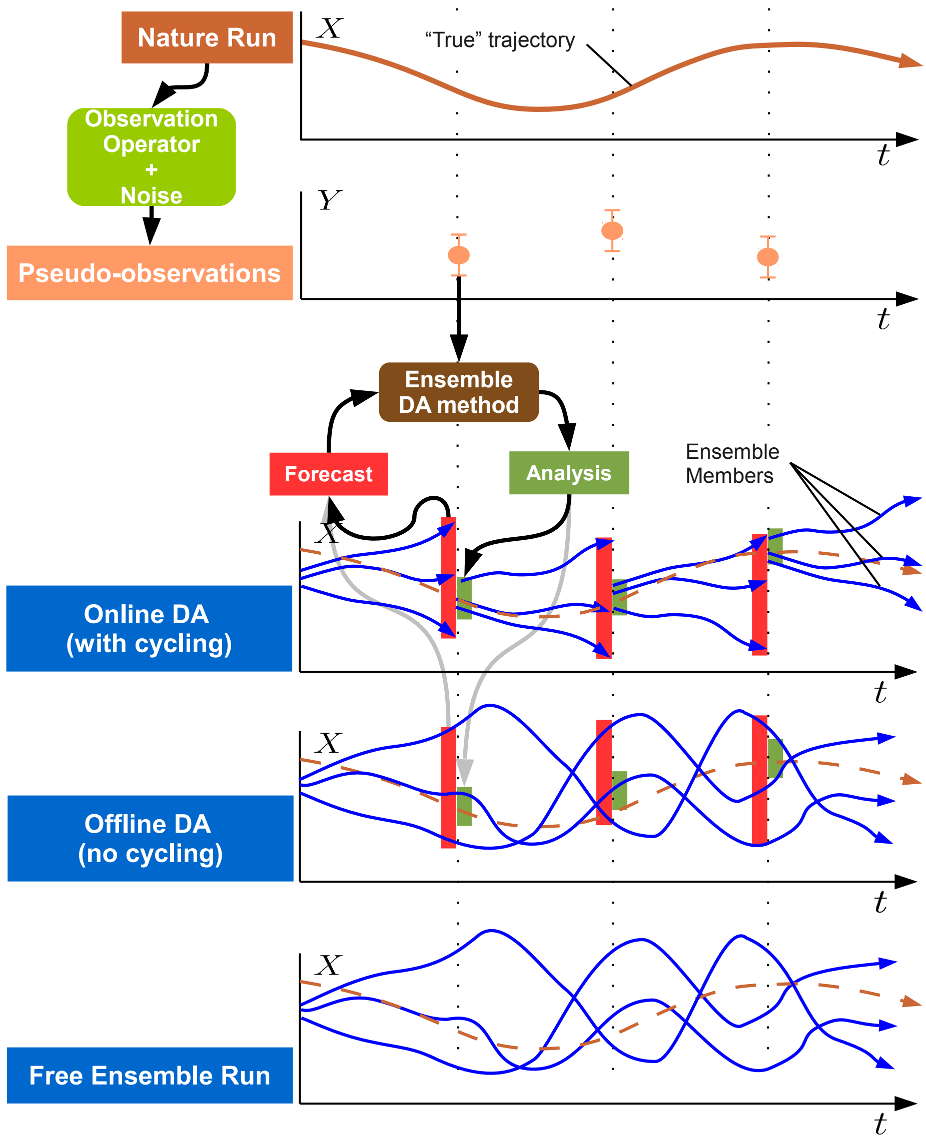


Figure 1. Schematic of a typical Observation System Simulation Experiment (OSSE) with ensemble "online" (with cycling) and "offline" (no-cycling) DA methods. t designates the time axis and $X(Y)$ denotes the model state (observation) space. Sharp (rounded) cornered boxes represent data (processes). Red (green) vertical shadings indicate the *Forecast* (*Analysis*) spread. Vertical dotted lines represent the assimilation steps.

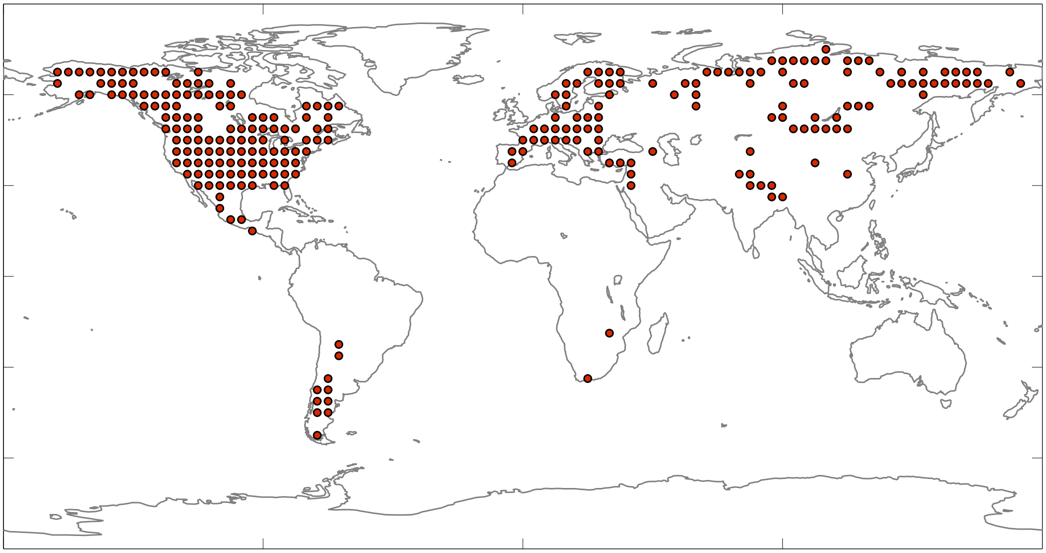
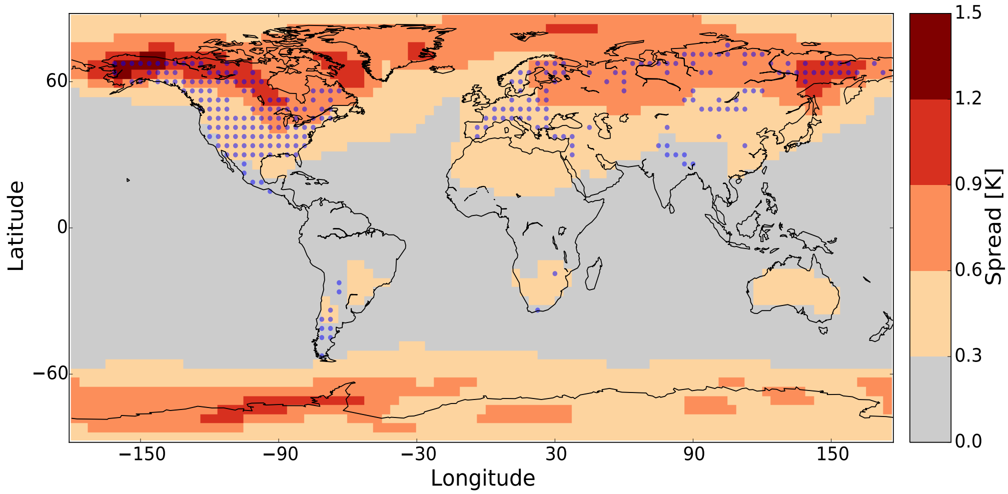


