Final answer to the Comments

Acevedo et al.,

January 17, 2017

Dear Prof. Goosse and Dear reviewers,

Thank you so much for your constructive comments. A track changes version of the manuscript is also added at the end of this answer(Red is deleted and blue is added). We reply to all your comments here:

1 Answer to Reviewer 1

We wish to thank you so much for your constructive review and very detailed comments. It would be our pleasure to do all the modifications and make the improvements you have suggested, in the next version of the manuscript. We answer your comments (*italic*) point by point (**Bold**):

My major comment for this paper is that while the language is very concise and direct, it is perhaps too concise; many sections of the paper leave the reader wanting more detail or are a bit confusing because they are so brief. For example, in the introduction, I felt as if I didn't have a clear grasp on what new problem the authors were looking to solve, existing gaps in the literature, and how they addressed these in a novel way.

There are also a lot of references to previous studies but without any additional infor- mation and the reader is left feeling lost. I have made notes in my comments below about the specific sections where more detail is needed. Very few equations describing the experimental design are given to orient the reader to the various components of the DA strategy and how you altered it for your specific set of tests. There are also some major relevant references that have come before this study that are missing and not discussed, which I have pointed out in my line-by-line comments below.

We completely agree. Therefore, we have reviewed the methods in more detail and added additional text to the introduction and methods as well as the results and the discussions (We expanded the manuscript from 26 pages to 35 pages). Data assimilation equations are also added in the methods. Subsections "Rationale, Kalman Filter, Ensemble Kalman Filter, Time Averaged Ensemble kalman Filter, OSSE, TRW forward models and VSL from the Fuzzy Logic Viewpoint" are fully explained and added to the new version. We

hope that the new version of the manuscript with all additional subsections and explanations is now clear for the readers.

In particular, I think this paper needs to cite and discuss previous findings of Dee, Steiger, Emile-Geay and Hakim 2016 (JAMES):

Dee, Sylvia G., et al. "On the utility of proxy system models for estimating climate states over the common era." Journal of Advances in Modeling Earth Systems (2016).

They have already employed 3 different proxy system models with DA and it would be helpful here to discuss how your study is different from the findings that have recently been outlined in that paper. It's clear that you are performing different tests in this study, but you have to acknowledge that this is not the first piece of work to include forward proxy models with DA (as written you assert this).

We agree on this and the point that it is not the first study considering the forward proxy models with DA. We have also cited the paper by [Dee et al., 2016] in the new version and modified the Introduction accordingly, acknowledging this study. Given that this novel paper [Dee et al., 2016] was published first on 10^{th} August 2016, we missed that during the production process of our manuscript. This paper is a perfect reference and back-up for our similar strategy in paleoclimate reanalysis.

From a science perspective, and as I've highlighted below, I think there are some de- sign problems with the VS-Lite application: namely, you used prescribed soil moisture fields when you can instead use time-varying precipitation. Why use a climatological average that is not time-varying for DA when you can use dynamically-updated precip? This makes no sense to me, and I feel it detracts from your results.

The soil moisture used in our experiments is not just a climatological average but a monthly climatology, meaning that the soil moisture value does depend on the month and is therefore time varying, allowing the seasonal competition of limiting factors to develop. Using SPEEDY precipitation as input for VS-Lite model is indeed a possibility to add time variability to the soil moisture, however this would imply to use the CPC Leaky Bucket [Huang et al., 1996a] Model in order to generate moisture time series out of temperature and precipitation time series. We have implemented the Leaky Bucket Model and repeated two simulations. Please see the answer to the Editor or Figures 12-14 and subsection 3.2.4 of the new manuscript.

In general, with some revisions to the text giving more description, more background, and much more motivation, this paper should be suitable for publication in CoP. Note: The way this manuscript is numbered makes it challenging to give line by line comments. Can you please revise this?

Thanks for the comment. We have used the CoP's LATEX tem-

plate for compiling the manuscript. Maybe this issue can be suggested to the Editorial Support, providing a new template for CoP.

Page 1, Line: 4: appeared = appears (active voice) 13: revise "the so called" to "the usage of paleoclimate proxy records." 14: Revise: "Nonetheless, these natural archives. . ." 16: Revise: "is still an open question" to "can often remain opaque." 17: Delete "To the," rephrase "At present, many. . .. 18: comma after hindcasts, 22: and cite Dee et al., JAMES 2016 in addition to other citations.

Done!

Page 2 5: rephrase last sentence "Finally, the use of a particle filter has been tested..." 11: cite Dee et al., 2015 (JAMES)—PRYSM along with Evans (review of existing forward models). 12-15: Need to cite and discuss previous findings of Dee, Steiger, Emile-Geay and Hakim 2016 (JAMES) here: Dee, Sylvia G., et al. "On the utility of proxy system models for estimating climate states over the common era." Journal of Advances in Modeling Earth Systems (2016).

Done!

17: comma after AC15, delete now, comma after scenario. "were" = "where." 20-25: Back to my major comment above: to a person who is not already quite familiar with the technical details of Data Assimilation, these objectives are opaque. We need more background and you haven't yet defined prior, posterior, etc. There hasn't been any prior introduction of the DA equations so all of this comes out of nowhere.

Done!

25: filed = 'field' 28: yes it has already been explicitly investigated, in Dee et al., 2016—as I mentioned, you'll have to discuss this and potentially change the language in your introduction accordingly. 29: rephrase "the TRW forward model, and the climate model.."

We have deleted this sentence and acknowledged Dee et al., 2016.

Page 3 3: rephrase: "accuracy, relatively user-friendly implementation, and computational expense." 5: what do you mean by "adjoint model"? This is not clear. 9: rephrase "have historically been prohibitively expensive...", hyphen in high- dimensional. Change "However" to "Thus" 10: toy models—is this a common phrase in DA? You have not defined it. I think you should change to "perfect model studies," which is a more widely recognized term for this type of study. Or, "pseudoproxy tests."

We changed the text accordingly.

13: If I am a person unfamiliar with DA, how do I know what the 'observation operator' is? We could really use some equations here: lay out the DA equations for us so we know what the 'observation operator' is. The DA community will follow you, but most others will not. 13: You have not defined "TA" yet.

Several sections ans subsections have been added to the manuscript to cover your comment. We also reordered many parts of the manuscript to describe every detail of the methodology.

15: grammar is incorrect in last part of this sentence. Perhaps you mean: "We study the impact of....using the assimilation of TA linear observations as a reference." 19: what is fuzzy logic? Please cite and explain in detail. 2.2.1 Spell out "V-S-Lite Model" 20: change "limiting factors" to "model inputs for VSL are ..." and put parentheses around (T, M). 21: 'variables' (add s) and rephrase "variables influence tree growth..." delete period after gm, just continue sentence "using a piece-wise" and put colon after Tolwinski citation:

Done!

Page 4 1: no indent, no capitalization of Where, change to "denote minimum thresholds for temperature and moisture below which there is no grown, and TU and MU are upper thresholds above which tree growth is optimal" 2: are you sure it's optimal and not too hot/dry?

Done! TU and MU are defined as optimal growth limits and not the hottest or driest limits.

2.2.2 the reader does not know what Fuzzy Logic is because you have not introduced it in the text, nor have you cited it. We need more context. 9: delete 'have, the' ... and what is PLF? Have you defined this yet? 10-15: this is too brief and we need more motivation here about your experimental design and what you're testing.

We added a complete section describing the Fuzzy Logic concept.

Page 5 1: delete 'the' before version 32 1-7: be a bit careful here with text—this reads awfully similar to Molteni 2003 6: rephrase "The latter makes SPEEDY..." 7: change 'presented in this paper' to 'necessary for this study' 15-16: not enough information for a non-DA specialist. 21: 'where' = were 23: huge = large, change 'are' to 'were' – also, what is the fallout of this? 24: change 'as the following' to 'as follows:' 25: delete comma after deviation 2.3.3. change to 'Simulations' instead of Runs characteristics, which is not grammatically correct.

Done. We think that the new section added will cover some basics of the DA implementation.

Page 6, Line: 2 consist = 'consists' 6 'from the equilibrium' —not enough detail. Do you mean it's already spun up, or it's a control simulation? Be more clear. 7 should not be a new paragraph, change 'affordability' to 'efficiency' 8 "minimum" and "product" Triangular norms come out of no where, we need an explanation, description, citation, and to not be lost by the first use of these terms 10 150 year (no 's') — and, by 'nature' run do you mean 'control' run? I have never seen the term 'nature' run. change wording. 11 change month to 'months' 12 nature = control, when you say 'different ensemble runs' are these *the ensemble of climate state vectors or 'prior' for the DA? be clear. Change 'driving' to 'forcing SPEEDY' 16 change 'added to the clean' to 'imposed on the TA observations' — also I think you should spell out TA and not abbreviate. It's a short acronym and it's confusing when there are already so many other acronyms flying around. Delete comma after 'observations so as to obtain'..*

Done

17 10 seems like a very high and unrealistic SNR for a pseudo proxy test. See previous literature on this topic, and the Smerdon et al. 2012 review.

Thanks a lot for this comment. We have redone the offline DA using VSL-min for 24 different SNR values (from $SNR =$ 0.03 to 11) and plotted the time-averaged global RMSEs. We have added Figure 11 and a subsection for SNR to the manuscript. As can be seen the plot shows an elbow around value $SNR = 1$ and reaches the Free run at around $SNR = 0.03$ where almost all of the observations are neglected in DA..

20 So, this seems very unsatisfying. Even though SPEEDY has a climatological mean soil moisture field, precipitation, by contrast, is varying. You can run VSLite with Precipitation and a parameterization that goes from precip to soil moisture—people run VS Lite this way all the time, and I don't think it make sense not to in this case. I would redo all the pseudo proxy analysis with time-varying precipitation instead of time- invariant climatological mean soil moisture....I have seen this mistake before with VSLite and it causes an unphysical response for the trees.

We already answered this on page 2 of this answer.

30 citation needed after 'internal variability'

Done!

Page 7, Line: 1 rephrase to "Our results are presented in three sections: 1)..." 2-4 This is confusing—what do you mean by the word 'selection'? Elaborate. Add 'the' before 'temperature' 8 rephrase "disentangled to some extent by considering atmospheric variability to be a superposition.." 24 change but present... to "stationary and fluctuate over longer time scales. These low-frequency"... 25 occur should be 'occurs' 26 reverse order of wording to read 'modes of variability' 27 which annular modes? this is a very offhand reference. 28 change to 'displacements of the jet stream' 29 no comma after SPEEDY, nature=control

Done.

Page 8, Line: 2 again I think nature should be control throughout. Larger comment for Section 3.2.1: We need more information on the experimental design—perhaps a graphic showing a schematic of your experimental design and the PSM vs. no PSM simulations, online vs. offline, showing the full scope of *the research you performed for this paper. What is the point of the control run in this context? It's just not very clear in the current text. How did you use it?*

Figure 1 is illustrating the schematic of our experiment. Here we refer to the figure 1 in the new manuscript and import the Appendix in the main part of the paper along with figure 1.

15 change 'there exists a DA skill' — awkward wording, revise for clarity 18 rephrase to 'proxy record locations', and the comma after Northern hemisphere should be a semi-colon (;) 20 no comma after 'skill' 24 rephrase: "constrain temperature with considerably larger skill than TRW sites in South Africa. This finding may prove useful for the design of optimal TRW chronology networks...." 25 you need to cite Comboul et al., 2015 here which is also about optimizing observing networks in paleoclimate data, and discuss their findings (using coral pseudo proxies) in relation to yours: " CITATION: Comboul, Maud, et al. "Paleoclimate Sampling as a Sensor Placement Problem." Journal of Climate 28.19 (2015): 7717-7740. 29 citations are out of chronological order, change last bit of sentence from 'is currently' to 'is generally termed 'offline Data Assimilation.' 30 rephrase end 'using assimilation, the prior...'

Done.

Page 9, Line: 1 can you remind the reader about the differences between the two pseudo proxy schemes here ? MIN vs PROD? Give us a brief description to re-orient, as well as your hypothesis for how the two will differ. 5 delete comma after VSL-Min, add 'as a TRW observation ...' 6 rephrase "analysis, as demonstrated in Figure 6b. The expected value of the RMSE shifts significantly toward lower values.." 8 change present to 'shows' 10 revise "performs with slightly better skill" Note: there's no discussion of the pseudo proxy design here..... 17 What is TA DA???? Just write it out. 18 change to 'applied in parallel and independently of any specific..." 25-30 again cite Comboul et al., here: Comboul, Maud, et al. "Paleoclimate Sampling as a Sensor Placement Problem." Journal of Climate 28.19 (2015): 7717 7740.

Done.

—there is an official term for this kind of work, and it's optimal sensor placement (OSP)—much literature here in the pseudo proxy community and forward modeling/proxy system modeling that you need to work through in this discussion. Also, be careful with your language here.....is this really a fair statement to make when you didn't use time-variant soil moisture? if you are going to make the claim that your method can be used to design OSSEs you should probably give a walk-through, thorough example of this and associated caveats. Show a map of where the trees capture the most climate variability, etc. Also, the claim that you can apply this method to any proxy with 'stable time resolution' needs to be clarified. Do you mean annual resolution? It would be difficult to do this with lower frequency climate data like sediment cores or speleothems. So, this comment seems a bit far-reaching.

We have explained that in the new version of the manuscript.

We cited the OSP [Ancell and Hakim, 2007, Hakim and Torn, 2008, Mauger et al., 2013, Comboul et al., 2015] here with a discussion on the caveats.

Page 10, Line: 5 delete comma after provided, delete 'the' before results, delete "huge amount of" 6-7 revise language for clarity—'undiscriminated' — I think you mean 'indiscriminate' ? 9 change 'In addition to the classical DA approaches used in paleoclimate studies..." 10 and cite Dee et al., 2016 as well, which also uses this approach AND PSMs... 18 change "In this conditions" which is grammatically incorrect to "Under these conditions..." 19 what is meant by 'climatological levels?' 20 delete 'model' after SPEEDY, and the phrase "it is not surprising to enter the offline ..." is confusing and needs to be revised for clarity 22 delete "In this state of affairs" and change to Thus, it seems unlikely ... 23 constraint = constraints 24 this is too brief and we need examples—of course there is climate variability on time scales longer than 1 year. The obvious one is ENSO, but you need to give more examples and more citations. 25 rephrase "Accordingly, we expect that it should be possible to obtain...." and change 'skills' to skill. 27 rephrase "It is not clear if whether we can employ this technique with SPEEDY to properly estimate..." 28 comma after In particular,

Done.

Page 11, Line: 4 rephrase "conducted with SPEEDY support results obtained..." 9 delete colon (:) 10 rephrase "contained in them and the..." 11-15 it's not clear from the current text what point you're making here. Revise for clarity. 18 'response saturation'—what is this? The paper is jargon-y, as I mentioned. We need more description of these terms. 22 be careful here.....VSLite is not very Gaussian either. There is a brief discussion of this in Dee et al., 2016 in the TRW section. What is the fall out of this? General: we need a concrete summary of your findings—there isn't a conclusion section that gives us a summary and broader implications of your work. Needs to be added.

The comments are considered in the new manuscript. "response saturation" is changed to "the threshold, for temperature or moisture, after which the growth response does not change". Non-Gaussianity of the VSL is also a challenge for EnKF. A complete subsection have been added at the end of the manuscript to cover a summary and broader implications of our work.

Page 12 Appendix—you need to spell out the meaning of OSSE on first use—cannot abbreviate.

We moved the appendix to the main text after figure 1.

2 Answer to Reviewer 2

We wish to thank you so much for your positive review and constructive comments. We answer your comments (*italic*) point by point (**Bold**):

One important point which I feel is not treated adequately in this paper is *the observa- tion error. The authors use a signal-to-noise ratio of 10. Typical pseudoproxy experiments use ratios of 0.25-1. Although the authors mention in the last part of their paper that their signal-to-noise ratio is optimistic, the reader is left wondering what the effect could be.*

This has been also asked by the first reviewer and new Figure 11 is explaining this issue. A block of text also is added to the new version of manuscript explaining this issue.

Furthermore, the error model is not well explained. Why white noise? What would be the effect of a spatial error structure? What would be the effect of system- atic spectral biases in the tree rings? Even more importantly: Was the error assumed to be known perfectly? These questions would be very important for the community and would probably deserve a dedicated paper, but to the extent to which they could interfere with some of the results presented, I think some discussion should be added.

The signal to noise ratio (SNR) is expressed as the ratio of the standard deviation of the nature (true) run time-series to that of the additive white noise. The measurements' error is assumed not to be correlated in time (no memory), therefore the white noise is usually used in such studies (for example see [Dee et al., 2016] or [McShane and Wyner, 2011]). Some pieces of text are added to explain the model's error in subsection Observation generation.

A second point concerns the model description, which is rather short. In particular, the boundary conditions are not well discussed (e.g., greenhouse gases, volcanic aerosols, etc.). I am aware that this is a Observation System Simulation Experiment, nevertheless I would be interested in the effects of boundary conditions. What are the climatological maps from ECMF used for? And maps of what quantities? The paper is sufficiently short; some more explanations could be added here.

Done. We added some explanations about the boundary conditions.

The authors use many acronyms (TRW, PLF, DA, SNR, VSL, GCM, TA, EnKF, CFR, OSSE) which may be familiar to some readers but not to others. Again, I don't think that the paper is too long, and some of the acronyms could be spelled out for the sake of better readability.

