Review of "Technical Note: Calculating state dependent equilibrium climate sensitivity from palaeodata", Köhler et al.

The authors, in a recent paper (Köhler et al., 2015), found evidence for state dependence of climate sensitivty in the paleoclimate record. In the present work, the authors explore an important consequence of this state-dependence, namely that point-wise and slope-based definitions of climate sensitivity can differ. This technical issue can present complications, and the authors discuss potential outcomes of these complications, along with ways of reconciling estimates made using one of these definitions with the other. The authors are performing needed and useful work by raising and addressing these complications, as the answers to the questions that this paper aims to present would be of use the broader community looking to study climate sensitivity and its state dependence. Unfortunately, I believe that there is a fundamental problem in how the authors approach their study of this state dependence.

Measuring how the climate sensitivity changes with state requires a formal definition of what we mean by "state-dependence". Although climate state can refer to a number aspects of the climate, it is principally taken to refer to the surface temperature. For example:

- "Climate sensitivity may also be time-dependent and state-dependent; for example, in a much warmer with little snow and ice, the surface albedo feedback would be different from todays." (Knutti and Hegerl, 2008)
- "Consistent intercomparison is crucial to detect systematic differences in sensitivity values — for example, due to changing continental configurations, different climate background states, and the types of radiative perturbations considered. These differences may then be evaluated in terms of additional controls on climate sensitivity, such as those arising from plate tectonics, weathering cycles, changes in ocean circulation, non-CO_2 greenhouse gases (GHGs), enhanced water-vapour and cloud feedbacks under warm climate states. Palaeoclimate data allow such investigations across geological episodes with very different climates, both warmer and colder than today."; "...only ancient records provide insight into climate states globally warmer than the present."; "...it may be more useful to consider past warm climate states as test-beds for evaluating processes and responses.." (PALAEOSENS, 2012)
- "The available evidence suggests that climate sensitivity depends considerably on the reference climate state... In particular, estimates of the ECS from climates much warmer than today, such as the Paleocene-Eocene, would naturally yield higher values..."; (Meraner et al., 2013)
- "The climate sensitivity decreases as the system warms after forcing is stabilized at S2050 and S2100 levels." ("Climate sensitivity and climate state," Boer and Yu (2003))

There is a physical reason for giving pride of place to surface temperature, as suggested by the above statements: the feedbacks that determine the climate sensitivity, such as the water vapor, surface albedo, and cloud feedbacks are expected to change with the surface temperature, and so will make climate sensitivity state-dependent. As a result, many papers make the assumption the most direct way to capture the state-dependence of climate sensitivity in a simple model is to add a polynomial dependence of the climate feedback parameter on temperature (e.g., Roe (2009), Colman and McAvaney (2009), Jonko et al. (2012)).

Rather than taking this course, the present authors test the polynomial dependence of climate sensitivity on radiative forcing. While radiative forcing and surface temperature change are expected to change proportionally for small forcings, for large forcings this proportionality breaks down, and during bifurcations the relationship between the two can become poorly defined, or at least discontinuous. As a result, for these larger forcings, analyses of state-dependence that measure dependence on radiative forcing rather than temperature can give quite different answers than the temperature-dependent approach. They can less faithfully capture the effects of changes in feedback strength, and generally cannot represent the effects of bifurcations. Worse still, they can give unphysical answers. I will discuss examples of these three issues below, but first I will more formally describe the temperature-dependence approach taken by previous authors.

1 The temperature-dependent approach

The climate sensitivity formalism is built around the idea that Earth can be modelled by a single ordinary different equation, $cdT/dt = N$, where N is the net top-of-atmosphere radiative flux, c is some thermal inertia of the Earth and T is the surface temperature. The value of N depends on various forcing agents such as the CO_2 concentration (or, depending on the partition between feedbacks and forcing, land ice), which we will denote as F, as well as on aspects of the general state of the atmosphere and surface that are expected to change with the surface temperature, T. In other words, $N(T, F)$, and for a given forcing, N is simply a function of T. This latter dependence has been an essential assumption from the earliest energy balance models (e.g., North (1975): "In order to set up a simple energy-balance climate model it is necessary to assume that all energetic fluxes can be parameterized by the temperature at the earth's surface.")