Figure 2. Station set resembling real TRW network from Breitenmoser et al. (2014)

a) Free ensemble Spread



b) Free ensemble Error

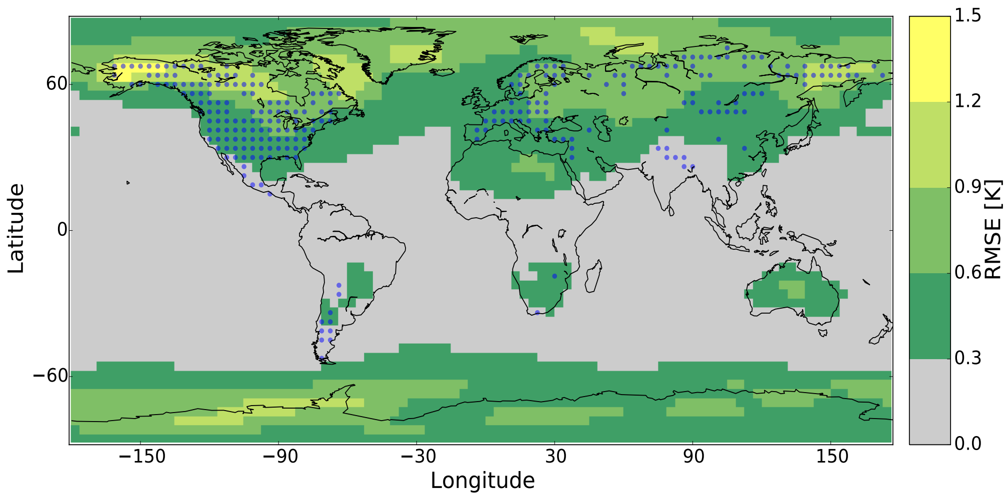
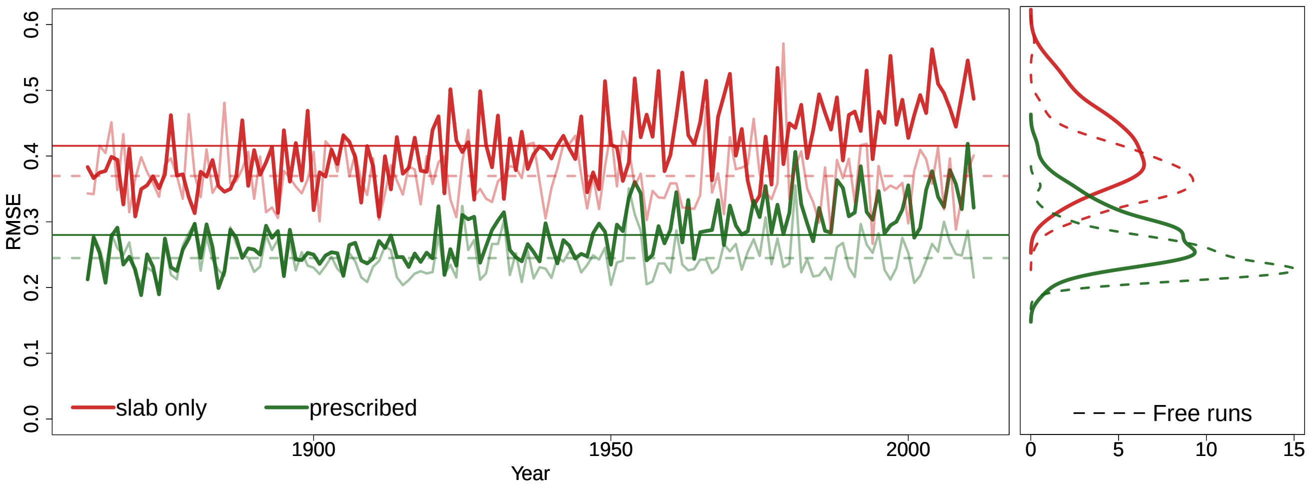


Figure 3. Free ensemble simulations for the SLAB experiment: a) Ensemble Spread [K] of near surface temperatures, b) Free ensemble RMSE [K].

a)



b)