We agree. We expanded the manuscript from 26 pages to 32 pages and spelled out many of the abbreviations.

The description not only of the methods, but also of the result is rather short.

We added additional text to the discussions as well as methods.

p. 3, l. 6: Or, covariance matrices may be blended from the ensemble and other estimations.

Done.

p. 4, l. 10: Explain t-norms.

It is fully explained now in the new manuscript.

p. 6, l. 28: "a fixed averaging period length of one year": How was that year defined? April to March?

Has been changed to "Given the annual resolution of TRW chronologies, we study the filter performance for yearly averaged values (near surface temperatures)."

p. 7: The reader might get confused with the terms "run" (nature run, free ensemble Discussion paper run) and experiment (PRESCRIBED, SLAB). The table does not help the confusion, but the Appendix does, it is very well written. Please refer at the appropriate places in the manuscript to the Appendix.

Appendix is moved after figure 1 and the OSSE is described there fully.

p. 7, l. 21: The low yearly internal variability in the tropics deserves some further attention. What does this mean in relation to real-world phenomena such as ENSO or PDO? This is particularly interesting as the authors discuss the PRESCRIBED set up and the SLAB but later note that fully coupled systems could/should be used. Would the result be completely different in the tropics?

This issue was raised also by reviewer 1. So we discussed this issue in the new subsection Outlook. Giving more examples of phenomena with larger time-scales than one year.

p. 7, l. 25: Just really minor: "Fig. 3a" is arguably more common than "figure 3.a"
Done.

p. 8, l.16 and elsewhere: Is the emphasis (bold italics) necessary? The authors use the term in the same way as the literature.

Done.

p. 9, l. 18: What do you mean with "any specific year"? Does that mean that the boundary conditions are disregarded? Can 1900 serve as a prior for 1999?

We deleted "any specific year". We used 1900 for 1900. It was meant that we could calculate several years at the same time not in a sequence. making the algorithm even faster.

p. 10, l. 13: "a more consistent"?

Changed to "realistic"

p. 10, l. 30: There is another important difference to traditional CFR techniques (by the way: spell out), namely that data assimilation at least formally does not require calibration and thus is less sensitive to stationarity issues.

Done. We added the comment.

p. 11, l. 1: "full atmosphere-ocean interaction".
Done.

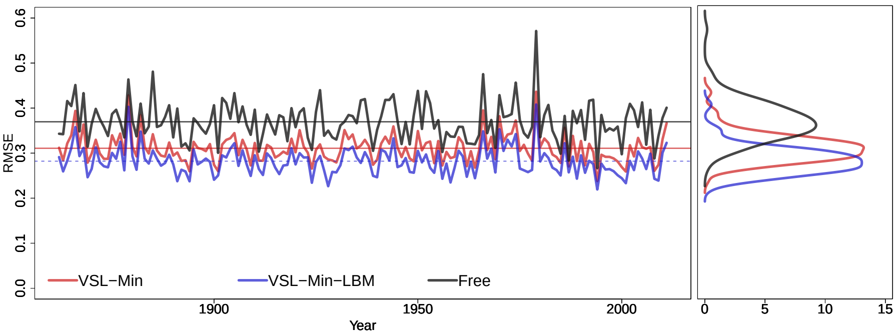
p. 11, l. 25: Not only model errors, also the observation error is an issue.
Given that we know the "true" state, the observation error is known in our OSSE. But in real word application this is true and is discussed in outlook.

3 Answer to Editor

Dear Authors, Thanks for posting your responses to the reviewers' comments. The reviewers raised substantial points but the suggestions included in your answers to take them into account appear reasonable to me at this stage (I have not checked the new version of the paper attached to one of your comments as the revised manuscript should be submitted separately). I would thus be happy to consider a revised version for publication in Climate of the Past. My only significant concern from your answers is the issue of the soil moisture. In particular, I do not understand your answer 'We consider that this approach would reduce the consistency of the simulations, given that the moisture values considered by SPEEDY parametrizations would be still the climatological ones and not the ones produced by the CPC Leaky Bucket'. Does it mean that you are using a climatological soil moisture in Speedy? Would this imply that in SPEEDY the soil moisture is not consistent with interannual variations in precipitation? This would then be very instructive to test what would be the impact on your results of using the precipitation from SPEEDY to compute the soil moisture appplied in VS LITE. Sincerely, Hugues Goosse

According to your and reviewer one's suggestions we implemented the Leaky Bucket Model ([Huang et al., 1996b]) in our DA code. The Leaky Bucket Model code was extracted from VS-lite v2_3 (`ftp://ftp.ncdc.noaa.gov/pub/data/paleo/softlib/vs-lite/`). Instead of using climatological soil moisture for VS-Lite, the precipitation and temperature output from SPEEDY is used by Leaky Bucket Model to produce the new set of soil moisture with interannual variations. In the next step we repeated the off-line data assimilation runs for two VS-Lite presentations (VSL-Prod and VSL-Min). Accordingly we will add the Figures 12, 13 and 14 (in this answer Fig.1-3) to the new version of manuscript. The results show that using the new set

a)



b)

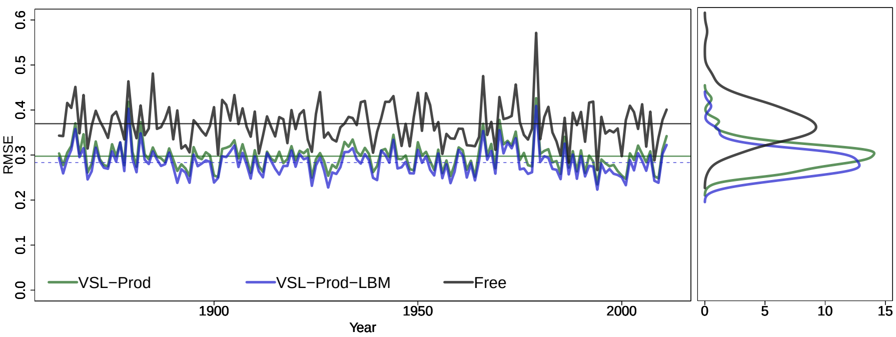
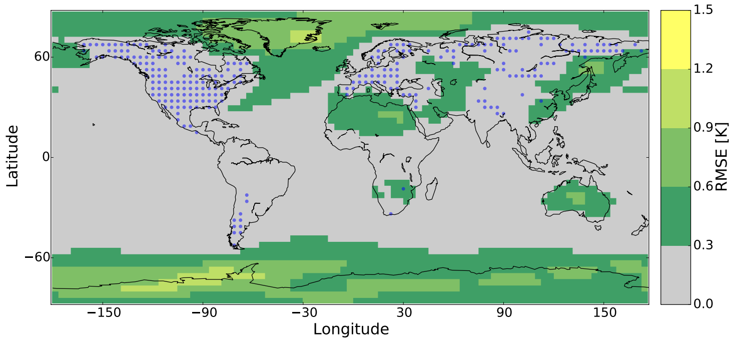


Figure 1: Global ensemble mean for analysis constrained by pseudo-TRW observations for a) VSL-Min with the climatological soil moisture (red), with the soil moisture computed by Leaky Bucket Model (blue) and free run (black); b) VSL-Prod with the climatological soil moisture (green), with the soil moisture computed by Leaky Bucket Model (blue) and free run (black). Horizontal lines exhibit the mean values. Right panels exhibit the histograms of the time-series.

of soil moisture has improved the error reduction of VSL-Min with minor improvement for VSL-Prod in both time evolution and maps of RMSE. Thus, the RMSE of VSL-Min reaches the one of VSL-Prod in the new runs. This is more clear in the RMSE maps (Fig. 2 of this answer). We added subsecton 3.2.4 to the new version of the manuscript.

Figure 3 shows the histograms of the RMSE time-series. The results show that the VSL-Min is more sensitive to the choice of soil moisture and using the calculated soil moisture with the Leaky Bucket Model improves the performance of the model. However, for VSL-Prod the improvement in error reduction is not significant when using the calculated soil moisture with the Leaky Bucket Model.

a) DA analysis for VSL-Min with Leaky Bucket Model and nocycling



b) DA analysis for VSL-Prod with Leaky Bucket Model and nocycling

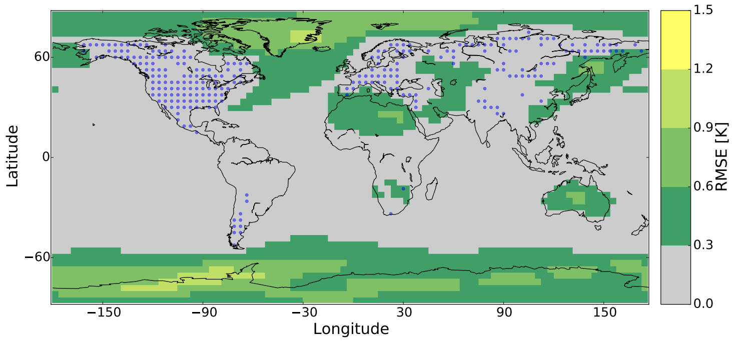


Figure 2: Time-averaged RMSEs of SLAB experiment for a) nocycling DA analysis using the VSL-Min with Leaky Bucket Model and b) the VSL-Prod with Leaky Bucket Model observation operator.

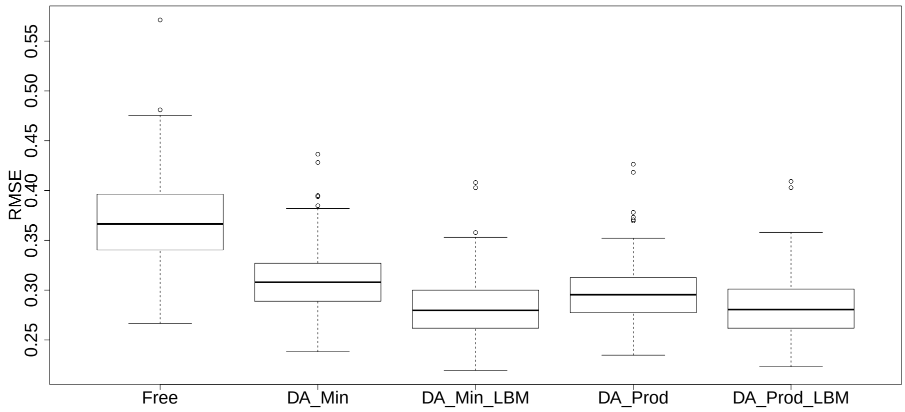


Figure 3: Histograms of global ensemble mean for analysis constrained by pseudo-TRW observations for Free run, DA run with VSL-Min, VSL-Prod using the climatological soil moisture and VSL-Min, VSL-Prod using the Leaky Bucket Model (LBM).

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[McShane and Wyner, 2011] McShane, B. B. and Wyner, A. J. (2011). A statistical analysis of multiple temperature proxies: Are reconstructions of surface temperatures over the last 1000 years reliable? *Annals of Applied Statistics*, 5(1):5–44.

Assimilation of Pseudo-Tree-Ring-Width observations into an Atmospheric General Circulation Model

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Abstract. We investigate the assimilation of Tree-Ring-Width (TRW) chronologies into an atmospheric global climate model using Ensemble Kalman Filter (EnKF) techniques and a process-based tree-growth forward model as observation operator. Our results, within a perfect-model experiment setting, indicate that the nonlinear response of tree-growth to surface temperature and soil moisture does deteriorate the operation of the time-averaged EnKF methodology. Moreover, this skill loss appears significantly sensitive to the structure of growth rate function, used to represent the Principle of Limiting Factor (PLF)s within the forward model. On the other hand, it was observed that the error reduction achieved by assimilating a particular pseudo-TRW chronology is modulated by the strength of the yearly internal variability of the model at the chronology site. This result might help the dendrochronology community to optimize their sampling efforts. In our experiments, the "online" (with cycling) paleao Data Assimilation (DA) approach did not outperform the "offline" (no-cycling) one, despite its considerable additional implementation complexity.

1 Introduction

The low-frequency temporal variability of the climate system can not be estimated from the available time span of instrumental climate records. Accordingly, paleoclimate reconstruction must necessarily rely on the usage of the ~~so-called~~ paleoclimate proxy records. ~~These~~ Nonetheless, these natural archives exhibit ~~, nonetheless,~~ several problematic features, e.g., low time-resolution, sparse and irregular spatial distribution, complex nonlinear response to climate and high noise levels. Therefore the proper extraction of the climate signal therein contained ~~is still an open question~~ can often remain opaque (Evans et al., 2013).

~~To the~~ At present, many different paleoclimate modeling ideas have been proposed, e.g., data-driven statistical techniques, climate model hindcasts, and Bayesian probabilistic methods (see Crucifix (2012) as a review). ~~AAmong~~ Among this plethora of ~~ap- proaches, DA~~ approaches, DA methodologies are today particularly appealing as they allow to systematically combine the information of paleoclimate records with the dynamical consistence of climate simulations (Oke et al., 2002; Evensen, 2003; Hughes et al., 2010; Brönnimann, 2011; Bhend et al., 2012; Hakim et al., 2013; Steiger et al., 2014; Matsikaris et al., 2015; Hakim et al., 2016; Dee et al., 2016).

Heretofore, several very diverse ~~paleo-DA~~ paleo-DA schemes have been investigated providing very encouraging results (see (Hughes and Ammann, 2009; Widmann et al., 2010) as reference): Pattern Nudging (von Storch et al., 2000) and Forcing

Singular Vectors (Barkmeijer et al., 2003; van der Schrier and Barkmeijer, 2005) techniques were designed to curb the atmospheric circulation towards a target pattern by means of an artificial term added to the model dynamics. 4D-Var methodology has been used to assimilate pseudo-proxies into an ocean model (Paul and Schäfer-Neth, 2005; Kurahashi-Nakamura et al., 2014). EnKF was adapted to time-averaged observations (Dirren and Hakim, 2005) and tested for a hierarchy of atmospheric models (Huntley and Hakim, 2010; Bhend et al., 2012; Pendergrass et al., 2012; Steiger et al., 2014). Finally, the use of a particle filter has been tested with an Earth system model of intermediate complexity (Annan and Hargreaves, 2012; Dubinkina et al., 2011; Dubinkina and Goosse, 2013; Mathiot et al., 2013).

A typical assumption in most of the paleo-DA studies so far conducted is that the climate-proxy relation is linear. Nonetheless, currently it is widely recognized that climate proxies are the result of complex recording processes, which can have physical, chemical and biological nature. Furthermore, several research groups have already developed and validated forward models for several proxy types (Evans et al., 2013; Dee et al., 2015). Hence, in order to increase the realism of DA-based climate reconstructions, it is relevant and pertinent to connect the climate state space to the proxy space by way of forward models ~~. Following this train of thought,~~ (Acevedo et al., 2015; Dee et al., 2016). Dee et al. (2016) applied three different nonlinear proxy system forward models in a DA framework and investigated the utility of paleoclimate observations for constraining climate simulations. They demonstrated that the linear-univariate models for tree ring width may not capture the GCM's climate, especially for regions where the tree's growth is dominated by moisture. The tree ring forward model used in our study is a modified version of the model used in Dee et al. (2016). Acevedo et al. (2015) [AC15, hereafter] evaluated the applicability of the process-based TRW forward model Vaganov-Shashkin-Lite (VSL) (Tolwinski-Ward et al., 2011) as observation operator within a simplified DA setting. Using a chaotic 2-scale dynamical system as a toy model, AC15 generated pseudo-TRW observations and assimilated them via the time-averaged -EnKF algorithm (Dirren and Hakim, 2005). This paper follows closely the rationale of AC15, but within a more realistic scenario, where an Atmospheric General Circulation Model (AGCM) is used as dynamical system and the observational network resembles the currently available TRW chronologies.

In addition to the classical DA approaches used in paleoclimate studies, a so-called "off-line" DA-based climate reconstructions is presented by (Steiger et al., 2014; Dee et al., 2016; Hakim et al., 2016). In an off-line approach the climate model is not re-initialized at the observation time steps (no initialization cycle or "no-cycling").

The main objectives of this study are to shed light on the following four fundamental questions :

1) Can paleo-DA improve the skill of the model for the ~~*forecast*~~ forecast (prior) state?

2) Can paleo-DA improve the skill of the model for the ~~*analysis*~~ analysis (posterior) state?

3) Can an on-line ("with cycling") DA outperform an "off-line" ("nocycling") one (see Sec.4 for the definition of "off-line")?

4) How does the nonlinear response of tree-growth to surface temperature and soil moisture affect the performance of the time-averaged EnKF-DA method?

The third question is one of the most important challenges in the ~~field of the~~ paleo-DA ~~. Considering only~~ field, given that the computational expenses of an on-line DA scheme with a realistic coupled GCM is far beyond the affordable limits of today's computers. On the other hand, state-of-the-art climate models have little or no predictive skill on the long timescale of proxy records (Hakim et al., 2016). ~~The fourth question, to our knowledge, has not been yet explicitly investigated in any of the paleo-DA studies.~~

In section 2 we describe the DA technique, the TRW forward model ~~,~~ and the climate model as well as the experimental setting used. Our numerical results are shown in section ~~3~~3, followed by a discussion in section ~~4.~~ 4.

2 Materials and Methods

2.1 ~~Data Assimilation method~~Data Assimilation Basics

2.1.1 Rationale

The knowledge about the climate is drawn from observations and the physical laws governing the evolution of the climate system. Numerical models apply the latter to estimate the state of the flow. DA is a process which applies both available information sources to estimate the state of the climate (Talagrand, 1997).