For a preindustrial state T_{pre} and F_{pre}, $N(T_{pre}, F_{pre}) = 0$. For small enough changes, the cross terms between T_{pre} and F_{pre} will be small[1], and we will have $N(T, F) = N(T, F_{pre}) + \Delta R$, where ΔR, the radiative forcing, can be defined to be $N(T_{pre}, F) - N(T_{pre}, F_{pre})$. If we impose a positive radiative forcing, it makes N positive, causing the Earth to warm. $\partial N/\partial T|_{pre}$ typically has a negative slope, so that the resulting warming compensates for the radiative forcing until $N(\Delta T + T_{pre}, \Delta R + F_{pre}) = 0$ and the system is in equilibrium.

[1] These cross terms can be thought of as representing how changes to the forcing elements effect the ability of the surface to set the top-of-atmosphere energy flux. For example, for a planet with a richer CO_2 atmosphere, the same change in surface temperature might have less of a direct Planck effect on the outgoing radiation.

In other words, the climate sensitivity (the change in surface temperature ΔT due to a given radiative forcing ΔR) is determined by the function $N(T, F_{pre})$ — the dependence of top-of-atmosphere fluxes on surface temperature. As described in Lorius et al. (1990), "The response of the system to an increase of radiative forcing would be a change in [equilibrium surface temperature] T_e necessary to restore radiative equilibrium." We typical linearize this dependency, taking $N(\Delta T + T_{pre}, \Delta R + F_{pre}) = \lambda \Delta T + \Delta R$ where $\lambda = \partial N / \partial T$, so that the climate sensitivity $S = -1/\lambda$. However, the climate sensitivity might change with climate state, because the dependence of top-of-atmosphere flux on surface temperature changes with climate state. (This is what is meant by the change in feedbacks discussed above). The most natural way to capture this change is to take the next term in the Taylor expansion of $N(T, F_{pre})$, namely $N(\Delta T + T_{pre}, \Delta R + F_{pre}) = \lambda \Delta T + a \Delta T^2 + \Delta R$ (e.g., supplementary materials for Roe and Baker (2007), Roe (2009), Colman and McAvaney (2009), Jonko et al. (2012)). This Taylor expansion could potentially be expanded further.

2 Advantages of fitting temperature-dependent approach over the state-dependent approach

For these reasons, even though it may seem natural to think of ΔT as a function of ΔR and to take a polynomial fit of ΔT to ΔR, we do not expect the relationship $\Delta T(\Delta R)$ to be polynomial, or even necessarily a function. As a result, diagnosing state-depenence of climate sensitivity with a polynomial fit of $\Delta T(\Delta R)$ can lead to a number of problems:

2.1 $\Delta T(\Delta R)$ is not necessarily a well-defined function

If there are any sorts of bifurcations in the Earth's climate — and given the Snowball Earth and the runaway greenhouse effect, we expect there are — $\Delta T(\Delta R)$ will not be a well-defined function, since for these cases the same radiative forcing can lead to very different levels of warming depending on the path taken by that forcing. Bifurcations need not be large events like Snowball Earth or the runaway greenhouse; a example of a more modest jump in temperature is given in previous work by some of the authors, Heydt et al. (2014), which uses a simple four-box model of the Earth (Gildor and Tziperman (2001)) to explore the behavior of glacial/interglacial cycles. I will illustrate my point about $\Delta T(\Delta R)$ by using this jump as an example.