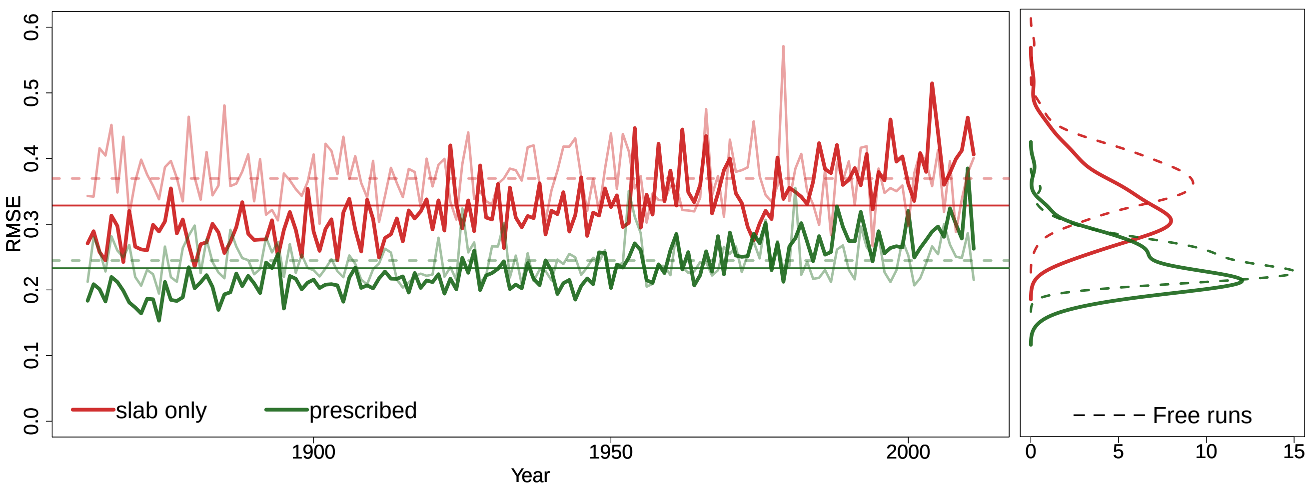
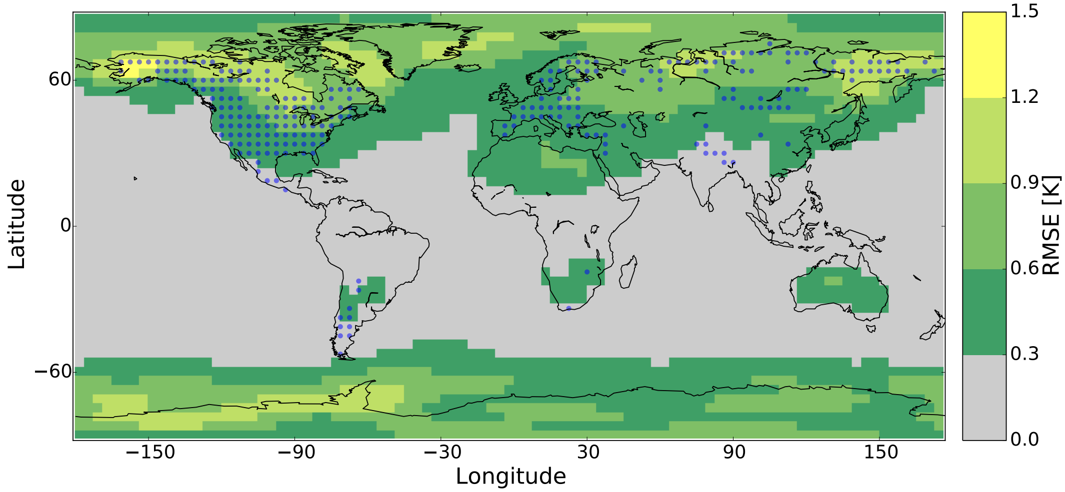


Figure 4. Global ensemble mean for a) Forecast constrained by VSL-T pseudo-TRW observations (bold lines) and Free run (thin lines); b) Analysis (solid lines) and Free run (thin lines). Horizontal lines exhibit the mean values. Right panels exhibit the histograms of the time-series.

a) DA forecast for VSL-T



b) DA analysis for VSL-T

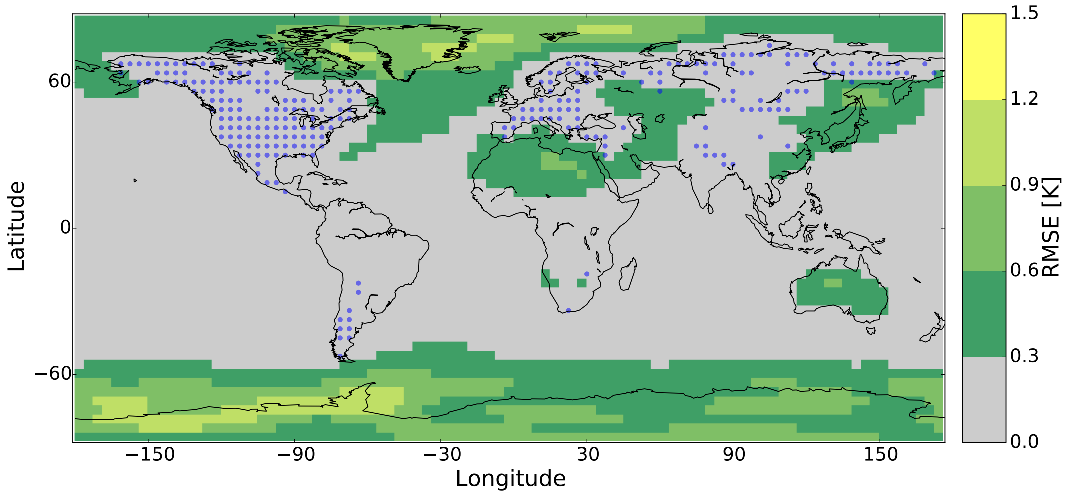


Figure 5. Time-averaged RMSEs of SLAB experiment for a) DA forecast and b) DA analysis using the VSL-T observation operator.