In a typical *sequential* DA scheme, a climate model is integrated in time steps over which observations are available. The predicted state at an observed instant (forecast), is used as "background" for the DA scheme. Furthermore, the forecast is "updated" or "corrected" by observations to form the analysis. The model is then reinitialized from the analysis state and propagates in time to reach the next observed instant. The analysis step is determined by availability of observations, their time scales and computational resources. DA methods have evolved from very empirical approaches, such as Newtonian relaxation, to probabilistic ones that estimate the state Probability Density Function (PDF) conditional to the observations (see Kalnay (2003) and Lahoz et al. (2010) for review).

Among all the available ~~DA techniques, EnKF~~ DA techniques, EnKF (Burgers et al., 1998) offers an appealing trade-off between accuracy, ~~implementation easiness~~ relatively user-friendly implementation and computational expenses. ~~EnKF~~ EnKF works robustly for very sparse observation networks and moderate number of ensemble members (Whitaker et al., 2009). Its implementation does not require adjoint model (DA calculations are outside the model code) and uncertainty estimates can be directly obtained from the ensemble spread (Hamill, 2006). The main disadvantage of ~~EnKF~~EnKF, within a paleoclimate setting, is its inability to handle strongly non-Gaussian ~~Probability Density Functions (PDFs)~~PDFs, which can result from the nonlinearities of climate models and observation operators. Nonetheless, it is very difficult to remove this limitation, given that strictly ~~nonGaussian DA techniques have been so far~~ non-Gaussian DA techniques have historically been prohibitively expensive to run for ~~high dimensional systems. However, validation of the different DA schemes in toy models or/and realistic climate models with pseudo observations or different types of proxies is of particular importance .~~ high-dimensional systems.

~~Following~~

2.1.2 Kalman Filter

Given that the model's state is $\mathbf{x}(t) \in \mathbb{R}^n$, the ~~rationale used in the experiments of AC15~~Kalman Filter (KF) (Kalman, 1960) assumes that the PDF of forecast state is given by a Gaussian function of mean $\mathbf{x}^f$ and covariance $\mathbf{P}^f \in \mathbb{R}^{n\times n}$:

$$p(\mathbf{x}) \propto \exp(-\frac{1}{2}(\mathbf{x}-\mathbf{x}^f)^T(\mathbf{P}^f)^{-1}(\mathbf{x}-\mathbf{x}^f)). \tag{1}$$

The observations $\mathbf{y}(t_j) \in \mathbb{R}^k$ are also assumed to have Gaussian errors and therefore the conditional probability of the observation vector $\mathbf{y}$ given the state $\mathbf{x}$ is:

$$p(\mathbf{y}|\mathbf{x}) \propto \exp(-\frac{1}{2}(\mathbf{y}-\widehat{\mathrm{H}}\mathbf{x}^f)^T\mathbf{R}^{-1}(\mathbf{y}-\widehat{\mathrm{H}}\mathbf{x}^f)), \tag{2}$$

where $\widehat{\mathrm{H}}$ and $\mathbf{R} \in \mathbb{R}^{k\times k}$ are the observation operator and the observation covariance matrix, respectively. Following the Bayes theorem, the conditional probability of the state given the observations, i.e., the analysis PDF, is:

$$p(\mathbf{x}|\mathbf{y}) \propto \exp(-\frac{1}{2}(\mathbf{x}-\mathbf{x}^f)^T(\mathbf{P}^f)^{-1}(\mathbf{x}-\mathbf{x}^f) - \frac{1}{2}(\mathbf{y}-\widehat{\mathrm{H}}\mathbf{x}^f)^T\mathbf{R}^{-1}(\mathbf{y}-\widehat{\mathrm{H}}\mathbf{x}^f)). \tag{3}$$

Assuming the $\widehat{\mathrm{H}}$ is a linear function, equation 3 has also a Gaussian PDF (Eq.1 and Eq.2 are Gaussian). Therefore, its mean and covariance can be calculated by the so called Kalman update equations (Lorenc, 1986):

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}(\mathbf{y}-\widehat{\mathrm{H}}\mathbf{x}^f), \tag{4}$$

$$\mathbf{P}^a = (\mathbf{I}-\mathbf{K}\widehat{\mathrm{H}})\mathbf{P}^f; \tag{5}$$

where the Kalman gain matrix $\mathbf{K}$ is given by:

$$\mathbf{K} = \mathbf{P}^f\widehat{\mathrm{H}}^\dagger(\widehat{\mathrm{H}}\mathbf{P}^f\widehat{\mathrm{H}}^\dagger + \mathbf{R})^{-1}. \tag{6}$$

2.1.3 Ensemble Kalman Filter (EnKF)

In a realistic model setting, the calculation of the covariance matrices are numerically very expensive. Evensen (1994) have used an ensemble of model states ($\mathbf{X}(t) = (\mathbf{x}_1, \ldots, \mathbf{x}_m)$) to approximate the KF equations. Following this approach the best state estimate and its uncertainty are presented by the ensemble mean and ensemble spread. The ensemble spread is given by the standard deviation of the ensemble around its mean. Thus, an EnKF cycle consists of an ensemble forecast step which provides the empirical mean and covariance for approximation of the KF equations:

$$\langle\mathbf{X}_f\rangle = \frac{1}{m}\sum_{i=1}^{m}\mathbf{x}^f{}_i, \;\; \mathbf{P}^f = \frac{1}{m-1}\mathbf{X}_f{}'(\mathbf{X}_f{}')^T. \tag{7}$$

Here $\mathbf{X}_f{}' \in \mathbb{R}^{n\times m}$ denotes the forecast ensemble deviation matrix:

$$\mathbf{X}_f{}' = \mathbf{X}_f - \langle\mathbf{X}_f\rangle\mathbf{e}^T. \tag{8}$$

where $\mathbf{e} = (1,\ldots,1) \in \mathbb{R}^m$. ~~The impact of the representation of the PLF on~~ The analysis ensemble whose covariance satisfies equation 5 can be generated in different ways. Two main kinds of KFs are stochastic and deterministic filters (Hamill, 2006). In the stochastic approach an observational ensemble $\mathbf{Y}$ is generated by adding a set of realization of the observational noise to the observation vector $\mathbf{y}$. The analysis ensemble is created by the following updating equation:

$$\mathbf{X}_a = \mathbf{X}_f + \mathbf{K}\left(\mathbf{Y} - \widehat{\mathrm{H}}\mathbf{X}_f\right). \tag{9}$$

In the deterministic updating scheme, instead of creating an ensemble of observations, the analysis mean ($\overline{\mathbf{X}_a}$) and deviations $\mathbf{X}_a{}'$ are calculated by using different update formula (Tippett et al., 2003).

Due to the limited ensemble size, the forecast uncertainty is usually underestimated in EnKF and after several assimilation cycles the observations may completely be ignored. This situation is known as "filter divergence" and can treated by multiplying the ensemble spread by a constant greater than one (covariance inflation).

For the experiments presented in this paper, we employed ensembles of 24 members (limited by the number of CPUs) and constant multiplicative inflation of 1% after the ensemble update. As the consequence of limited ensemble size, any observation may present spurious correlations with the distant ones and the filter performance ~~is studied using as reference of the assimilation of TA linear observations~~ may be affected. Therefore, the elements of the observation error covariance matrix are multiplied by a function that increases exponentially with distance and an infinite error is assigned to the distant observations (R-localization (Hunt et al., 2007)). This is achieved using the following formula:

$$R_{loc} = R * \exp\left((r_h/2\lambda_h)^2 + (r_v/2\lambda_v)^2\right) \tag{10}$$

where r_h and r_v stand for the horizontal and vertical distances, respectively. Their corresponding scaling parameters were set to the values $\lambda_h = 500$ Km and $\lambda_v = 0.4\ln p$.

2.2 ~~TRW Forward Model~~

~~Here, we review the forward operator formulation which was embedded by AC15 into the framework of Fuzzy Logic (FL)theory~~

2.1.1 Time-Averaged Ensemble Kalman Filter (EnKF)

Usually the time scale of the measured system is sufficiently longer than the response time of the sensor and the measurements can be assumed to be instantaneous. However, this assumption can not be applied for precipitation gauges, wind meters and proxy records. Proxies have averaged recording time spans ranging from months to decades. Time-averaged observations contain information of a segment of the model state trajectory instead of an instant of the model evolution.

The time-averaged ensemble background fields are updated by the EnKF and the instantaneous deviations from the mean remain unchanged. This approach is based on the fact that the observations can only contain time-averaged information (Dirren and Hakim, 2005).

2.1.2 ~~VSL Model~~

2.1.2 Observational System Simulation Experiments

Given a prediction system comprising a dynamical model and a DA scheme, forecast and analysis errors arise from many different sources, e.g. model imperfections, inadequacy of the DA strategy and insufficiency of observational information, which interact with each other in practice. In order to disentangle the effects of these error sources, a DA scheme is typically tested under simplified conditions by means of numerical experiments, currently known as Observation System Simulation Experiments (OSSE), whose realism level is gradually increased.

~~The limiting factors in the VSL model for~~ An OSSE consists of (i) a single model trajectory $\mathbf{x}^{\textsc{Nature}}$, typically referred to as "true" run or "nature" run, that is used as prediction target, (ii) pseudo-observations created by applying the observation operator to $\mathbf{x}^{\textsc{Nature}}$ and adding simulated observational noise, and (iii) an observationally constrained run $\mathbf{X}^{\textsc{DA}}$, obtained by performing a sequence of analysis cycles where the pseudo-observations are assimilated (see Fig. 1).

The nature run is normally generated by running the dynamical model starting from a random sample of the model climatology. Notice that thanks to the availability of the truth model evolution for an OSSE, the forecast and analysis skill of the observationally constrained run can be directly assessed, using for example the Root Mean Square Error (RMSE) of the ensemble mean:

$$\mathbf{RMSE}(\langle \mathbf{X}^{\textsc{DA}} \rangle) = \left(\overline{\left(\mathbf{x}^{\textsc{Nature}} - \langle \mathbf{X}^{\textsc{DA}} \rangle\right)^2} \right)^{\frac{1}{2}}, \qquad (11)$$

where $\overline{}$ and $\langle\ \rangle$ denote the time and ensemble mean operators, respectively.

An additional run frequently performed for OSSE involving ensemble DA methods, is a free ensemble run $\mathbf{X}^{\textsc{Free}}$, where no observations are assimilated and then the ensemble just freely evolve under the action of the model dynamics. $\mathbf{X}^{\textsc{Free}}$ is intended to provide a benchmark of performance, against which it is possible to asses the the added value of the DA scheme.

2.2 TRW Forward Modeling

2.2.1 VSL Model

The VSL model for TRW chronologies offers an intermediate complexity approach between ecophysiological and completely data-driven models (Tolwinski-Ward et al., 2011; Tolwinski-Ward, 2012), where the climate-driven component of tree-ring growth is parametrized by way of a simple representation of the PLF (Fritts, 1976). This biological concept states that the pace at which a plant develops is controlled by the single basic growth resource, typically either energy or water, that is in shortest supply. Within VSL the limiting factors considered are near-surface air temperature (T) and soil moisture (M). These variables influence tree growth by means of "growth response" functions g_T and g_M using a piece-wise linear "standard ramp" function

(Tolwinski-Ward et al., 2014):

$$\Psi(u) = \begin{cases} 0 & \text{if} \quad 0 \geqslant u \\ u & \text{if} \quad 0 < u \leqslant 1 \\ 1 & \text{if} \quad \phantom{0<{}} u > 1, \end{cases}$$

VSL's growth responses at a particular time is expressed as:

$$g_T = \Psi\left(\frac{T - T^L}{T^U - T^L}\right) \tag{12}$$

and

$$g_M = \Psi\left(\frac{M - M^L}{M^U - M^L}\right). \tag{13}$$

Where T^L and M^L denote minimum thresholds for temperature and moisture below which there is no grown, and ~~TU and MU~~ T^U and M^U are upper thresholds above which tree growth is optimal. ~~The corresponding~~ Afterwards, the growth rate G_{MIN} is determined by the smallest growth response, i.e.,

$$G_{MIN} = \min\{g_T, g_M\}, \tag{14}$$

The yearly TRW values W are obtained as following:

$$W_n = \int_{t_n - \tau}^{t_n} G_{MIN}(t) I(t)\, dt. \tag{15}$$

Where I is the relative local insolation.

2.2.2 VSL ~~in~~ from the ~~framework of Fuzzy Logic~~Fuzzy Logic (FL) Viewpoint

The term FL was coined by Zadeh (1975) and refers to a mathematical theory which has been very successful at modeling complex systems involving imprecise data and vague knowledge of the underlying mechanisms. Since its introduction, FL has greatly influenced many applied disciplines, most notably control theory (Nguyen et al., 2002). Within the environmental sciences, FL has also found numerous applications, including ecological and hydrological modeling (Marchini, 2011; Salski, 2006; Se, 2009). Regarding climate proxy forward modeling, AC15 ~~have claimed that the~~ recently showed that VSL ~~'s formulation of the Principle of Limiting Factor (PLF) determines the fuzzy set of temperature-moisture conditions which optimize the~~ model can be completely embedded into the framework of FL. Within this reinterpretation, the growth response function g_T (g_M) correspond to the membership function to the set S_T (S_M) of optimal temperature (moisture) conditions for tree growth. ~~Accordingly, we have selected three different t-norms:(i) smallest growth response or original or :~~ Temperature (moisture) values lying below T^L (M_L) present null values for g_T (g_M) and accordingly do not belong to S_T (S_M). On the other hand, temperature (moisture) values lying above T^U (M_U) lead to g_T (g_M) values equal to 1, meaning they belong completely to S_T (S_M). All the other temperature (moisture) conditions present growth responses between 0 and 1 and consequently they are considered to belong partially to S_T (S_M). This idea of partial membership is the basis of fuzzy logic and the sets defined this way are called fuzzy sets. Furthermore, the intersection of the fuzzy sets S_T and S_M is again a fuzzy set $S_{T\wedge M}$, whose membership function can be calculated by evaluating the minimum between G_T and G_M:

$$\cancel{G}g_{\cancel{MIN}T\wedge M} = \min\{g_T, g_M\}\cancel{,} \tag{16}$$

~~(ii)~~

Equation 16 is completely equivalent to the equation 15 and then VSL's growth rate function can be interpreted as the membership function for the fuzzy intersection set $S_{T\wedge M}$. In FL theory, the minimum function (Eq. 16) is one of the most popular representations of the intersection operation, however it is not the only, existing actually a whole family of appropriate functions referred to as t-norms (see Nguyen et al. (2002)). In AC15 a number of t-norms was tested as replacement for VSL's growth rate function within a highly simplified paleo-DA setting. In particular it was found that the product t-norm $g_{T\wedge M} = g_T \cdot g_M$ might improve significantly the performance of the time-averaged EnKF technique. Accordingly, beside the minimum t-norm we consider also in this paper the product growth response ~~or~~ VSL with Product t-norm (VSL-Prod):

$$G_{PROD} = g_T \cdot g_M \tag{17}$$

~~(iii) temperature growth response or :~~

$$\cancel{G_T = g_T}. \tag{18}$$

2.3 ~~Experimental Setting~~Atmospheric General Circulation Model

2.3.1 ~~SPEEDY model~~

The Simplified Parametrizations, primitivE-Equation Dynamics (SPEEDY) model (Molteni, 2003) is an intermediate complexity ~~AGCM~~ AGCM comprising a spectral dynamical core and a set of simplified physical parametrizations, based on the same principles as state-of-the-art ~~AGCM~~ AGCM but tailored to work with just a few vertical levels.

SPEEDY's dynamical core solves the hydrostatic primitive equations by means of the spectral transform developed by Bourke (1974), which uses absolute temperature, logarithm of the surface pressure, specific humidity, divergence and vorticity as basic prognostic variables. The time stepping is performed via a leapfrog scheme with an standard Robert–Asselin filter (Robert, 1966). The sub-grid scale processes parametrized in speedy are convection, large-scale condensation, clouds, short- and long-wave radiation, surface fluxes, and vertical diffusion.

In this paper we employ ~~the~~ version 32 of SPEEDY, ~~featuring seven levels in the vertical~~ with seven vertical levels (L7) and standard Gaussian grid of 96 by 48 points in the horizontal ~~, which correspond to a triangular spectral truncation at total~~

~~wave number 30~~ (T30). The top and bottom layers ~~are meant to~~ represent the stratosphere and the planetary boundary layer, respectively. Despite of its low resolution and the relative low complexity of its parametrizations, SPEEDY still captures many observed global climate features in a realistic way, while its computational cost is at least one order of magnitude lower than the one of sophisticated state-of-the-art ~~AGCMâ~~AGCM's at the same horizontal resolution (Molteni, 2003). ~~This latter virtue,~~ DIFdelend The latter makes SPEEDY specially suitable for studies involving long ensemble runs, like the ones ~~presented in this paper~~necessary for this study.

2.4 Experimental Setting

Following the rationale used in the experiments of AC15, pseudo-TRW observations are generated using Vaganov-Shashkin-Lite (VSL) (Tolwinski-Ward et al., 2011, 2013) as observation operator. Afterward, the time-averaged state of the atmosphere is estimated via EnKF approach Dirren and Hakim (2005). The impact of the representation of the PLF on the filter performance is studied using the assimilation of time averaged linear observations as a reference.