Consider Figure 1, which uses Figure 1f from Heydt et al. (2014) (reproduced here as 1a) to estimate $N(T, F)$ for that model (given in 1b):

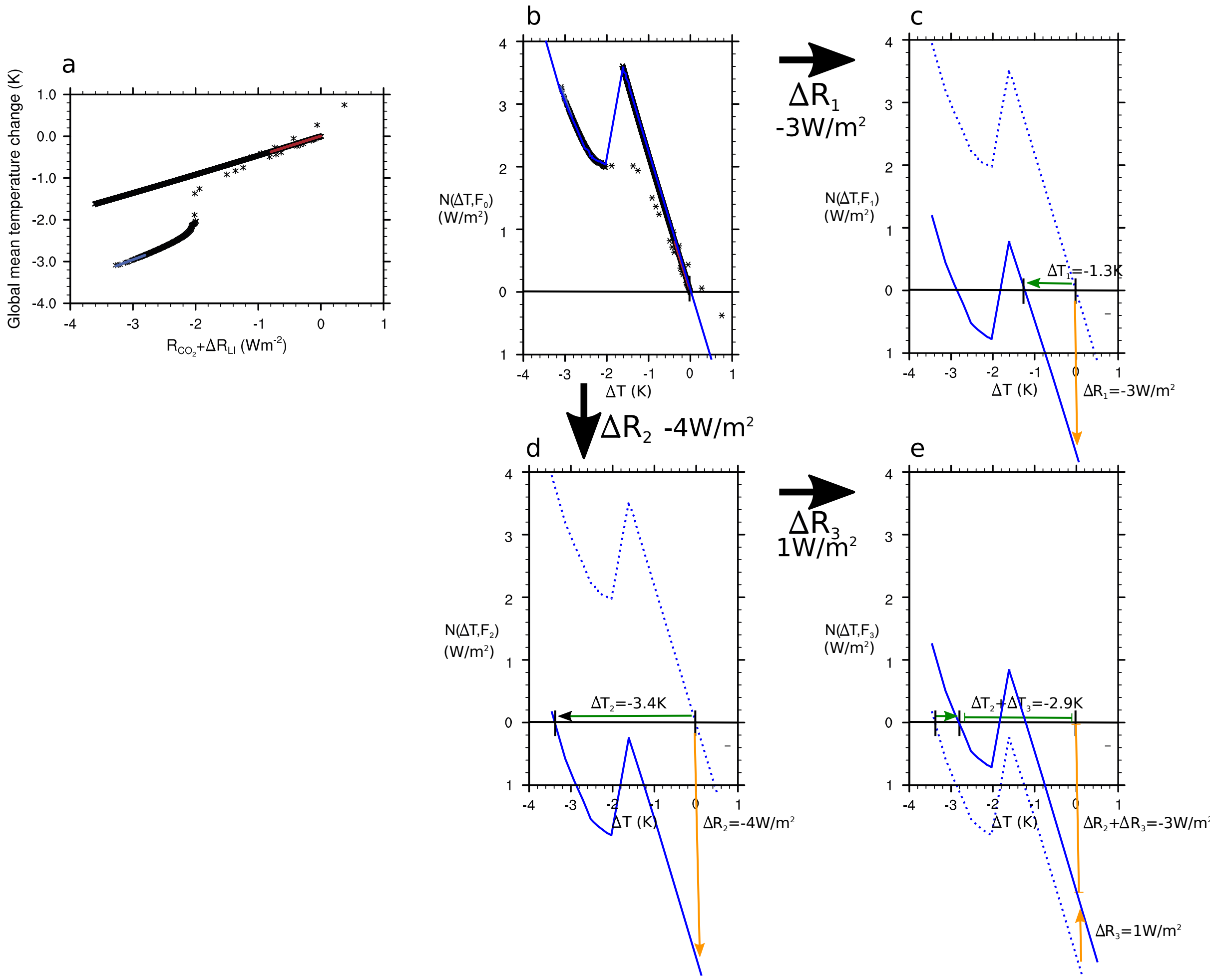

Figure 1: The presence of bistability can allow the same quantity of radiative forcing to cause different changes in surface temperature depending on how the forcing is administered. In an example drawn from Heydt et al. (2014), a direct forcing of $-3W/m^2$ from preindustrial causes $1.3K$ of cooling (panel c), while a direct forcing of $-4W/m^2$ (panel d) followed by a subsequent forcing of $1W/m^2$ (panel e) causes a total $2.9K$ of cooling.