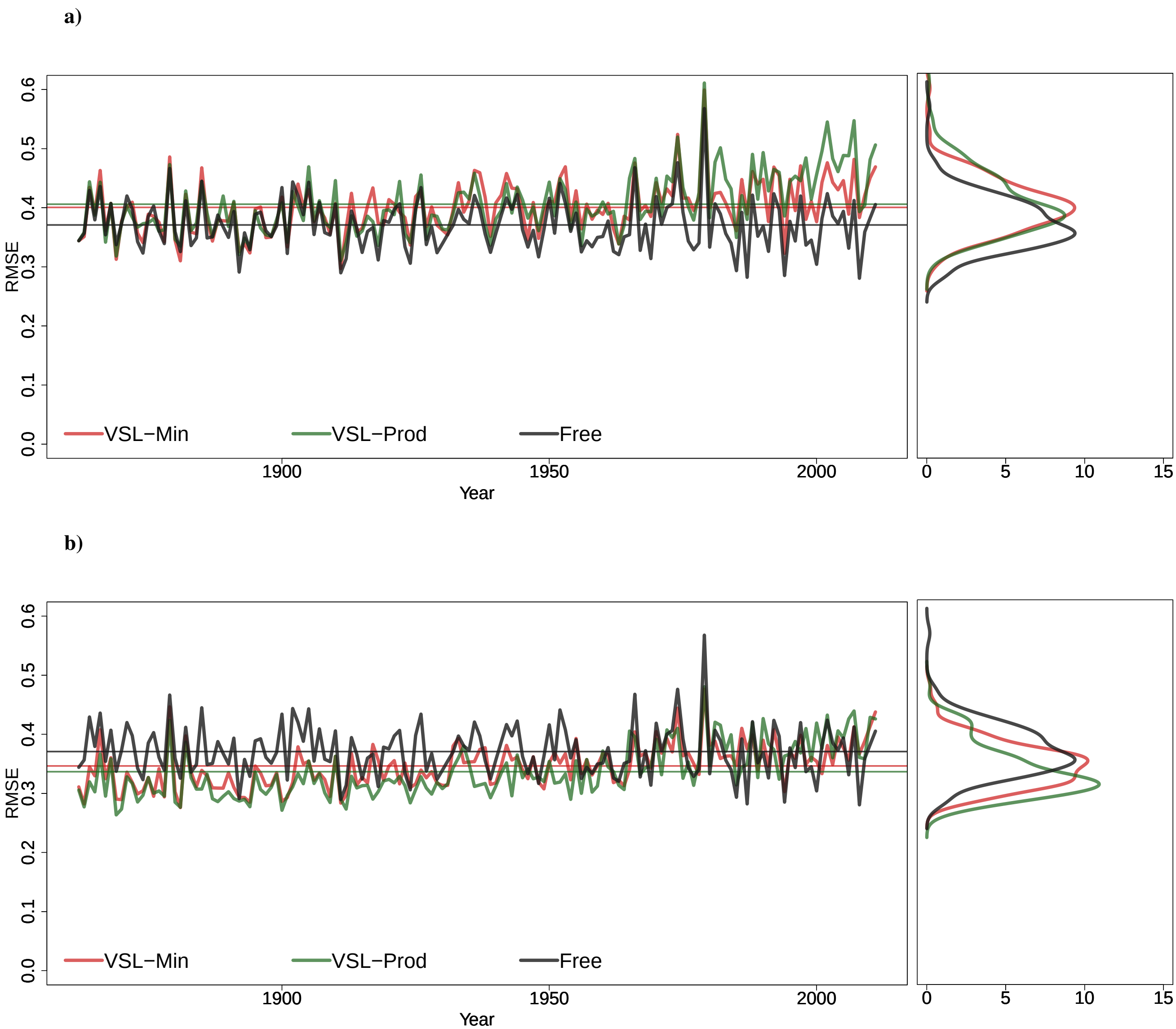
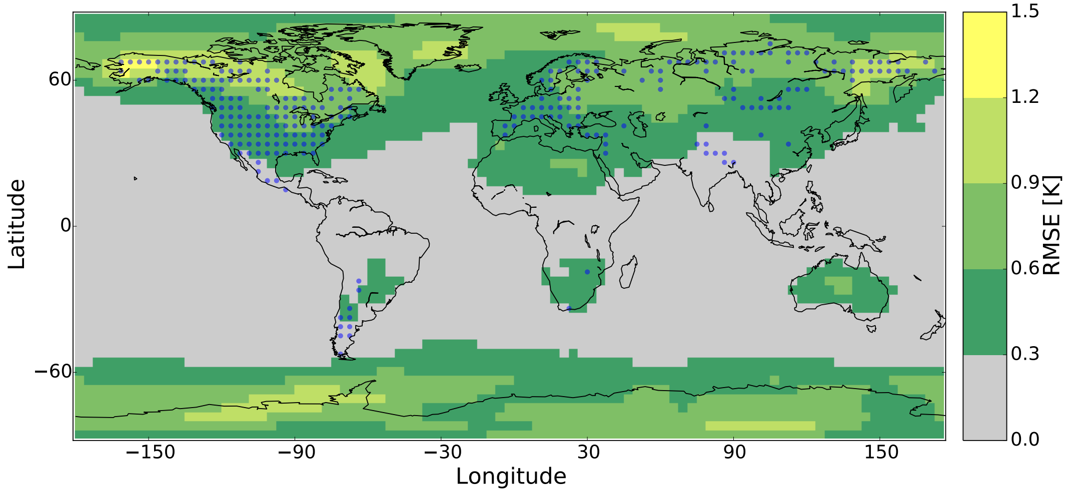


Figure 6. Global ensemble mean for a) forecast and b) analysis constrained by VSL-Min (red) and VSL-Prod (green) pseudo-TRW observations and free run (black). Horizontal lines exhibit the mean values. Right panels exhibit the histograms of the time-series.