2.4.1 Filter ~~implementation~~Implementation

The SPEEDY model is embedded by Miyoshi (2005) into the ensemble ~~DA~~ DA framework using the ~~Local Ensemble Transform Kalman Filter (LETKF)~~ Local Ensemble Transform Kalman Filter (LETKF) (Hunt et al., 2007), the so called ~~SPEEDY-LETKF~~ SPEEDY-LETKF framework. The parallel FORTRAN 90 implementation of the ~~LETKF~~ LETKF is promising for high resolution model given that the calculation of the analysis for a particular grid point requires only the information of the neighboring grid points. Therefore, ~~LETKF~~ LETKF offers outstanding scalability properties. ~~SPEEDY-LETKF~~ SPEEDY-LETKF is an open-source software which have already been widely used for several DIFdelbegin ~~DA~~ DA studies (Li et al., 2009; Miyoshi, 2010; Lien et al., 2013; Ruiz et al., 2013; Amezcua et al., 2014). Here, ~~SPEEDY-LETKF~~ SPEEDY-LETKF was extended for the assimilation of time averaged linear observations and ~~pseudo-TRW~~ pseudo-TRW observations. This was done by (i) modification of the model time cycling, (ii) addition of the ~~TA-Up updating approach~~ time-averaged updating approach of Dirren and Hakim (2005) and (iii) development of the ~~VSL-like~~ VSL-like observation operator.

~~For the experiments presented in this paper, we employed ensembles of 24 members (limited by the number of CPUs) and constant multiplicative inflation of 1% after the ensemble update. R-localization is achieved using the following formula:~~

$$\cancel{R_{loc} = R * \exp(r_h/2\lambda_h)^2 + (r_v/2\lambda_v)^2}$$

~~where r_h and r_v stand for the horizontal and vertical distances, respectively. Their corresponding scaling parameters where set to the values $\lambda_h = 500$ Km and $\lambda_v = 0.4 \ln p$.~~ Additionally, in order to avoid catastrophic filter divergence (ref. sec. 2.1.3), observations with ~~huge~~ large divergence from their corresponding predicted values ~~are neglected. The observation quality control strategy within the SPEEDY-LETKF is as the following :~~ were neglected. Moreover, in order to avoid the crash of the model after assimilation steps, the following quality control criterium is applied: The observations whose corresponding innovation vector norm (absolute mismatch regarding the forecast observation) is bigger that 10 times its error standard deviation ~~,~~ are discarded.

2.4.2 ~~Runs~~Simulations' ~~characteristics~~Characteristics

The modified version of ~~SPEEDY-LETKF~~ SPEEDY-LETKF is utilized to carry out a set of standard "perfect model" ~~Observation System Simulation Experiments~~ OSSEs (Fig. 1)~~. The model can be coupled with a shallow ocean which contains a 50 m deep thermodynamic slab ocean layer~~. First the VSL with Temperature t-norm (VSL-T) representation of the PLF is utilized for two sets of experiments under different ocean conditions:

- *PRESCRIBED experiment* is forced by the boundary conditions included in the version 41 of the code, which comprises the sea surface temperature (SST) anomalies from 1854 to 2010 with respect to the period 1979 to 2008 derived from NOAA_ERSST_V3 dataset (Smith et al., 2008; Xue et al., 2003), as well as climatological maps derived from input data of the ~~ECMWF~~European Centre for Medium-Range Weather Forecasts (ECMWF)'s reanalysis (Gibson et al., 1997). ~~The latter consist of soil wetness, land temperature~~ At the surface boundaries model requires the climatological maps of sea surface temperature, sea ice fraction, surface temperature at the top of the soil, moisture in the top soil layer and the root-zone layer, snow depth, ~~ice cover, vegetation cover and albedo monthly maps~~bare-surface albedo, fraction of land-surface vegetation. At the top of the atmosphere, the model calculates the flux of incoming solar radiation from astronomical formulae (Molteni, 2003). The solar radiation absorption by ozone in the stratosphere follows empirical functions with seasonal variability. The latitudinal variability of the optical depth depends on the daily averaged zenith angle (Molteni, 2003). The climatological fields are derived for the period 1981-1990 to have a better balance for warm and cold El Niño-Southern Oscillation (ENSO) events (Molteni, 2003). This procedure follows the AMIP-type experiments (Herceg Bulić and Kucharski, 2012).

- *SLAB experiment* is coupled with a slab ocean model ("q-flux adjusted mixed layer model") forced by climatological ocean dynamics and no initialization is used. The model starts from ~~the equilibrium. Then, for the sake of simplicity and computational efficiency, only two~~ a spun up state.

Two representations of the PLF are considered: the ~~"minimum" and the "product" Triangular Norms (t-norms) . This selection is based on the results of AC15 experiments.~~

"minimum" (G_{MIN}) and the "product" (G_{PROD}) t-norms. Initially, a one-year long spin-up run is performed for all experiments, starting from January 1st, 1860. The final state of this model trajectory is subsequently used as initial condition for a 150 ~~years long nature~~ year long nature ("true") run. The ensemble runs with and without ~~DA~~ DA are identically initialized from a set of states gathered daily from the last two ~~month~~ months of the spin-up run (lagged 2 day initialization). Notice that the nature ("true") run and the different ensemble runs ~~are generated driving SPEEDY~~ (priors) are generated with the same time varying forcing fields.

2.5 ~~Observation generation~~

~~Pseudo-TRW~~

2.4.1 Observation Generation

Pseudo-TRW observations are produced following ~~VSLâ~~VSL's formulation, plus a final white noise addition step, where random draws from a Gaussian distribution are ~~added to the clean TA observations, so as to obtain a Signal to Noise Ratio (SNR)equal to 10.0~~imposed on the time averaged observations. The measurements' error is assumed not to be correlated in time (no memory), therefore the white noise is used in this study (McShane and Wyner, 2011; Dee et al., 2016). Surface temperature data was extracted from the lowest level of the state vector, while soil moisture was taken from the surface boundary conditions. Notice that temperature is a prognostic variable of the model, whereas soil moisture is a prescribed variable with yearly periodicity. It is worthwhile to mention that although soil moisture is not a prognostic variable of SPEEDY, it does affect prognostic variables, such us humidity, through the parametrizations.

Regarding the geographical distribution of observations, we place a station at every grid box where at least one actual ~~TRW~~ TRW chronology from the database of Breitenmoser et al. (2014) is present. This strategy yields an observational network comprising 257 stations (see figure ~~2).~~ 2). Concerning the configuration of the observation operator, for our experiment involving SPEEDY we focus on the effect of the first ~~VSLâ~~VSL's nonlinearity, i.e., the shifting of recorded variable. Consequently, we configure ~~VSL~~ VSL so that no thresholding takes place. This is done by setting the upper and lower response thresholds to the maximum and minimum values during the nature (true) run, respectively, so that the response functions reduce to linear rescaling operators (ref. AC15).

2.4.2 Diagnostic Statistics

2.5 ~~Diagnostic statistics~~

Our results are presented in three sections: 1) time-series of globally averaged temperature RMSE, 2) histograms of these time-series and 3) maps of time-averaged (150 years) temperature RMSEs. We show the analysis of the temperature variable due to its larger error reduction compared to other variables (*eg,* humidity, u-wind, v-wind) when DA is applied.

3 Results

Given the annual resolution of ~~TRW~~ TRW chronologies, we study the filter performance for ~~a fixed averaging period length of one year~~yearly averaged values (near surface temperatures). We monitor the behavior of ensemble runs by means of ~~Root Mean Square Error (RMSE)~~ RMSE for the near surface ~~Temperature (see Appendix A)~~temperature. SPEEDY presents spatially heterogeneous internal variability (Molteni, 2003). Due to this feature, for a particular time averaging length, there will typically be regions with very low internal variability (~~eg.,~~ *eg.,* equatorial regions for temperature) for which ~~RMSE~~ RMSE shows very low values.

~~The results are illustrated by three kinds of presentations: 1) time-series of globally averaged temperature RMSE, 2) histograms of these time-series and 3) maps of time-averaged (150 years) temperature RMSEs. We note that the selection of temperature variable is due to its larger error reduction compared to other variables (*eg,* humidity, u-wind, v-wind) when DA is applied.~~

4 ~~Results~~

3.1 Free Ensemble Run

3.2 ~~Free ensemble run~~

~~An AGCM~~ An AGCM is an example of non-autonomous system and accordingly the evolution of its state is determined by both the atmospheric dynamics and the external forcing. The influences of these two distinct factors can be ~~disentangle to a good extend by considering the atmospheric variability as a linear~~ disentangled to some extent by considering atmospheric variability to be a superposition of an internal component, caused by the intrinsic dynamics, and an external one, resulting from the variations of the boundary conditions (Deza et al., 2014). Under this assumption, internal and external variability can be separated by way of a free ensemble run, using the ensemble mean as an estimate of the forced component. The magnitude of the internal variability can then be estimated from the ensemble spread. Note that using an ensemble ~~DA~~ DA method is only beneficial in the presence of internal variability, given that the forced variability can be well described by an unconstrained ensemble run (free ensemble run).

3.1.1 Free ~~ensemble spread~~ Ensemble Spread and ~~error~~Error

The time averaging operator acts as a low pass filter that reduces the amplitude of fluctuations with time scales shorter than the averaging period. Subsequently, geographical areas dominated by fast processes, compared to averaging period~~(τ_{aver})~~, tend to present constant mean values, or equivalently no internal time averaged variability. The climatology and the formulation of the SPEEDY model is fully described in Molteni (2003). Therefore, we focus only on the results of the ~~DA~~ DA approach, without considering the systematic errors of the model. In the case of ~~TRW~~ TRW chronologies, the characteristic one-year averaging period is long for atmospheric phenomena, and as consequence several areas show very low yearly internal variability for certain variables. A clear example of this is temperature around the equator (see figure ~~3.~~3a) where the temperature variability is dominated by the daily cycle and accordingly is strongly attenuated by the yearly averaging. On the other hand, planetary scale patterns are not completely stationary ~~but present fluctuations over long~~ and fluctuate over longer time scales. These ~~slow~~ low-frequency processes introduce internal variability in the yearly means, as can be seen in figure ~~3.~~3a. Maximum temperature spread ~~occur~~ occurs near the surface at high latitudes around ~~± 70~~ $\pm 70°$. These yearly internal variability maxima can be related to leading ~~variability modes of~~ modes of variability of the global circulation, such as the ~~"annular modes"~~"annular modes" (e.g., acenso) (Thompson and Wallace, 2000), migrations of the ~~ITCZ~~ Inter Tropical Convergence Zone (ITCZ) (Schneider et al., 2014), as well as ~~jet stream displacements~~ displacements of the jet stream (Woollings et al., 2011).

An important consequence of the spatially heterogeneous yearly internal variability of SPEEDY ~~,~~ is that the nature (true) run variables at geographical areas with low internal variability can be well predicted by the ensemble mean of the free ensemble run, as it can be seen in figure ~~3.~~3b for the tropical surface temperature. On the other hand, ~~RMSE~~ RMSE extremes take place in areas of maximal internal variability (compare figures ~~3.a and 3.~~3a and 3b). Generally, the error of the free ensemble run, used as a predictor of the nature (true) run, is essentially the projection of the nature (true) run trajectory on the internal variability component DIFdelbegin ~~. Figure 3~~ (see schematic in Fig. 1). Figure 3 exhibits the results for the SLAB experiment~~and the~~. The PRESCRIBED experiment presents very similar behavior.

3.1.2 ~~Assimilating TA linear observations~~

~~The assimilation of TA linear observations leads to low temperature RMSE levels in all geographical areas adjacent to the observation network (see figure **??**). The situation for moisture is very similar, however in this case there exist few areas with considerable error despite the presence of a nearby observational station, e.g., south-west China and Texas. On the other hand, u-wind and v-wind (not shown) analysis fields present no noticeable improvement in any part of the globe.~~

~~A low RMSE value of DA runs for the yearly mean of a particular variable might be due to observational constraint or/and to lack of internal variability at that time scale. In these conditions, the error reduction with reference to the free run, $\varepsilon^{\text{Free}}_{\text{Reduction}}$, appears as a very convenient quantity to assess the real benefit of performing DA. Figure **??** shows the existence of considerable error reduction for temperature and moisture in some geographical areas, whereas for u-wind $\varepsilon^{\text{Free}}_{\text{Reduction}}$ exhibits negligible values as it is expected from the analysis of figure **??**. This absence of observational constraint on the wind variables was common to all of the simulations and accordingly wind-related quantities will not be analyzed hereafter.~~

~~An important aspect of our results concerning the DA skill, when yearly averaged linear observations are assimilated, is that the error reduction regarding the free ensemble run appears modulated by the magnitude of the yearly internal variability of the particular variable at a specific site (compare figures **??** and **??**). As a consequence, stations located in areas of strong yearly internal variability are more efficient than the others at reducing the error of the TA state estimate. An example of this are the stations located in Alaska which constrain temperature considerably more strongly than the ones laying on south-east USA, south America or south Africa. This finding can then be utilized as guidance for the design of optimal TRW chronology networks, in particular, and proxy networks in general (see the discussion at the end of this paper).~~

~~An additional relevant feature of figure **??** is that, both for temperature and humidity, the error reduction is strongly localized around data-rich areas. This behavior can be explained by the negligible error reduction obtained for all the forecast variables (not shown). This situation, where a DA method presents TA analysis skill for averaging periods where the TA forecasting skill is completely lost, has been previously observed in studies applying EnKF techniques on TA quantities . DA performed under these circumstances is currently labeled as ?off-line?. This term is used to indicate that, under the randomizing action of chaotic model dynamics, at assimilation time, the prior ensemble is completely decorrelated from the previous analysis state. As a consequence, the observational information cannot accumulate over time, as opposed to the typical application of DA for short-range prediction. This complete absence of observational constraint on the forecast implies that our DA experiments are performed in an "offline" regime.~~

3.2 Assimilating ~~pseudo-TRW observations~~Pseudo-TRW Observations

3.2.1 Temperature-based PLF ~~representation~~ Representation (VSL-T)

As described in Sec. 2.4.2, we investigate two different experiments using SLAB and PRESCRIBED ocean conditions (see Table 1). For the sake of simplicity, we set up the sensitivity experiments using the simple observation operator VSL-T to investigate the effect of the SLAB ocean model. The use of SLAB ocean is motivated by the fact that the coupled ocean may lend predictability to the atmosphere as a slow component of the climate system. On the other hand, the climate of the PRESCRIBED experiment may follow the trends of the forced ocean conditions instead of the terrestrial proxy records. Therefore, the PRESCRIBED experiment's spread and error are expected to be smaller than the SLAB experiment. Figure 4 supports this hypothesis, showing a reduction in globally averaged free ensemble error in PRESCRIBED compared with the SLAB.

Figure 4~~.~~a illustrates that no error reduction is obtained for the forecasted temperatures. The expected value of the ~~RMSE~~ RMSE is slightly larger than the free ensemble simulation for both SLAB and PRESCRIBED. However, ~~there exist a DA skill for the ***analysis***~~ the analysis state has skill (Fig. 4~~.~~b), especially prior to 1950s. The existence of the trend in the ~~RMSE~~ RMSE time-series may arise from cycling (reinitialization) of the ensemble or the choice of observation operator (more details in Sec. 4.2). The distribution density of the proxy record locations are biased to the Northern hemisphere; therefore, the error reduction of the ~~***analysis***~~ analysis is more evident in the ~~RMSE~~ RMSE maps (Fig. 5).

An important aspect of our results concerning the ~~DA~~ DA skill when yearly averaged linear observations are assimilated, is that the error reduction regarding the free ensemble run appears modulated by the magnitude of the yearly internal variability of the particular variable at a specific site (compare figures 4 and 5). As a consequence, stations located in areas of strong yearly internal variability are more efficient than the others at reducing the error of the ~~TA~~ time-averaged state estimate. An example of this are the stations located in Alaska which constrain temperature with considerably larger skill than ~~TRW~~ TRW sites in South Africa. This finding ~~can then be utilized as guidance~~ may prove useful for the design of optimal DIFdelbegin ~~TRW~~ TRW chronology networks, in particular, and proxy networks in general (see Comboul et al. (2015) and the discussion at the end of this paper).

This situation, where a ~~DA method presents TA ***analysis***~~ DA method presents time averaged analysis skill for averaging periods where the ~~TA***forecast***~~ time averaged forecast skill is completely lost, has been previously observed in studies applying EnKF techniques on ~~TA~~ time averaged quantities (Huntley and Hakim, 2010; Bhend et al., 2012; Pendergrass et al., 2012; Steiger et al., 2014). ~~DA~~ DA performed under these circumstances is generally termed "offline Data Assimilation". This term is used to indicate that, under the randomizing action of chaotic model dynamics, ~~using assimilation,~~ the prior is completely decorrelated from the previous ~~***analysis***~~ DIFdelend analysis state. As a consequence, the observational information cannot accumulate over time, as opposed to the typical application of ~~DA~~ DA for short-range prediction. This complete absence of observational constraint on the ~~***forecast***~~ forecast implies that our ~~DA~~ DA experiments are performed in an "off-line" regime.

3.2.2 ~~Original~~ Minimum and Product ~~PLF representation~~ Growth Rate Functions (VSL with Minimum t-norm (VSL-Min) and VSL-Prod)

Here the performance of the two different growth functions within the VSL's formulation, the product growth response (G_{Prod}) and the minimum growth response (G_{Min}), are investigated. These formulations are tested for both "online" (with cycling) and "offline" (no-cycling) data assimilation set-ups. In a simple DA experiment, AC15 have shown that the G_{Prod} performs slightly better than G_{Min}.

3.2.2.1 DA with cycling

Considering the SLAB experiment, we compare the two nonlinear ~~PLF~~ PLF representations in our ~~DA setting (VSL-Min and VSL-Prod~~DA setting (VSL-Min and VSL-Prod). As illustrated in figure ~~6.~~6a, the DA ~~***forecast***~~ forecast presents no skill in the globally averaged temperature for both of the representations. However, the use of ~~VSL-Prod~~VSL-Prod, instead of ~~VSL-Min, as TRW observation operator~~ VSL-Min appears beneficial to the filter performance for the ~~***analysis*** as it can be seen in figure 6.~~analysis, as demonstrated in Figure 6b. The expected value of the ~~RMSE presents a significant shift towards~~ RMSE shifts significantly toward lower values for ~~VSL-Prod~~ VSL-Prod compared to the free ensemble run. Similar to the case of ~~VSL-T, the RMSE~~ VSL-T, the RMSE time-series ~~present~~ shows an increasing trend for both ~~VSL-Min and VSL-Prod~~VSL-Min and VSL-Prod.

The ~~RMSEs of DA~~ RMSEs of DA forecasts using different ~~VSL~~ VSL representations (figures ~~5.a, 7.a and 8.~~5a, 7a and 8a) do not indicate any improvement over the free ensemble run (Fig. ~~3.~~3b). The ~~***analysis*** of VSL-Prod performs a~~ analysis of VSL-Prod performs with slightly better skill than ~~VSL-Min~~ VSL-Min over Europe, the United States and Central Asia. Due to the strong nonlinear features of ~~VSL-Min and VSL-Prod~~VSL-Min and VSL-Prod, the performance of these filters is expected to be degraded with respect to the ensemble runs constrained with ~~VSL-T~~ VSL-T linear observation (see AC15). This behavior can be readily seen by comparing the figures ~~5.b, 7.b and 8.~~5b, 7b and 8b.

3.2.2.2 DA with no-cycling

Our experiments show that the DA ~~***forecast***~~ forecast has no skill over the model climatology. Several recent studies have applied a similar DA methodology (Steiger et al., 2014; Dee et al., 2016; Hakim et al., 2016). Dee et al. (2016) have performed paleoclimate reconstructions by using a physically based Proxy System Models (PSM) for three kinds of proxies (tree ring, coral $\delta^{18}O$ and ice core $\delta^{18}O$) and two isotope-enabled atmospheric general circulation models. Matsikaris et al. (2015) compared an off-line and an on-line ensemble-based DA and showed that the both methods outperform the model without DA. They concluded that the on-line method performs a more realistic temporal variability. Therefore, we investigate the idea of purely "off-line" DA. The free ensemble simulation at any individual year is used as the prior state vector for that year instead of the DA ~~***forecast***~~forecast. Following this methodology, the cycling step (reinitialization) of the ensemble is neglected and the time averaged DA technique is applied in parallel~~and independently of any specific year of the free ensemble run.~~

. A very interesting feature of figure 9 is that the increasing trends in the ~~RMSE~~ RMSE time-series of the ~~***analysis***~~ analysis have vanished. This indicates that the previously existing trends in the ~~***forecast***~~ forecast and consequently in the ~~***analysis***~~ analysis originated from the reinitialization step of the system but not the proxy records. Figure 10 also confirms that the performance of no-cycling DA can compete with the performance of the online DA.

~~The use of as observation operator appears compatible with the -based technique utilized, as it is evidenced by the low RMSE levels observed in figure **??** around the observational network. Nonetheless, due to the strong nonlinear features of VSL-Min, the performance of filter is expected to be degraded with respect to the ensemble runs constrained with TA linear observations (see AC15) . This behavior can be readily seen in figure **??**, which show considerable error increases due to observation nonlinearities. Regarding temperature,~~ $\cancel{\varepsilon^{\mathrm{NonLinObs}}_{\mathrm{Increase}}}$ ~~presents particularly high values over the Labrador peninsula, central Europe and Siberia. As for humidity, there are three spots of high error increase on the northwest USA, the band extending from the gulf of Mexico to the Great Lakes, and the stripe connecting the Baltic with the black sea.~~

~~An interesting feature of figure **??** is the presence of unobserved zones with negative~~ $\cancel{\varepsilon^{\mathrm{NonLinObs}}_{\mathrm{Increase}}}$ ~~values for temperature, e. g. , Antarctica and for humidity over the Arabian peninsula; Indian subcontinent and southeastern China, which implies that the estimation of the TA state is not optimal over these regions. This phenomenon might be attributed to significant non-Gaussianity for these variables in this geographical areas. As an example it is believed that the Indian monsoon presents a clear bimodal behavior .~~

3.2.3 ~~Product PLF representation ()~~

~~The use of VSL-Prod, instead of VSL-Min, as TRW observation operator appears beneficial to the filter performance as it can be seen in figure **??**, where~~

3.2.3 Signal to Noise Ratio

The Signal to Noise Ratio (SNR) is expressed as the ratio of the ~~error increase becomes in general lower and more homogeneous. Notice that for humidity there remains a spot of considerably positive~~ $\cancel{\varepsilon^{\mathrm{NonLinObs}}_{\mathrm{Increase}}}$ ~~values in southeastern USA.~~

~~An interesting behavior regarding humidity is the appearance of negative~~ $\cancel{\varepsilon^{\mathrm{NonLinObs}}_{\mathrm{Increase}}}$ ~~values over the relatively well observed area of central Siberia. This phenomenon makes again manifest the existence of non-negligible non-gaussian features for certain variables at particular geographical regions, which implies lack of optimality for the TA state estimation even when TA linear observations are assimilated.~~

3.3 ~~Globally averaged quantities~~

~~None of~~ standard deviation of the ~~experiments show any significant forecasting skill for the global quantities. Consequently,~~ nature (true) run to that of the ~~ensemble spread does not divaricate from the simulations without (Fig. **??**). However, the analysis spread is significantly reduced. This situation will be discussed in more details in the following section (sec. 4). Let us now highlight the differences between the experiments constrained with the two~~ additive white noise. We examined the performance of the off-line DA with different ~~pseudo-TRW observations (i.e., and). The global averaged forecast~~ SNRs (Fig. 11). Figure 11 exhibits that the time-averaged global RMSE ~~for shows an overall increase with respect to the free ensemble run (Fig. ??.a). Another unpleasant feature of~~ shows an elbow at values around $SNR = 1$ and reaches the error levels of Free run at $SNR = 0.03$, where all the pseudo-observations are ignored in the DA.

3.2.1 Time Variable Soil Moisture

To investigate the effect of using the time-varying soil moisture fields instead of climatological average in DA approach, we implemented the Climate Prediction Center (CPC) Leaky Bucket Model (LBM) (Huang et al., 1996) in our DA code. The LBM code was extracted from VSL v2_3 (ftp://ftp.ncdc.noaa.gov/pub/data/paleo/softlib/vs-lite/). Instead of using climatological soil moisture for VSL, the precipitation and temperature output from SPEEDY is used as input for LBM to produce the new set of soil moisture with interannual variations. In the next step we repeated the off-line data assimilation runs for two VSL presentations (VSL-Prod and VSL-Min).

The results show that using the new set of soil moisture has improved the error reduction of VSL-Min ~~observation operator pops up as a positive trend in the forecast and analysis~~ with minor improvement for VSL-Prod in both time evolution and maps of RMSE (~~Fig. ??~~Figures 12 and 13). The ~~experiment constrained with~~ RMSE of VSL-Min reaches the one of VSL-Prod ~~, however, presents a fair improvement for the forecast. The emergence of the positive trend in~~ when using the soil moisture calculated from LBM. Figure 14 shows the histograms of the RMSE time-series. The results show that the VSL-Min ~~may be due to its problematic exclusive growth limitations. AC15 discussed that the “response-switching” mechanism of~~ is more sensitive to the ~~minimum t-norm worsens~~ choice of soil moisture and using the soil moisture calculated by the LBM improves the performance of ~~. Given that the~~ the model. However, the improvement in error reduction for VSL-Prod ~~applies a smoother t-norm, allowing for both energy and moisture to modulate the tree growth, its performance remains stable. The response of the minimum t-norm parades a complete correlated trend with the forcing anomaly (Fig. ??).~~ is not significant when using the calculated soil moisture with the LBM.

4 Discussion and Conclusions

4.1 Error ~~reduction efficacy~~ Reduction Efficacy of ~~TRW chronologies~~TRW Chronologies

~~For the OSSE~~

For the OSSEs studied here, it was found that the ability of a particular ~~pseudo-TRW~~ pseudo-TRW chronology to reduce the error of the ~~EnKF-based~~ EnKF-based estimate of the ~~TA~~ time averaged state appears modulated by the strength of the yearly internal variability of the model at the chronology site. This ~~finding~~ methodology is termed Optimal Sensor Placement (OSP) and can in principle be employed to help the dendrochronology community to increase the effectivity of their sampling efforts by focusing on the sites with more potential to decrease reconstruction uncertainty (Ancell and Hakim, 2007; Hakim and Torn, 2008; Mauger et al., 2013). Furthermore, this approach can be directly applied to any proxy type with sufficiently stable time resolution (e.g., annual resolution)(Comboul et al., 2015). However, the application of this method for lower frequency climate data like sediment cores or speleothems has to be investigated. These results are likely to depend on the climate model, the proxy system model, the proxy network and their resolution (Comboul et al., 2015).

An evident caveat of the above mentioned rationale is that every model-based estimate of the climate internal variability strength for a particular time scale will necessarily exhibit the biases of the particular climate model used. We consider that this modeling subjectivity/imperfection issue can be ameliorated by means of multi-model and multi-physics approaches, which in principle should increase the robustness of the results and provide uncertainty estimates. In any case, we believe that provided ~~, the~~ results are analyzed cautiously taking into account the weaknesses of current climate models~~, the huge amount of~~ . The climate dynamics knowledge condensed into an Earth system model can certainly be used profitably to reduce the cost of a ~~undiscriminated~~ indiscriminated proxy sampling strategy.

4.2 Off-line ~~regime~~Regime

~~Additional to the classical DA approaches used in paleoclimate studies, a so-called “off-line” DA-based climate reconstructions is presented by . In their novel method, the same prior or background ensemble is used for each reconstruction time-step. compared an off-line and an on-line ensemble-based DA and showed that the both methods outperform the model without DA. They concluded that the on-line method performs a more consistence temporal variability. However, they suggested further investigations to evaluate their results.~~

Within our simplified perfect model ~~OSSE~~OSSE, the observed situation of simultaneously having significant ~~DA skill for *analysis*~~ DA skill for analysis quantities and none for ~~*forecast*~~ forecast quantities, currently referred to as off-line ~~DA~~ DA regime, can arise either from the dynamical model or from the ~~DA~~ DA scheme (answers to the questions 1 and 2 raised in Introduction).

Regarding the dynamical model, the most obvious reason to enter into the off-line regime is that the period between ~~con-secutive~~ consecutive observations exceeds the predictability horizon of the model. ~~In this~~ Under these conditions, as already discussed in AC15, the ensemble spread reaches climatological levels (spread of the Free ensemble run) before new observations are assimilated and the accumulation of observational information is essentially lost. For SPEEDY~~model~~, due to its purely atmospheric nature, it is ~~not surprising~~ likely to enter the off-line regime for a 1-year inter-observation period. This might be also the case for current operational (coupled) climate prediction systems, given their lack of useful lead times longer than one year. ~~In this state of affairs it looks~~ Thus, it seems unlikely to achieve effective observational ~~constraint~~ constraints on the forecast using proxy records with yearly time resolution. However, there is already evidence for the existence of potential sources of climate internal variability with time scales longer than 1 year ~~(Smith et al. , 2012)~~(Smith et al., 2012). The so called “annular modes” (Thompson and Wallace, 2000) may present internal variability in the high latitude areas. The latitudinal oscillation of the cell structure imposes variability at the fringes of the jet streams and oscillations of the ITCZ impacts the humidity (Holton and Hakim, 2013). ENSO affects a large portion of tropical and subtropical climate in time-scales larger than one year. Accordingly, ~~it is expected~~ we expect that it should be possible to obtain actual inter-annual predictability ~~skills~~ skill in the foreseeable future.

Regarding the ~~DA~~ DA scheme, a possible culprit for the onset of the off-line regime is the ~~Time-Averaged Update (TA-Up) strategy (AC15)~~time-averaged update strategy (Dirren and Hakim, 2005). It is not clear if ~~for SPEEDY model this technique is able~~ whether we can employ this technique with SPEEDY to properly estimate instantaneous quantities out of ~~TA~~ time averaged observations. In particular, complete decorrelation between time averaged and instantaneous variables is not guaranteed.

In any case, despite its lack of accumulation of observational information over time, off-line ~~DA~~ DA has already been shown to be more robust than traditional ~~CFR~~ Climate Field Reconstruction (CFR) techniques based on orthogonal empirical functions ~~(Steiger et al., 2014; Hakim et al., 2016)~~and stationarity assumptions (Steiger et al., 2014; Hakim et al., 2016). Moreover, the implementation and running of ~~offline DA~~ off-line DA schemes is remarkably cheaper than on-line approaches.

Following the idea of ~~(Steiger et al., 2014; Matsikaris et al., 2015; Hakim et al., 2016) for purely "offline" DA~~ (Steiger et al., 2014; Matsikaris et al., 2015; Hakim et al., 2016) for purely "offline" DA (no-cycling), our perfect model experiments indicate that the ~~"online"~~"online" scheme may not outperform the ~~"offline"~~"offline" one in either the temporal or the spatial error reduction (answer to the question 3 raised in Introduction). It should also be emphasized that our model set-up (with slab ocean) can not capture the ~~fully~~ full atmosphere-ocean interactions. Therefore, using a more realistic coupled atmosphere-ocean model may improve the skill of the ~~"online" DA~~"online" DA DIFaddend .

4.3 Filter ~~operation sensitivity~~ Operation Sensitivity to the ~~PLF representation~~Growth Rate Function

The results of the ~~DA~~ DA experiments conducted with ~~speedy model support in general the ones obtained for~~ SPEEDY support results obtained the two-scale ~~Lorenz (1996)~~ Lorenz (1996) model (AC15) regarding the influence of the ~~PLF~~ PLF representation on the filter performance. The efficacy of the ~~EnKF- based TA~~ EnKF-based time averaged state estimation strategy appeared to be significantly sensitive to the selection of the t-norm used to calculate the growth rate, with the product t-norm (~~VSL-Prod~~VSL-Prod) outperforming the minimum t-norm (~~VSL-Min~~VSL-Min) used in the original ~~formu- lation of VSL~~ formulation of VSL forward model.

Tolwinski-Ward et al. (2014) described trees as fundamentally lossy[1] recorders of climate, due to the integrated nature of the information ~~in them contained~~ contained in them and the standardization process used to minimize the non-climatic effects on growth. Growth is influenced by temperature and/or moisture and the transitions between limitation regimes may happen suddenly ("abrupt shifting") Acevedo et al. (2015). In the same vein, we argue that the ~~abrupt shifting~~ "abrupt shifting" of recorded variable ~~implied~~ (temperature or moisture)–implied by the minimum function used in ~~VSL original formulation~~VSL's original formulation– might constitute an additional source of lossyness (at least within a EnKF-based ~~DA~~ acda setting used), which can be substantially reduced by resorting to alternative ~~FL-based~~ Fuzzy Logic-based representations of the ~~PLF. Our DA~~ PLF. Our DA experiment indicates a higher skill performance with the ~~VSL-Prod for both "offline" and "online"~~VSL-Prod for both "offline" and "online" regimes compared to the ~~VSL-Min~~ VSL-Min (answer to the question 4 raised in Introduction).

[1]This adjective is currently used in the information technology area to designate data encoding methods that lead to information loss from the original version for the sake of reducing the amount of data needed to store the content.

4.4 Challenges to be ~~addressed~~Addressed

As a cautionary remark, we want to highlight the several important limitations of the experiments described in this paper. The generated ~~pseudo-TRW~~ pseudo-TRW observations lack a threshold for temperature or moisture after which the growth response does not change and their contamination with noise was performed assuming optimistically high ~~SNR levels. However, we examined the performance of the off-line with different signal to noise ratios (Fig. 11). Figure 11 exhibits that the time-averaged global RMSE shows an elbow at values around $SNR = 1$ and reaches the Free run at $SNR = 0.03$, where all the pseudo-observations are ignored in the .~~ SNR levels. Furthermore, the response thresholds were set in a completely homogeneous fashion for all the observational stations, whereas actual ~~TRW~~ TRW networks are strongly heterogeneous in that sense, comprising chronologies generated under highly dissimilar growth limitation regimes. Additionally, the efficiency of ~~EnKF~~ EnKF technique used relies on the Gaussianity of all the variables of the model. Nevertheless, in a climate model some variables can present strongly non-Gaussian properties –specially definite positive quantities such as humidity– and ~~then~~ their estimation should in principle be performed with more sophisticated strategies such a Gaussian anamorphosis (Bocquet et al., 2010; Lien et al., 2013). It is worth mentioning the necessity of explicitly addressing model errors by conducting imperfect model ~~OSSE~~OSSE. Finally, we note that our findings are based on a slab coupled ocean model and we encourage using a proper coupled atmosphere-ocean model in future studies.