a) DA forecast for VSL-Min



b) DA analysis for VSL-Min

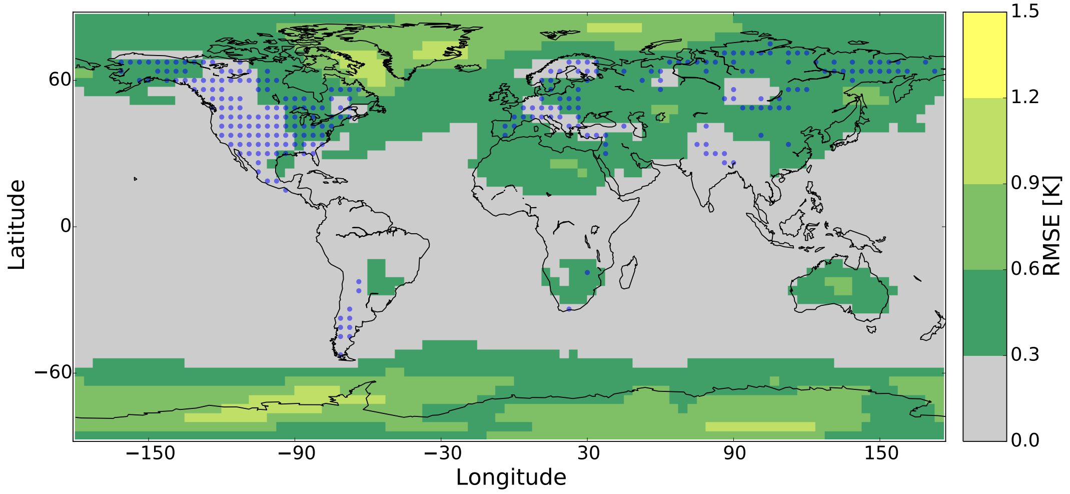
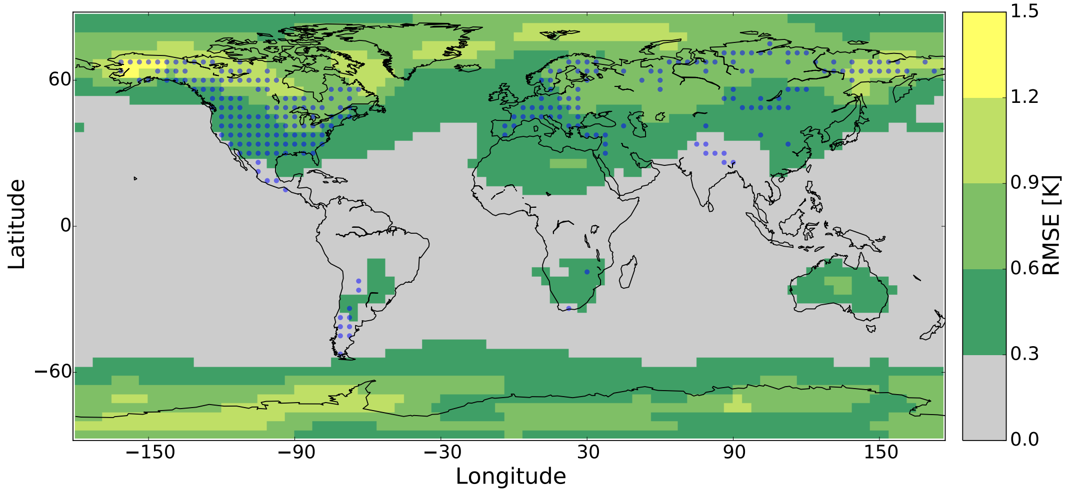


Figure 7. Time-averaged RMSEs of SLAB experiment for a) DA forecast and b) DA analysis using the VSL-Min observation operator.

a) DA forecast for VSL-Prod



b) DA analysis for VSL-Prod

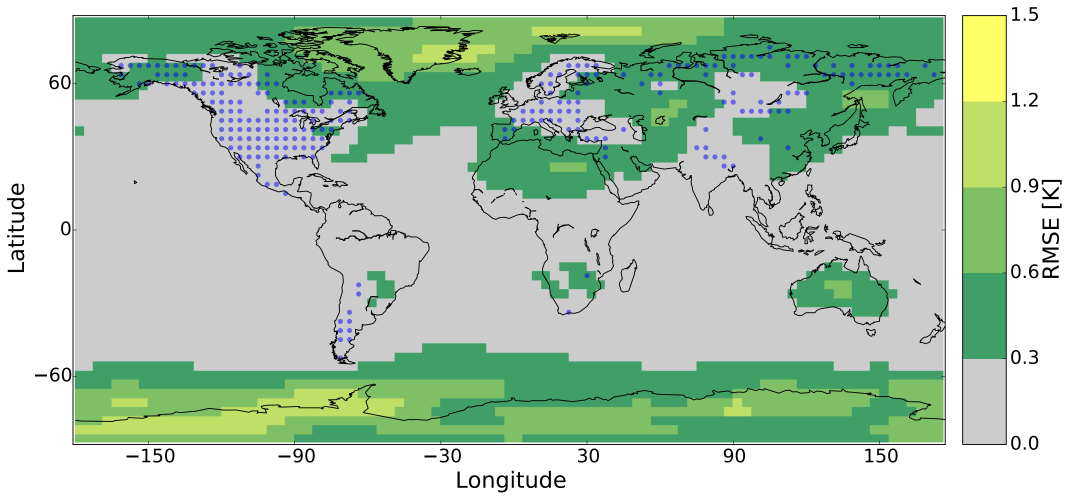


Figure 8. Time-averaged RMSEs of SLAB experiment for a) DA forecast and b) DA analysis using the VSL-Prod observation operator.