5 ~~Appendix: Observation System Simulation Experiments (OSSE)~~

~~Given a prediction system comprising a dynamical model and a DA scheme, ***forecast*** and ***analysis*** errors arise from many different sources, e.g. model imperfections, inadequacy of the DA strategy and insufficiency of observational information, which interact with each other in practice. In order to disentangle the effects of these error sources, a DA scheme is typically tested under simplified conditions by means of numerical experiments, currently known as Observation System Simulation Experiments (OSSE), whose realism level is gradually increased.~~

~~An OSSE consists of (i) a single model trajectory $\mathbf{x}^{\mathrm{NATURE}}$, typically referred to as "true" run or "nature" run, that is used as prediction target, (ii) pseudo-observations created by applying the observation operator to $\mathbf{x}^{\mathrm{NATURE}}$ and adding simulated observational noise, and (iii) an observationally constrained run $\mathbf{X}^{\mathrm{DA}}$, obtained by performing a sequence of ***analysis*** cycles where the pseudo-observations are assimilated (see Fig. 1).~~

~~The nature run is normally generated by running the dynamical model starting from a random sample of the model climatology. Notice that thanks to the availability of the truth model evolution for an OSSE, the ***forecast*** and ***analysis*** skill of the observationally constrained run can be directly assessed, using for example the RMSE of the ensemble mean: () = where $\overline{}$ and $\langle\ \rangle$ denote the time and ensemble mean operators, respectively.~~

~~As mentioned before, the development and evaluation of a DA setting should be carried out gradually, by way of a hierarchy of increasingly realistic OSSE. A typical first step is to create $\mathbf{x}^{\mathrm{NATURE}}$, $\mathbf{X}^{\mathrm{FREE}}$ and $\mathbf{X}^{\mathrm{DA}}$ using the same model, which leads to~~

4.2 Outlook

Our results appear useful for TRW chronologies in the sense that EnKF techniques are robust in the ~~so-called "perfect model" OSSE. In a real-world setting the dynamical model is always an imperfect representation of reality, then a natural next step is a "imperfect model" OSSE, where $\mathbf{X}^{\mathrm{FREE}}$ and $\mathbf{X}^{\mathrm{DA}}$ are performed using for example a simplified version of the model utilized to create $\mathbf{x}^{\mathrm{NATURE}}$~~face of two strong nonlinearities, i.e., "switching recording" (Acevedo et al., 2015). Thus, it is important to emphasize that the OSSE presented in this manuscript represents the first step of the long hierarchy of DA experiments to achieve an effective assimilation of proxy records into climate models using forward proxy models. We encourage further experiments using comprehensive earth system models with longer time scale processes to bring the proxy DA into an online regime. However, assimilation of proxies in an earth system model with different components may lead to inter-component DA pollutions.

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Table 1. Runs' characteristics.

No.	1	2	3	4
Forward Model	VSL-T	VSL-T	VSL-Min	VSL-Prod
Ocean	*SLAB*	*PRESCRIBED*	*SLAB*	*SLAB*

Simulations are 150 years long.

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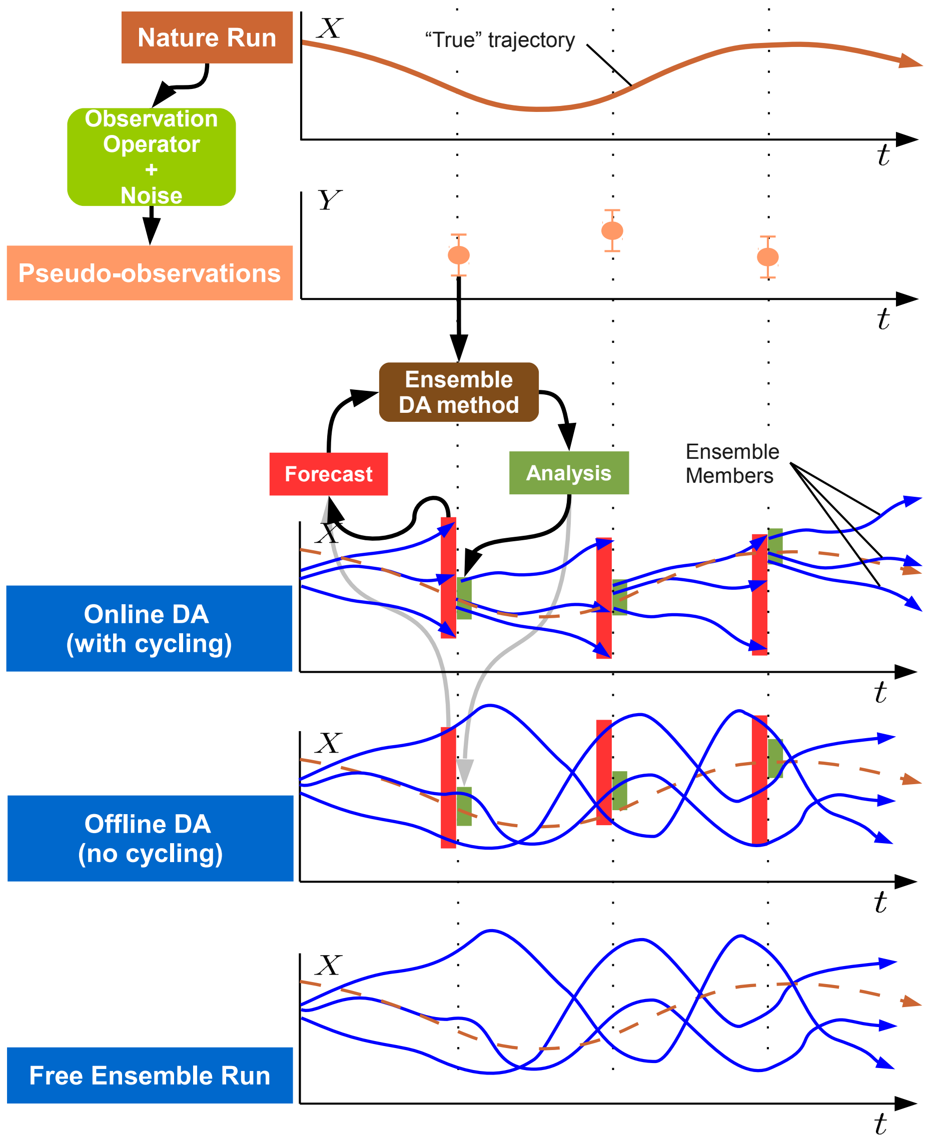


Figure 1. Schematic of a typical Observation System Simulation Experiment (OSSE) with ensemble "online" (with cycling) and "offline" (no-cycling) DA methods. t designates the time axis and $X(Y)$ denotes the model state (observation) space. Sharp (rounded) cornered boxes represent data (processes). Red (green) vertical shadings indicate the *Forecast* (*Analysis*) spread. Vertical dotted lines represent the assimilation steps.

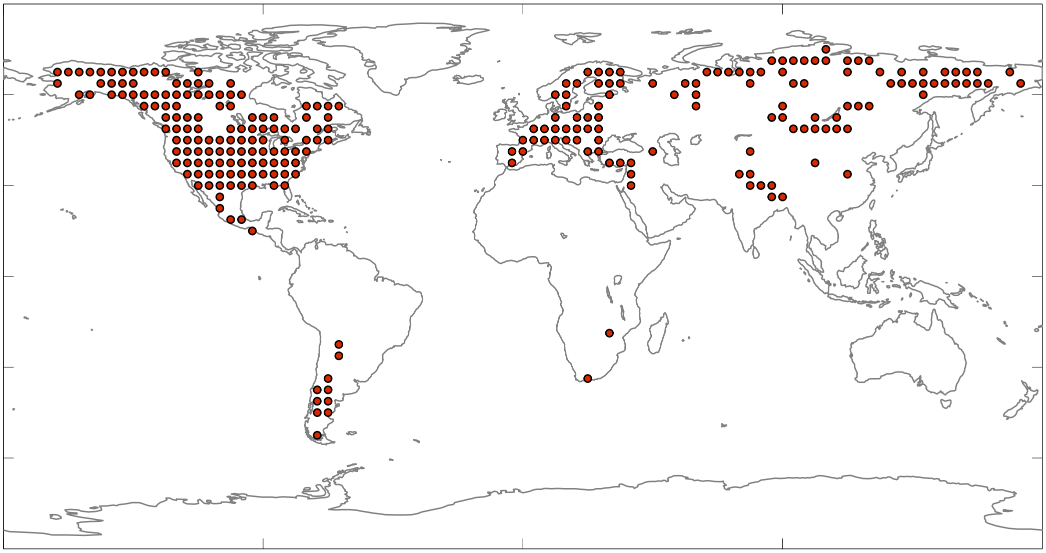
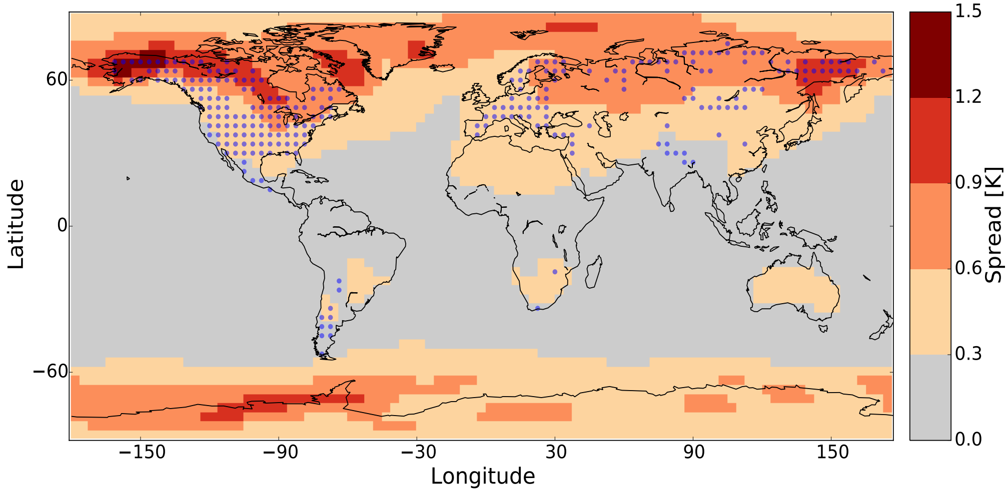


Figure 2. Station set resembling real ~~TRW~~ TRW network from Breitenmoser et al. (2014)

a) Free ensemble Spread



b) Free ensemble Error

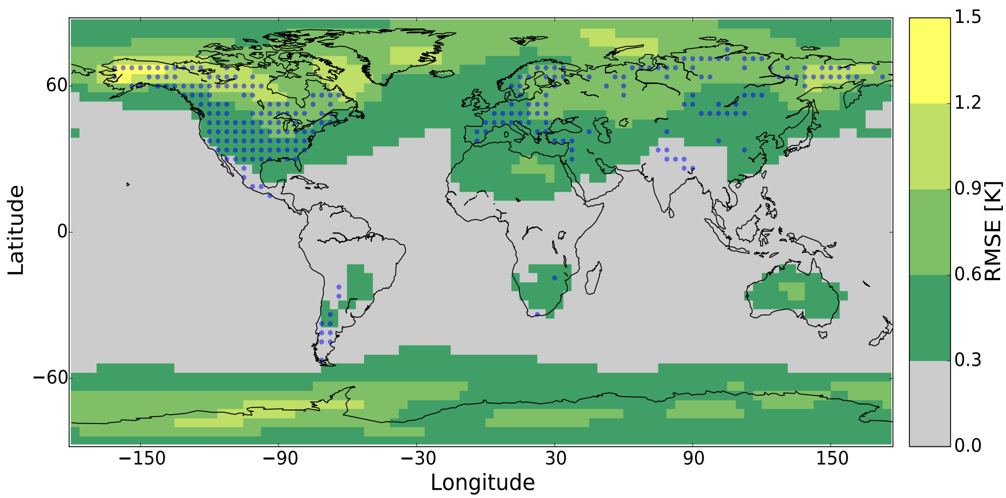
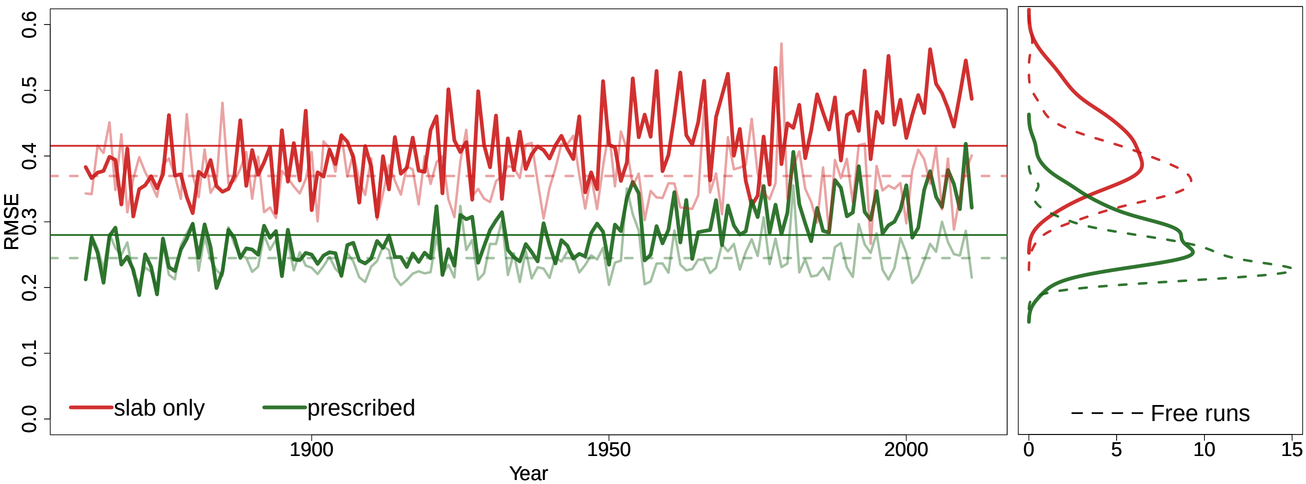


Figure 3. Free ensemble simulations for the SLAB experiment: a) Ensemble Spread [K] of near surface temperatures, b) Free ensemble ~~RMSE~~ RMSE [K].

a)



b)

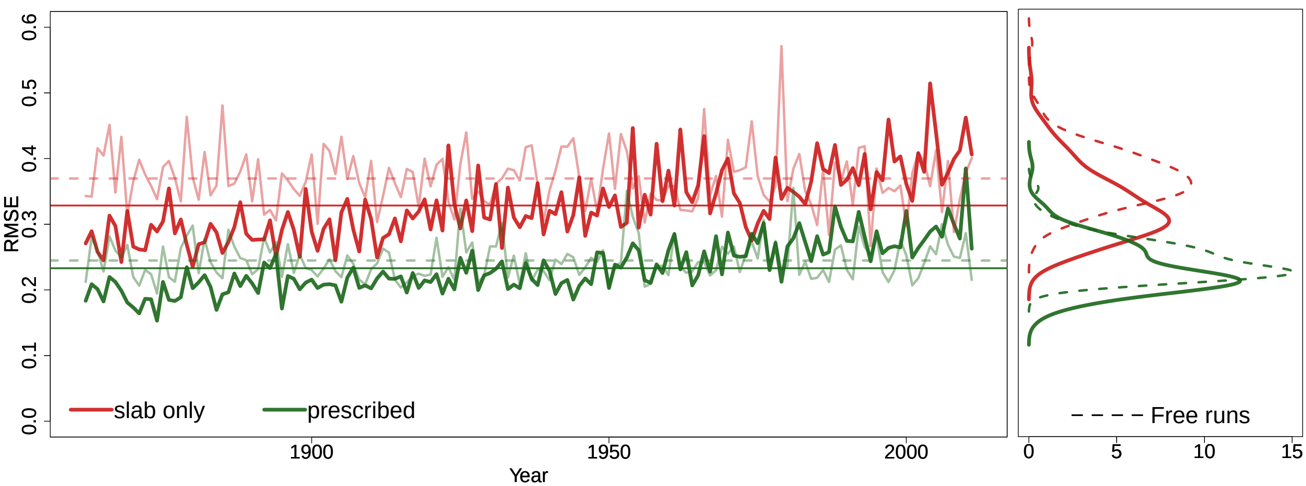
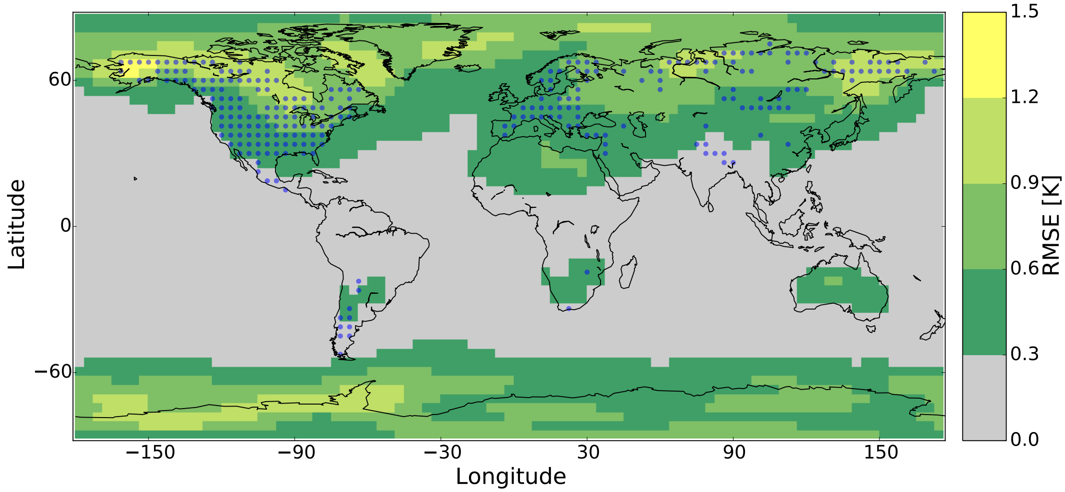