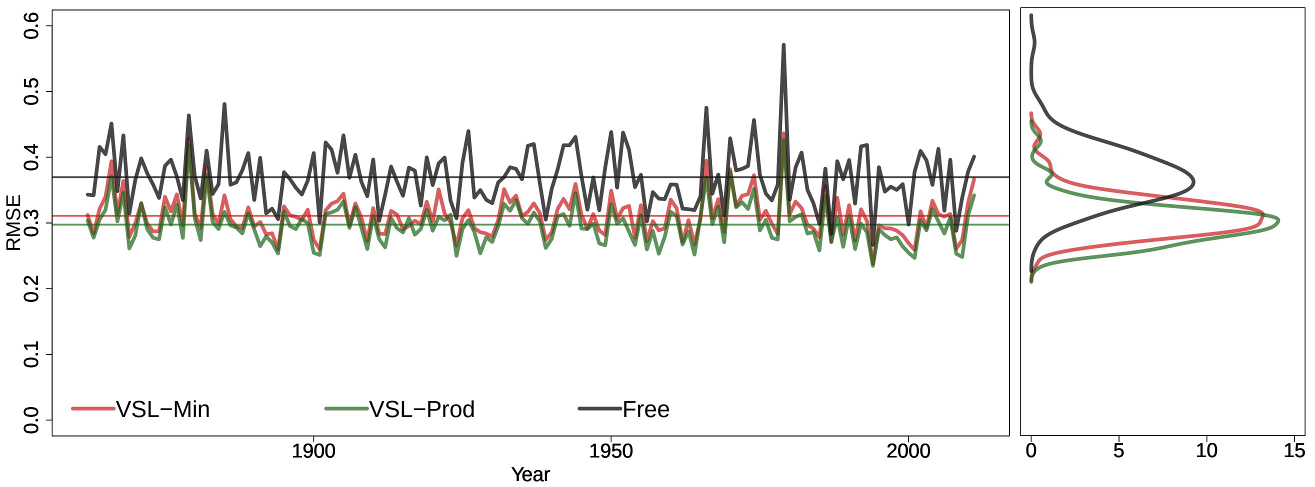
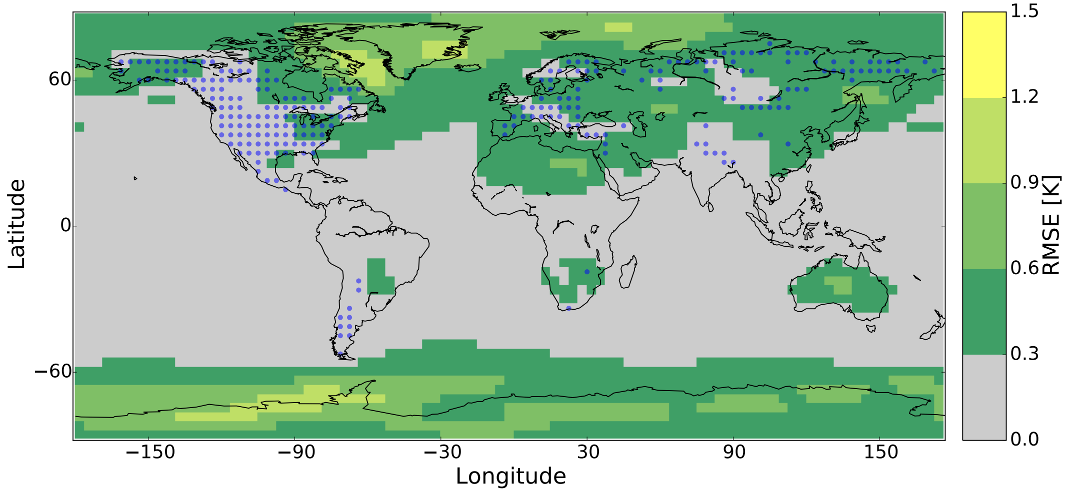


Figure 9. Global ensemble mean for analysis constrained by VSL-Min (red) and VSL-Prod (green) pseudo-TRW observations and free run (black). Horizontal lines exhibit the mean values. Right panel exhibits the histograms of the time-series.

a) DA analysis for VSL-Min with nocycling



b) DA analysis for VSL-Prod with nocycling

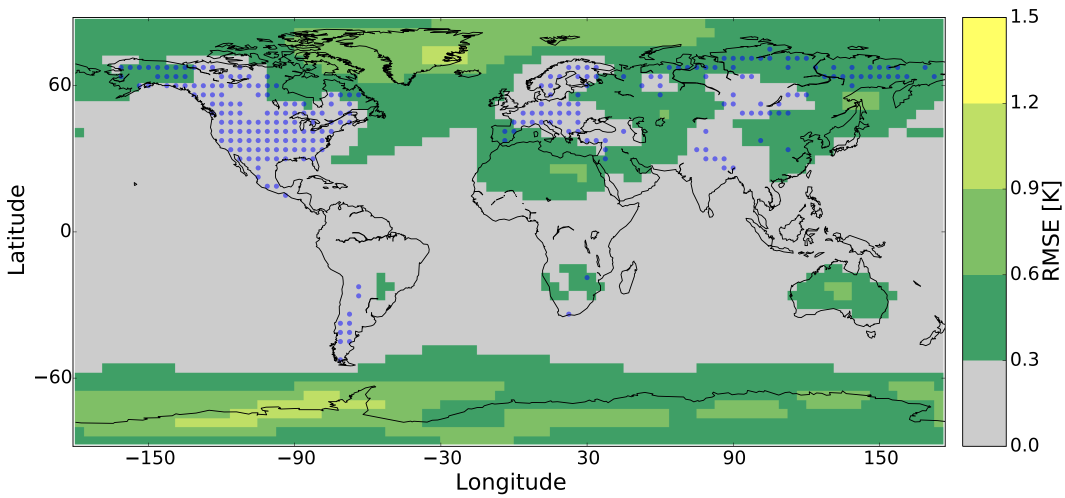


Figure 10. Time-averaged RMSEs of SLAB experiment for a) nocycling DA analysis using the VSL-Min and b) the VSL-Prod observation operator.