Figure 4. Global ensemble mean for a) Forecast constrained by VSL-T ~~pseudo-TRW~~ pseudo-TRW observations (bold lines) and Free run (thin lines); b) Analysis (solid lines) and Free run (thin lines). Horizontal lines exhibit the mean values. Right panels exhibit the histograms of the time-series.

a) ~~**DA *forecast***~~ **DA forecast for VSL-T**



b) ~~**DA *analysis***~~ **DA analysis for VSL-T**

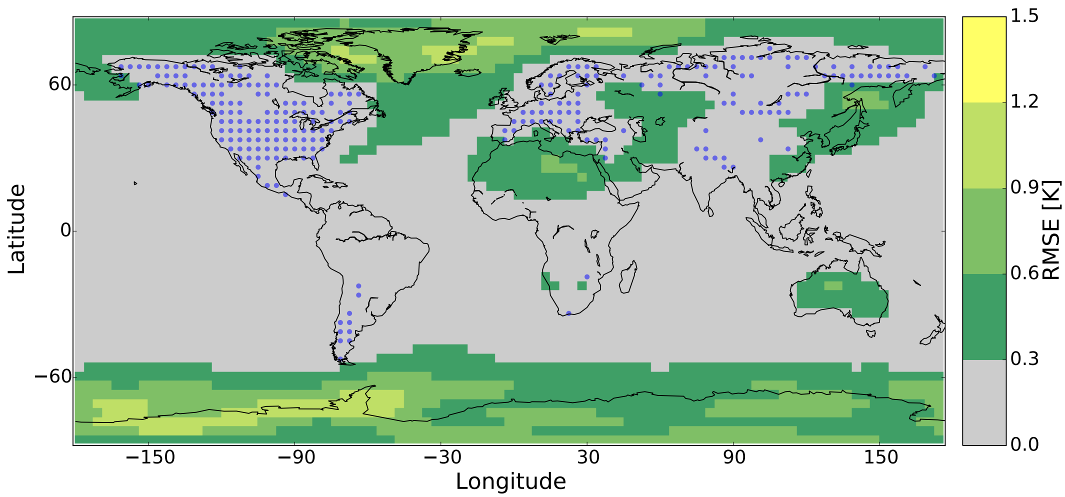
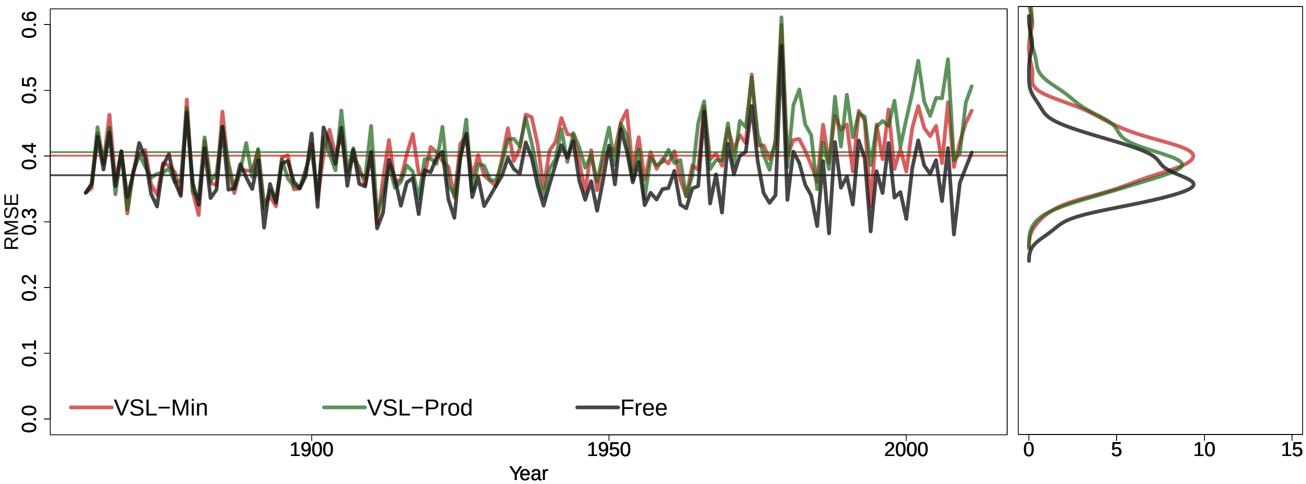


Figure 5. Time-averaged ~~RMSEs~~ RMSEs of SLAB experiment for a) ~~**DA *forecast***~~ DA forecast and b) ~~**DA *analysis***~~ DA analysis using the VSL-T observation operator.

a)



b)

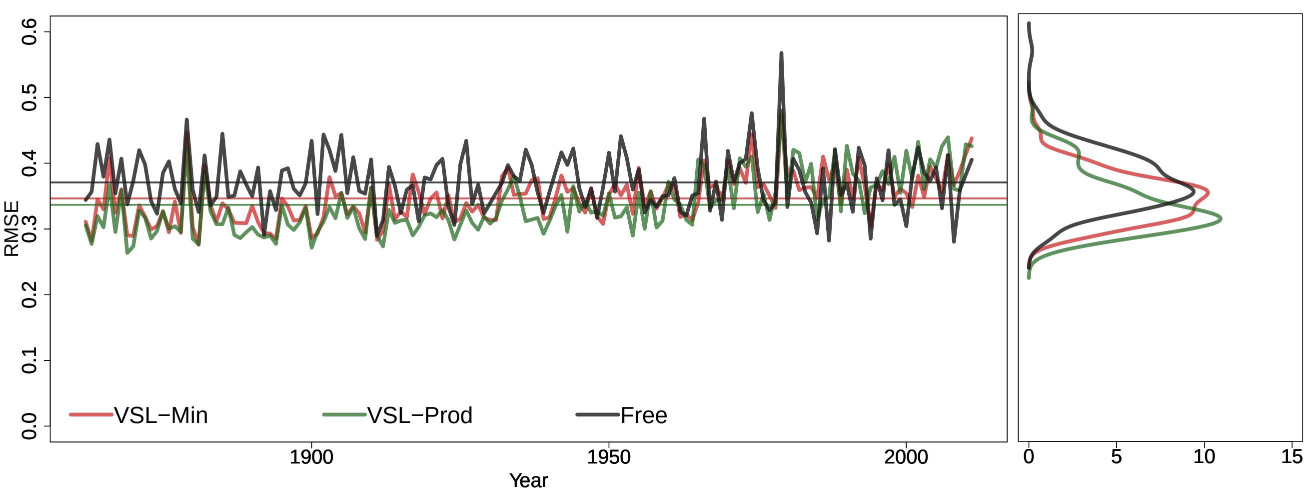
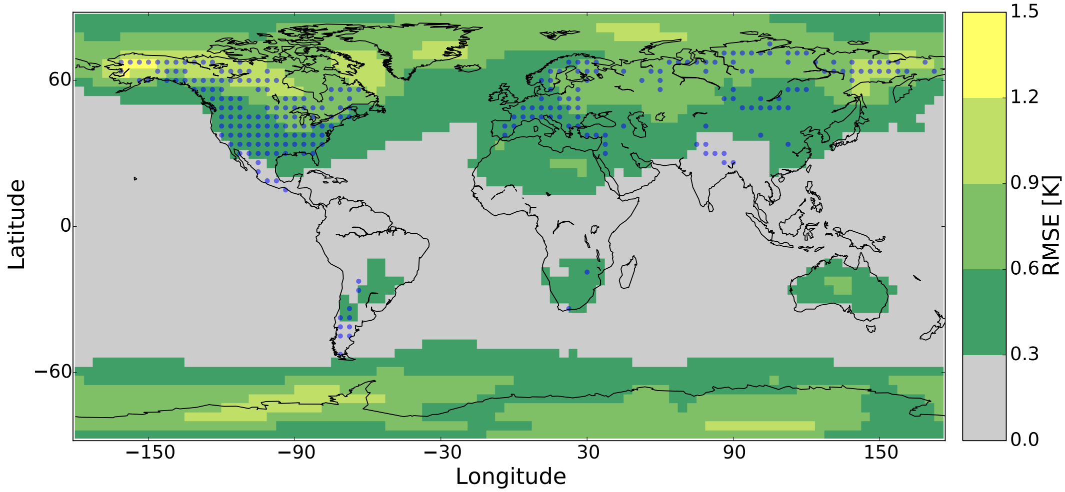


Figure 6. Global ensemble mean for a) ~~***forecast***~~ forecast and b) ~~***analysis***~~ analysis constrained by VSL-Min (red) and VSL-Prod (green) ~~pseudo-TRW~~ pseudo-TRW observations and free run (black). Horizontal lines exhibit the mean values. Right panels exhibit the histograms of the time-series.

a) ~~DA *forecast*~~ DA forecast for VSL-Min



b) ~~DA *analysis*~~ DA analysis for VSL-Min

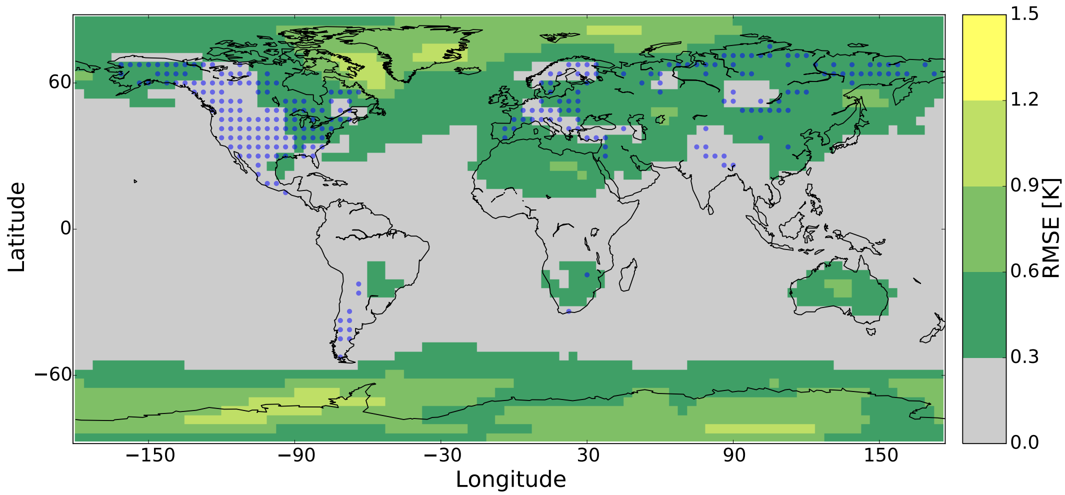
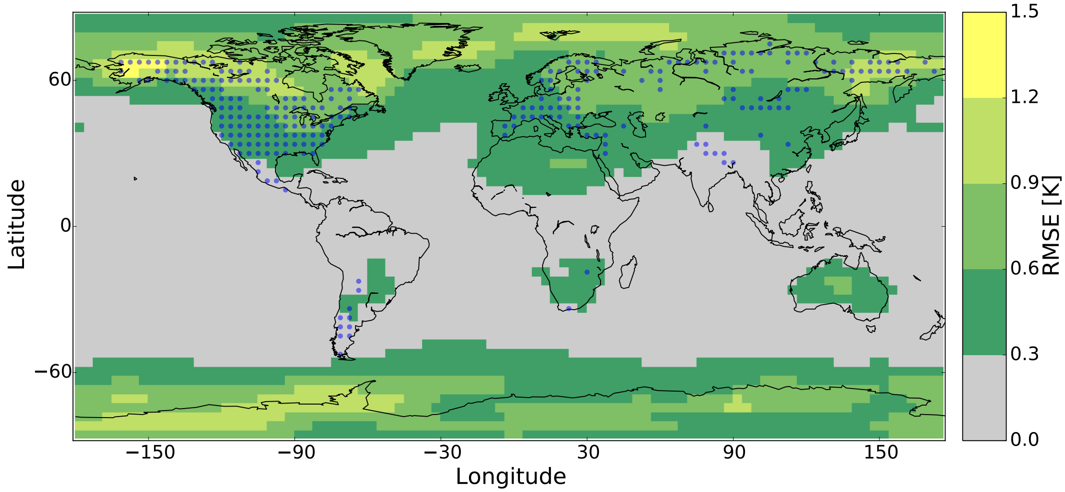


Figure 7. Time-averaged ~~RMSEs~~ RMSEs of SLAB experiment for a) ~~DA ***forecast***~~ DA forecast and b) ~~DA ***analysis***~~ DA analysis using the VSL-Min observation operator.

a) ~~DA *forecast*~~ DA forecast for VSL-Prod



b) ~~DA *analysis*~~ DA analysis for VSL-Prod

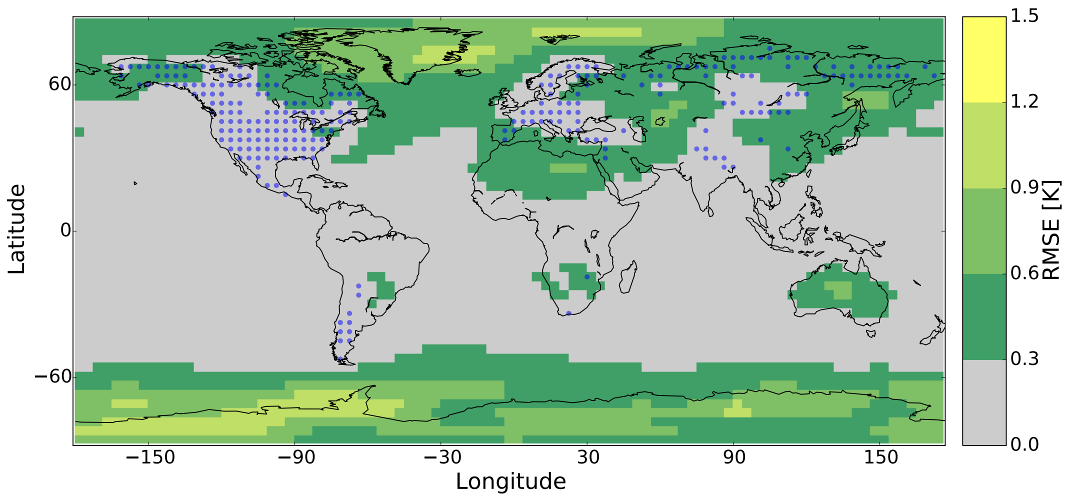


Figure 8. Time-averaged ~~RMSEs~~ RMSEs of SLAB experiment for a) ~~DA *forecast*~~ DA forecast and b) ~~DA *analysis*~~ DA analysis using the VSL-Prod observation operator.