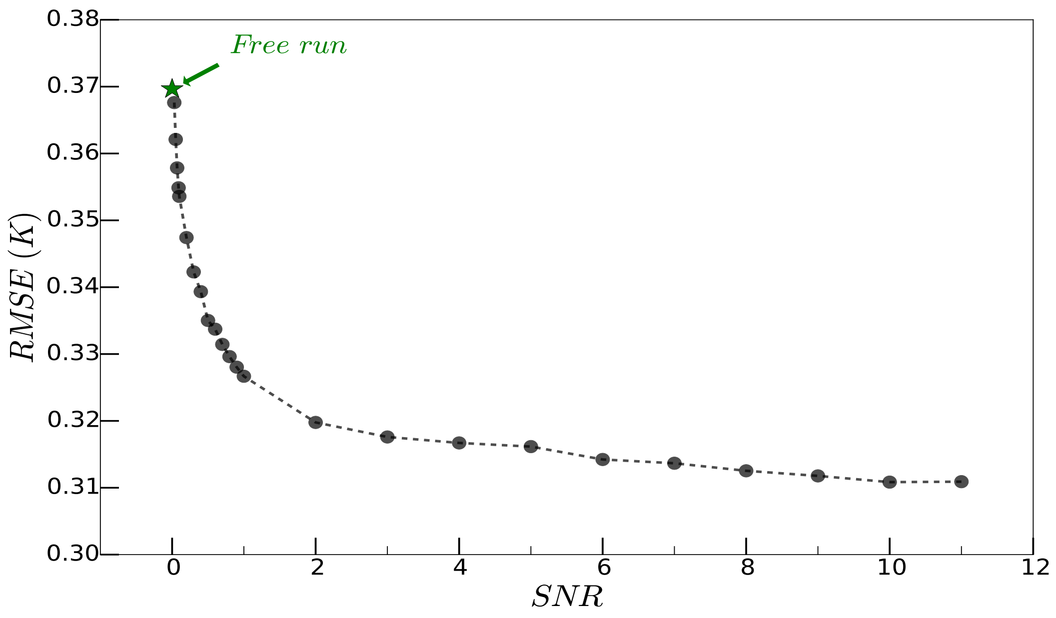
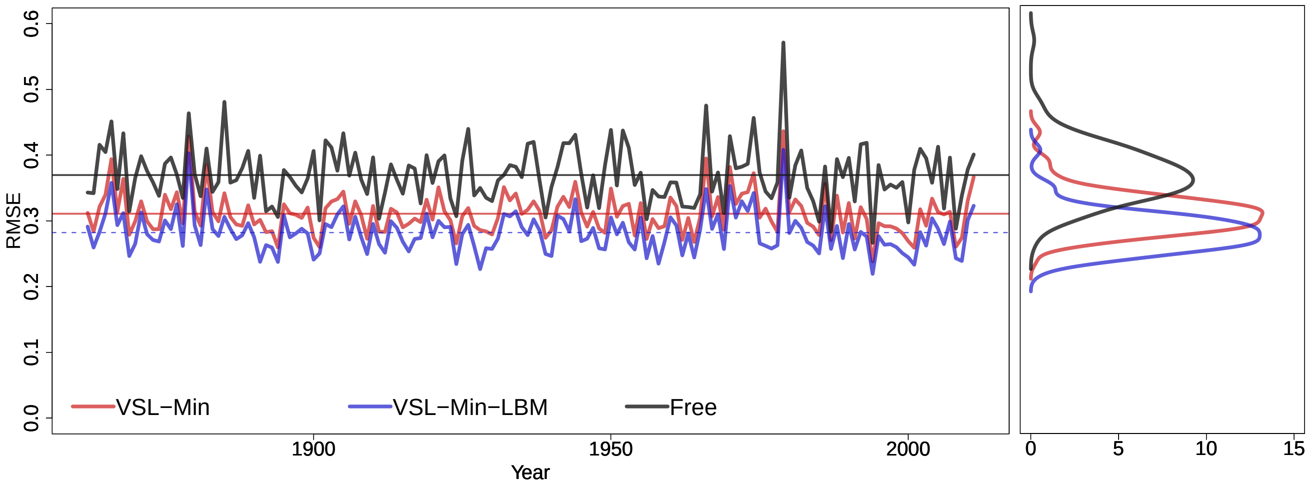


Figure 11. Time-averaged global RMSEs of SLAB experiment for nocycling DA using the VSL-Min and different signal to noise ratios. The Green star shows the Free run RMSE.

a)



b)

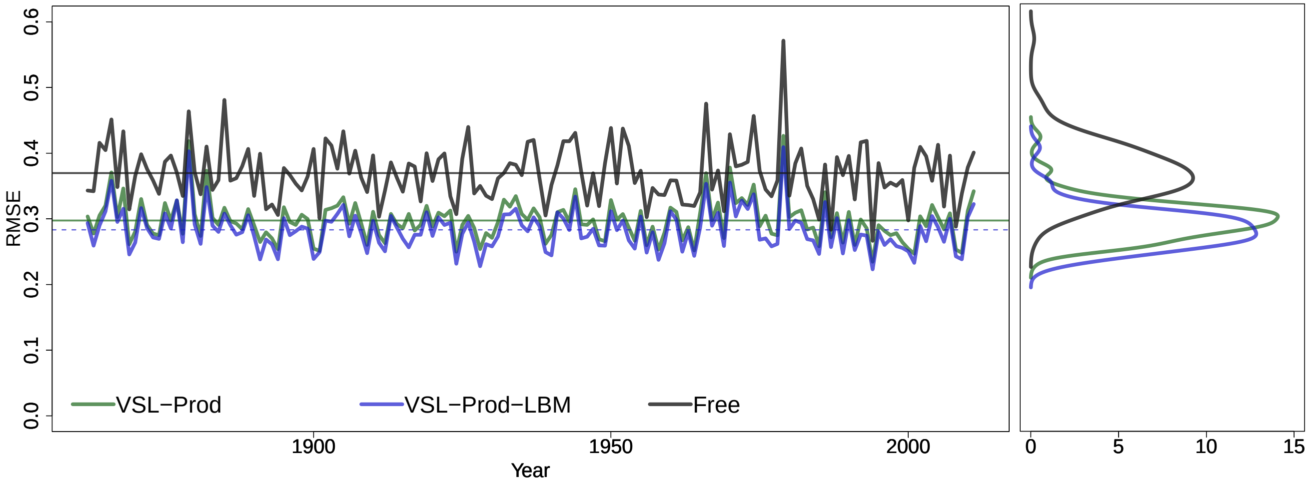
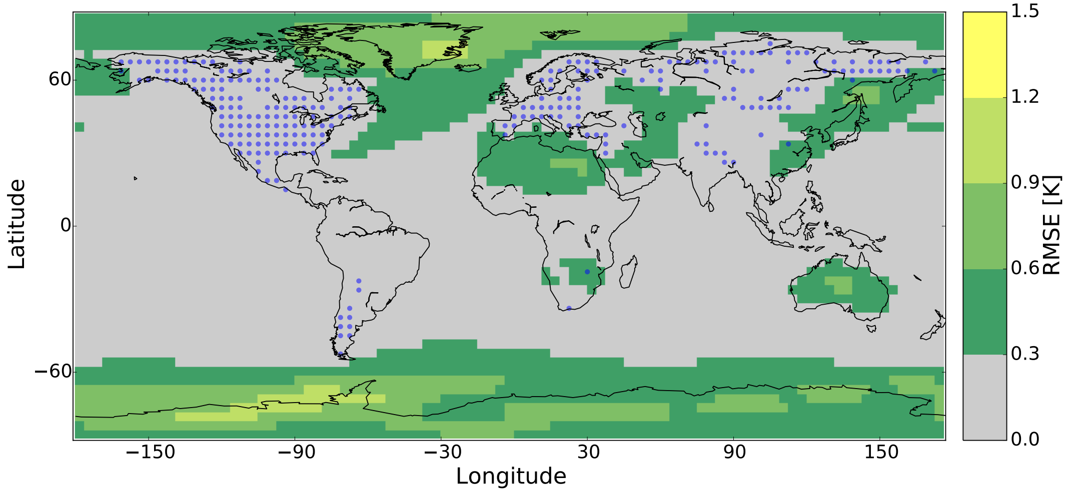


Figure 12. Global ensemble mean for analysis constrained by pseudo-TRW observations for a) VSL-Min with the climatological soil moisture (red), with the soil moisture computed by Leaky Bucket Model (blue) and free run (black); b) VSL-Prod with the climatological soil moisture (green), with the soil moisture computed by Leaky Bucket Model (blue) and free run (black). Horizontal lines exhibit the mean values. Right panels exhibit the histograms of the time-series.

a) DA analysis for VSL-Min with Leaky Bucket Model and nocycling



b) DA analysis for VSL-Prod with Leaky Bucket Model and nocycling

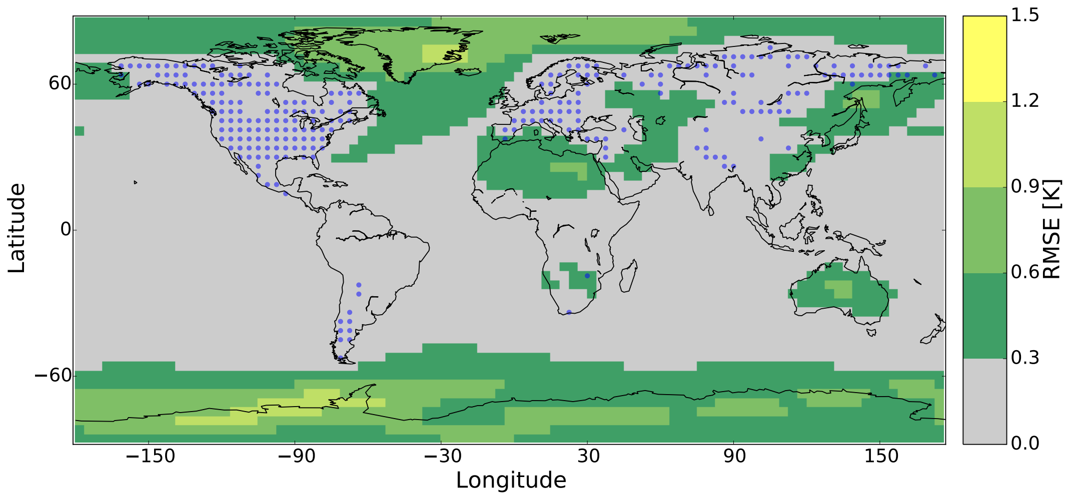


Figure 13. Time-averaged RMSEs of SLAB experiment for a) nocycling DA analysis using the VSL-Min with Leaky Bucket Model and b) the VSL-Prod with Leaky Bucket Model observation operator.

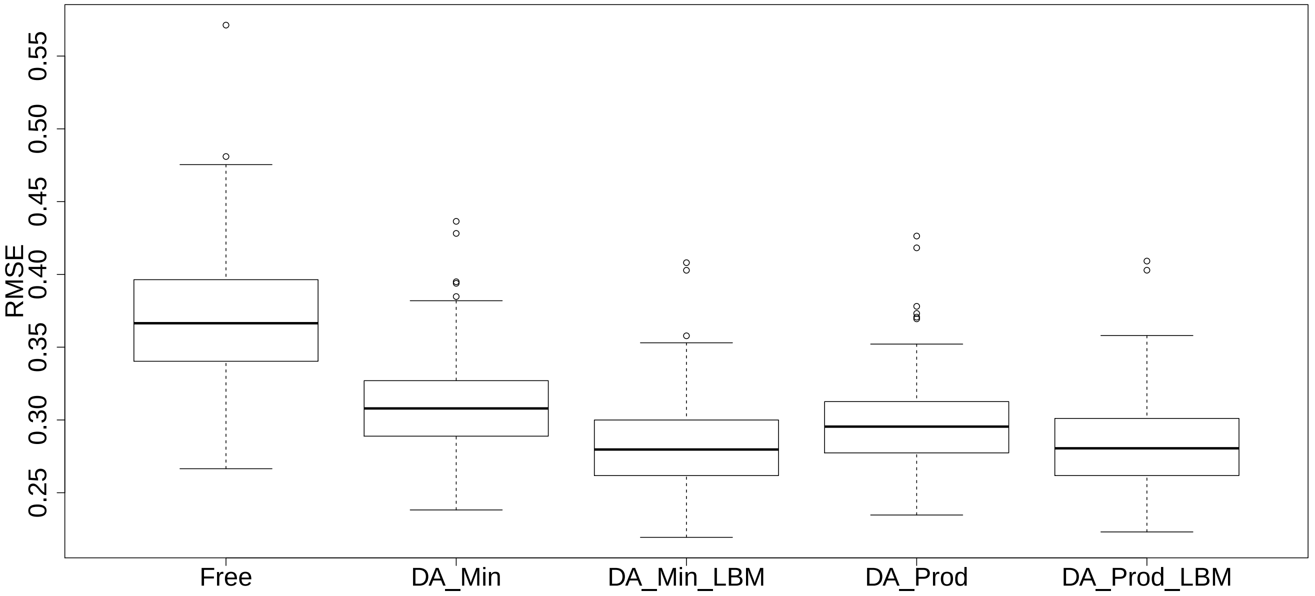


Figure 14. Histograms of global ensemble mean for analysis constrained by pseudo-TRW observations for Free run, DA run with VSL-Min, VSL-Prod using the climatological soil moisture and VSL-Min, VSL-Prod using the Leaky Bucket Model (LBM).