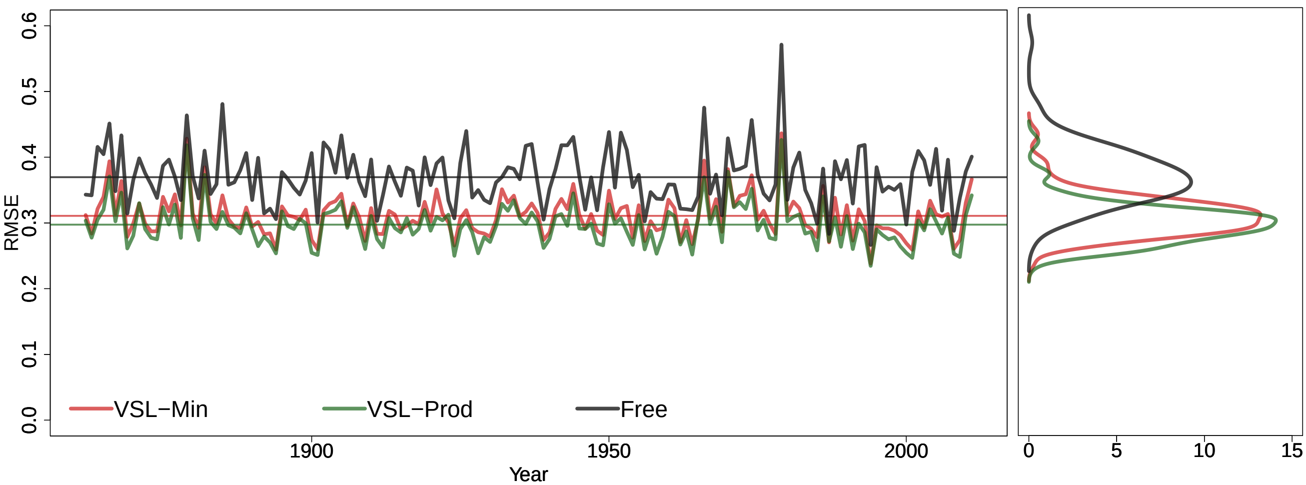
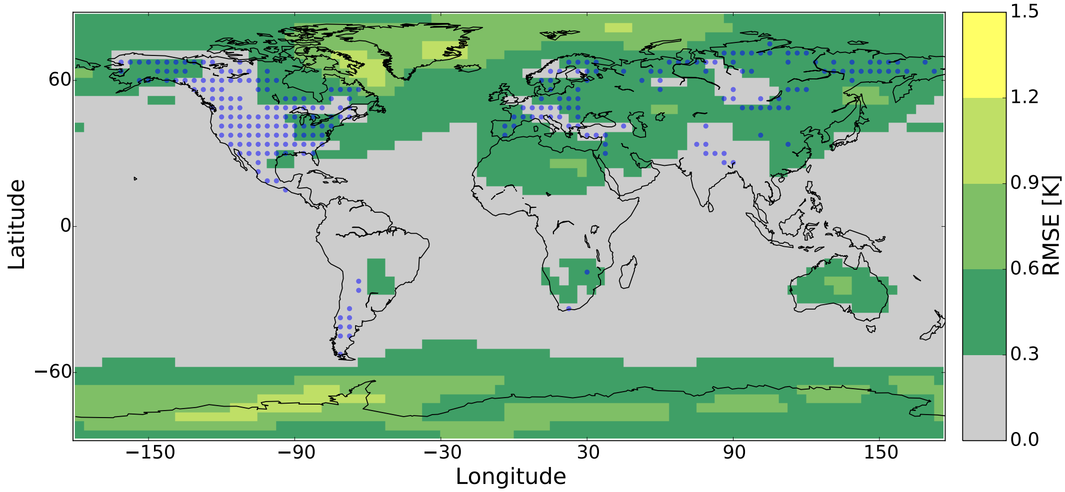


Figure 9. Global ensemble mean for ~~***analysis***~~ analysis constrained by VSL-Min (red) and VSL-Prod (green) ~~pseudo-TRW~~ pseudo-TRW observations and free run (black). Horizontal lines exhibit the mean values. Right panel exhibits the histograms of the time-series.

a) ~~DA *analysis*~~ DA analysis for VSL-Min with nocycling



b) ~~DA *analysis*~~ DA analysis for VSL-Prod with nocycling

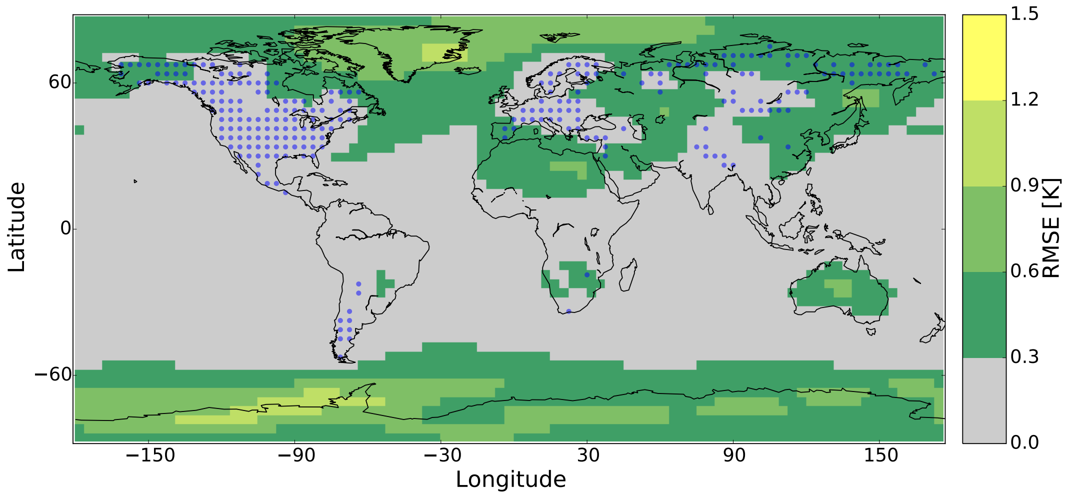


Figure 10. Time-averaged ~~RMSEs~~ RMSEs of SLAB experiment for a) nocycling ~~DA *analysis*~~ DA analysis using the VSL-Min and b) the VSL-Prod observation operator.

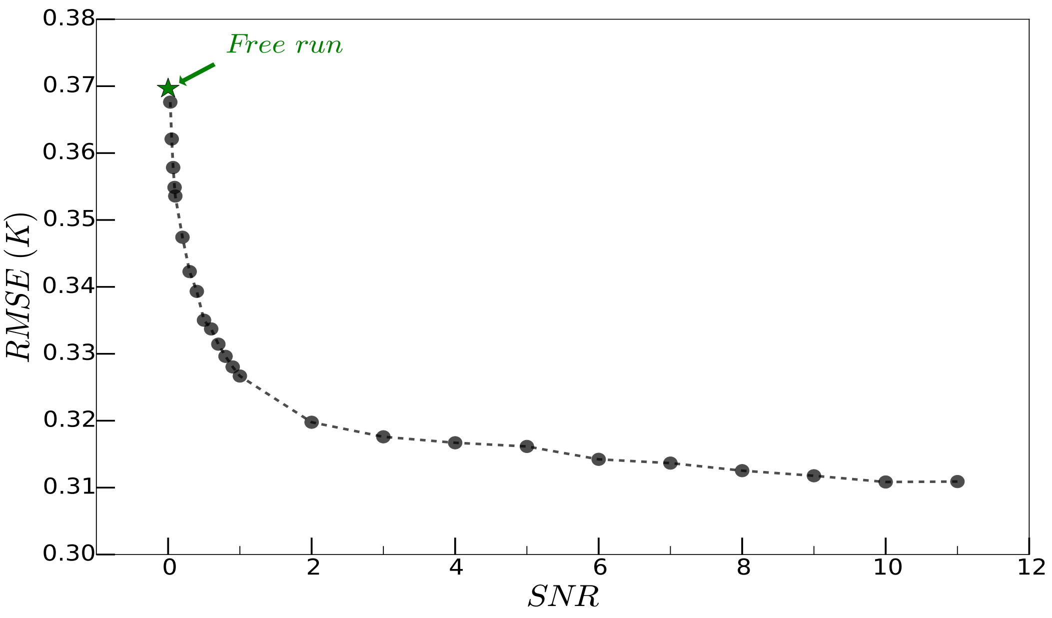
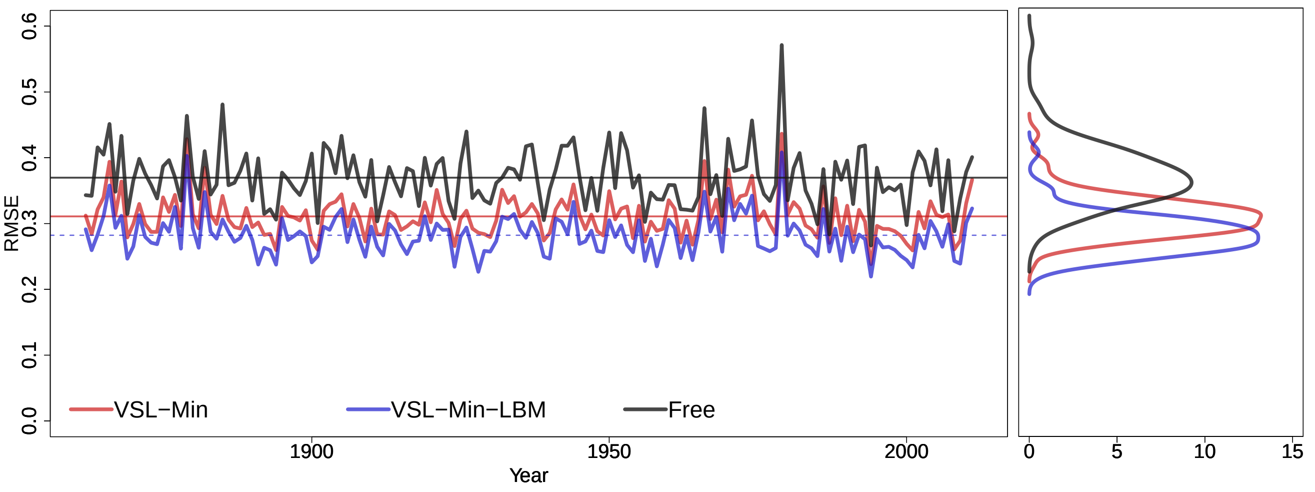


Figure 11. Time-averaged global RMSEs of SLAB experiment for nocycling DA using the VSL-Min and different signal to noise ratios. The Green star shows the Free run RMSE.

a)



b)

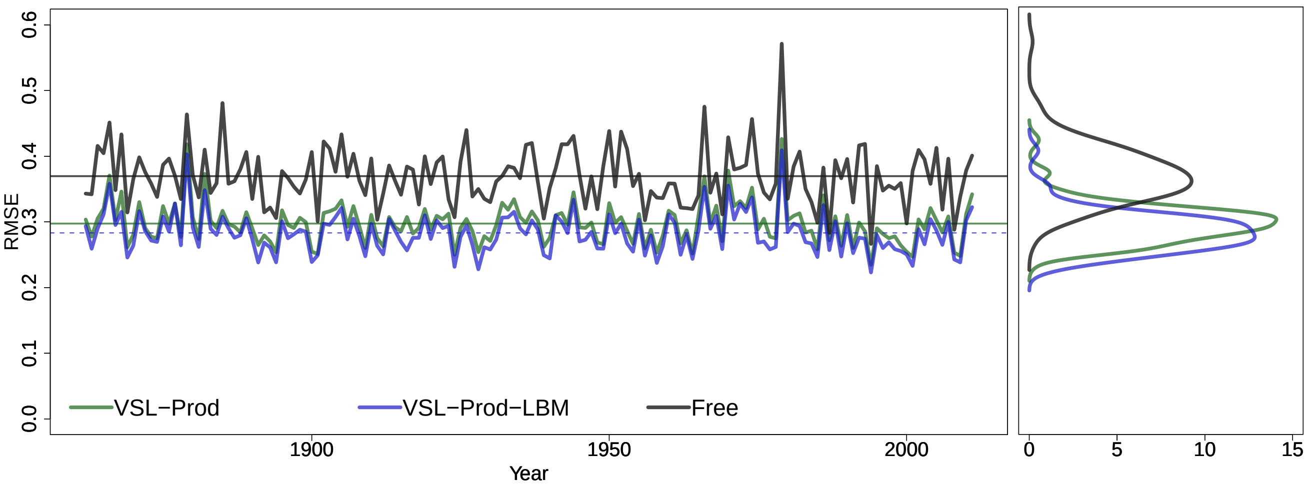
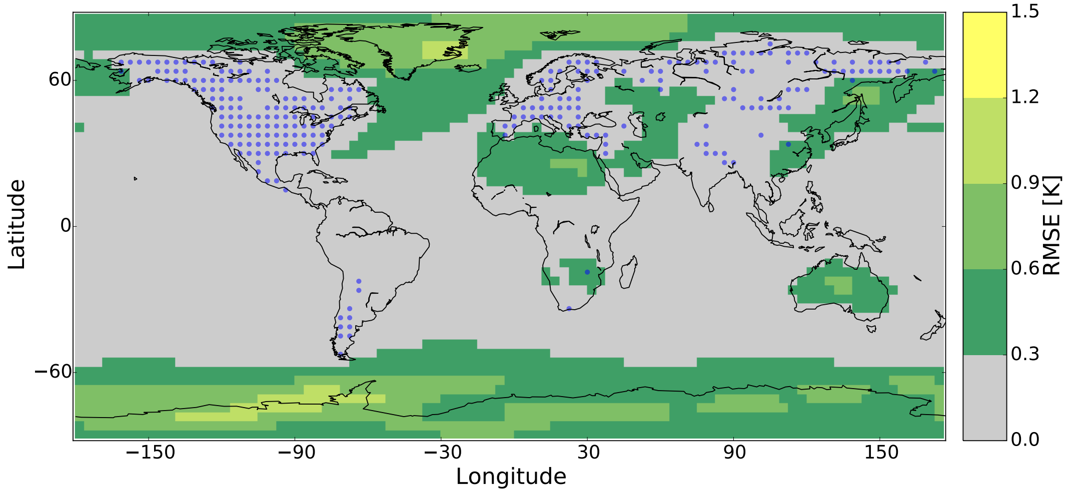


Figure 12. Global ensemble mean for analysis constrained by pseudo-TRW observations for a) VSL-Min with the climatological soil moisture (red), with the soil moisture computed by Leaky Bucket Model (blue) and free run (black); b) VSL-Prod with the climatological soil moisture (green), with the soil moisture computed by Leaky Bucket Model (blue) and free run (black). Horizontal lines exhibit the mean values. Right panels exhibit the histograms of the time-series.

a) DA analysis for VSL-Min with Leaky Bucket Model and nocycling



b) DA analysis for VSL-Prod with Leaky Bucket Model and nocycling

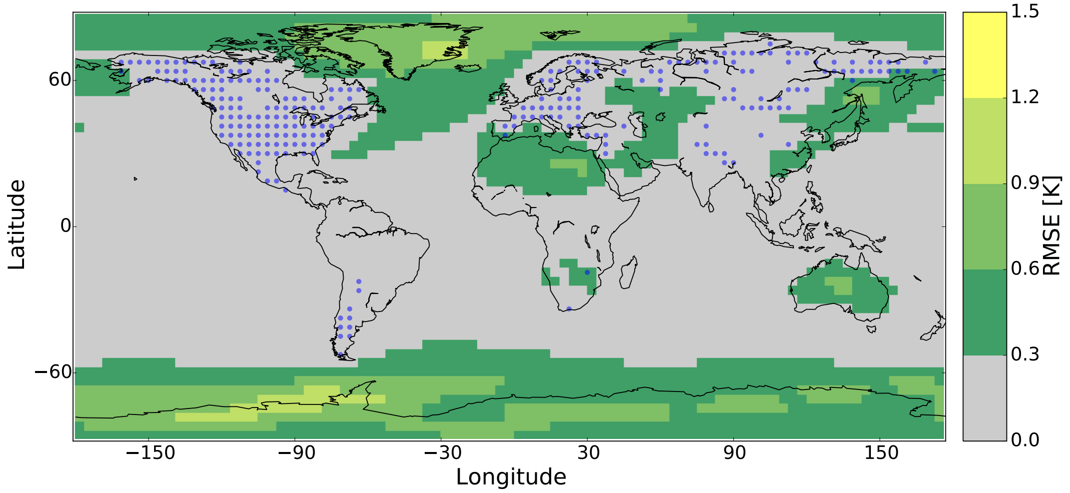


Figure 13. Time-averaged RMSEs of SLAB experiment for a) nocycling DA analysis using the VSL-Min with Leaky Bucket Model and b) the VSL-Prod with Leaky Bucket Model observation operator.

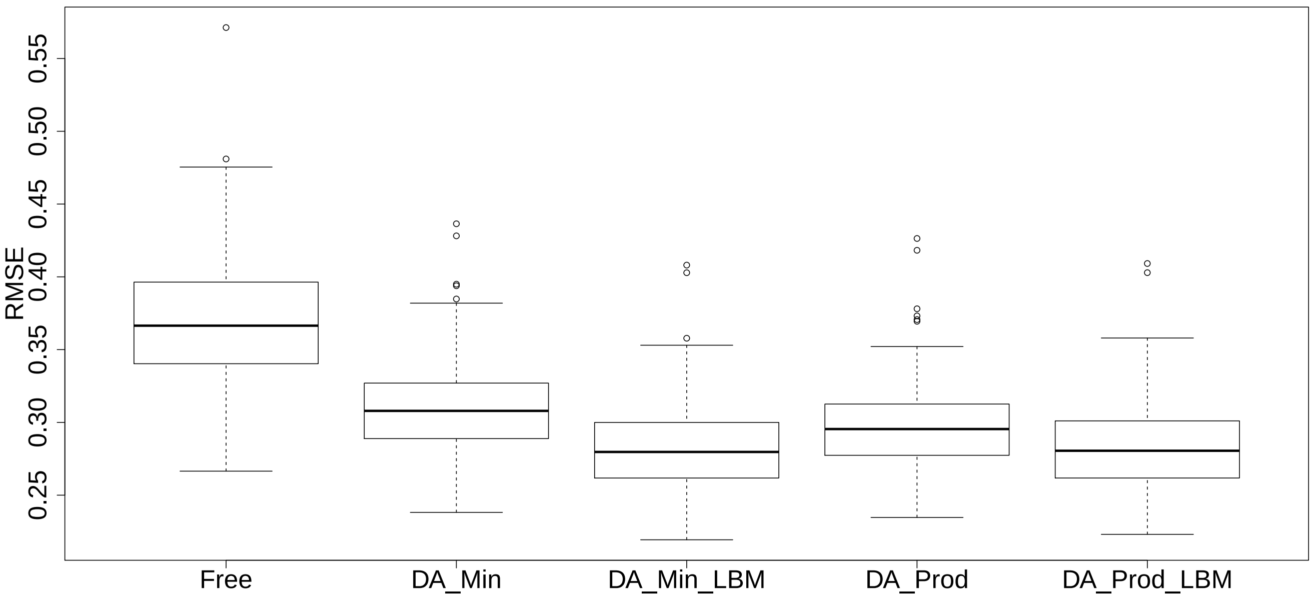


Figure 14. Histograms of global ensemble mean for analysis constrained by pseudo-TRW observations for Free run, DA run with VSL-Min, VSL-Prod using the climatological soil moisture and VSL-Min, VSL-Prod using the Leaky Bucket Model (LBM).