If we start in equilibrium at preindustrial conditions, apply a radiative forcing of $-3W/m^2$, and let the system cool until it reaches equilibrium, we get a cooling of $1.3K$ (1c). If we instead first apply a radiative forcing of $-4W/m^2$ and let the system equilibrate (1d), and then apply a radiative forcing of $1W/m^2$ and the let the system reequilibrate, the system will have cooled by $2.9K$, even though the overall forcing is still $-3W/m^2$.

This circumstance cannot be captured by a polynomial fit of $\Delta T(\Delta R)$, or by any sort of function $\Delta T(\Delta R)$ unless we assume that forcing can only move in one direction, which is inappropriate for glacial/interglacial cycles. On the other hand, a polynomial fit to $N(T, F)$ could capture this behavior easily (e.g. with a cubic function).

2.2 Different values of ΔR are very unlikely to be able to cause the same ΔT

Some of the polynomial fits in the present paper (the red curve in Figure 1 and Figure 3a, and the black curve in Figure 3b) and in Köhler et al. (2015) (Figure 7a, some of the curves in 7e) suggest that different values of radiative forcing ΔR can cause the same temperature change ΔT. I would argue this is almost certainly not a possibility.

Suppose there were two radiative forcings ΔR_1 and ΔR_2 that both cause the same ΔT, where ΔR_1 is less than ΔR_2. Suppose we first impose the ΔR_1 forcing, causing the planet to come to equilibrium after a temperature change of ΔT. Since we are in equilibrium, N is now 0 again. Now suppose that we impose a further $\Delta R_2 - \Delta R_1$ forcing. Since N is now $\Delta R_2 - \Delta R_1 > 0$, the Earth will warm. However, it has to eventually cool again to return to the same temperature it had before the $\Delta R_2 - \Delta R_1$ was imposed in order for ΔR_2 to also cause a total warming of ΔT. In order for this to happen, N has to go from being positive to negative at some point, in order for the planet to start cooling. However, this requires that at some point $N = 0$. As soon as it does, the Earth would be in equilibrium at a warmer temperature than ΔT, leading to a contradiction. Only when the overall zero-dimensional energy balance formalism breaks down (e.g. through having very different spatial temperature patterns with quite different $N(T, F_{pre})$ for the same value of T) could this contradiction be circumvented, an intriguing possibility but not one that I have seen much evidence for.

As a result, these polynomial fits are almost certainly unphysical. It should be noted that for Köhler et al. (2015), this unphysical behavior occurs not in regions of the fit that are extrapolations or have a small amount of data, but in the heart of the regions for which we have data.

2.3 Using temperature-dependence results in more plausible and coherent fits

To illustrate the practical significance of using temperature rather forcing-dependent estimates of sensitivity, I've recreated Figure 7 from Köhler et al. (2015), with polynomial fits performed using both methods. The data was obtained using the relative spatial position of the SVG elements from Figure 7 and then fitting them to the given values in the graph. As a result, the data is not necessarily precisely the same as that used in Köhler et al. (2015), but is good enough for our illustrative purposes. For the forcing-dependent fits (black lines), I've used the degree of the polynomial used in Köhler et al. (2015). For the temperature-dependent fits (blue lines), I fit $\Delta R = -N(T, F_{pre}) = -(\lambda \Delta T + a \Delta T^2 + b \Delta T^3)$:

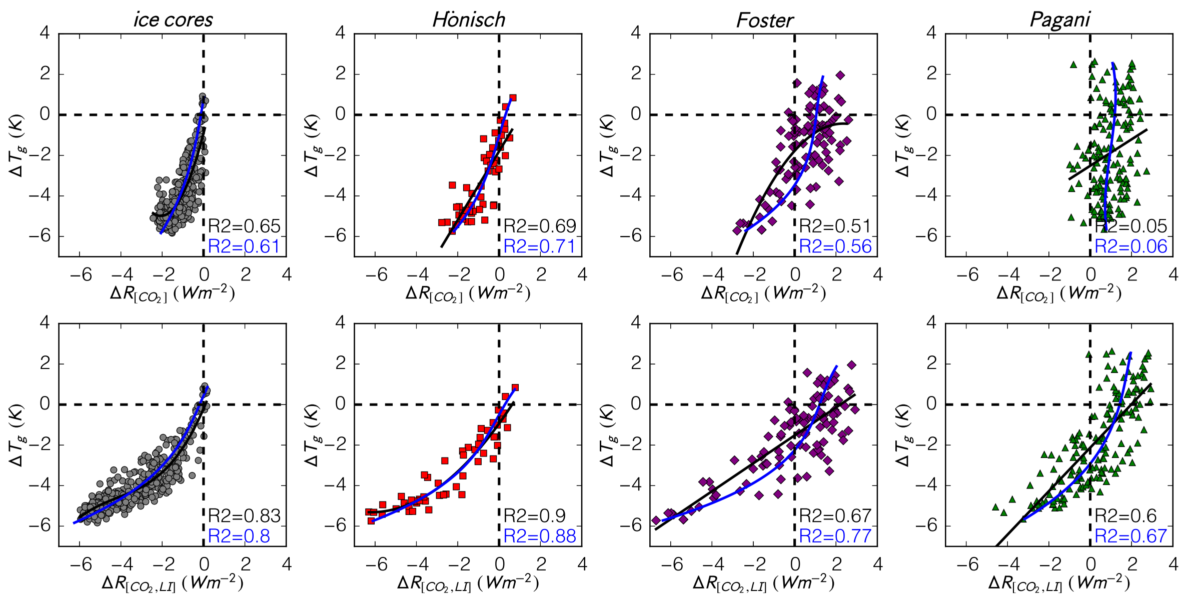

Figure 2: A reconstruction of Figure 7 from Köhler et al. (2015) (see text). The four columns correspond to four paleoclimate datasets. In the top row, land ice changes are considered a feedback on the climate system, while in the bottom row it is considered a forcing. Black lines show reconstructions of the forcing-dependent fits from Köhler et al. (2015), while blue lines show cubic fits to $\Delta R = -N(T, F_{pre}) = -(\lambda \Delta T + a\Delta T^2 + b\Delta T^3)$.

The fits both capture roughly the same amount of variance of their respective dependent variables (ΔR for the temperature-dependent fits, ΔT for the forcing-dependent fits) (though it should be noted that minimizing errors in ΔR, as is done by the temperature-dependent fit, can be much more significant than minimizing errors in ΔT, since $S = \Delta T / \Delta R$). Further, for the Hönlisch and ice core datasets (which are also used in the present article), there is not much difference in the nature of the fits (except for change in the sign of the slope of the $\Delta T(\Delta R)$ slope in the upper ice cores figure, which I argued above was unphysical).

However, the two Foster analyses and the bottom Pagani analysis give quite different answers depending on what method you use. Note that these methods do show improvements in R^2, and do appear visually to better capture the shape of the underlying data. However, there's an even more significant result of this change: for the temperature-dependent approach, **all the datasets which can be fit (all except the upper Pagani figure) agree that the Earth has a lower sensitivity for colder periods**, and in the case where the land ice changes are treated as a forcing rather than a feedback, the sensitivity is significantly smaller. This is seen in Figure 3, which compares the climate sensitivities ($d\Delta T/d\Delta R$) and climate feedback parameters ($d\delta R/d\Delta T$) for the four datasets (different colors) and different approachs (forcing-dependent approach, dashed lines; temperature-dependent approach, solid line). The temperature-dependent approach, with the exception of the top Pagani data, gives a similar estimate of sensitivity and its state-dependence for some of the temperature range, while the forcing-dependent fits disagree about the size of the sensitivity and how it changes in a colder/less-forced world.

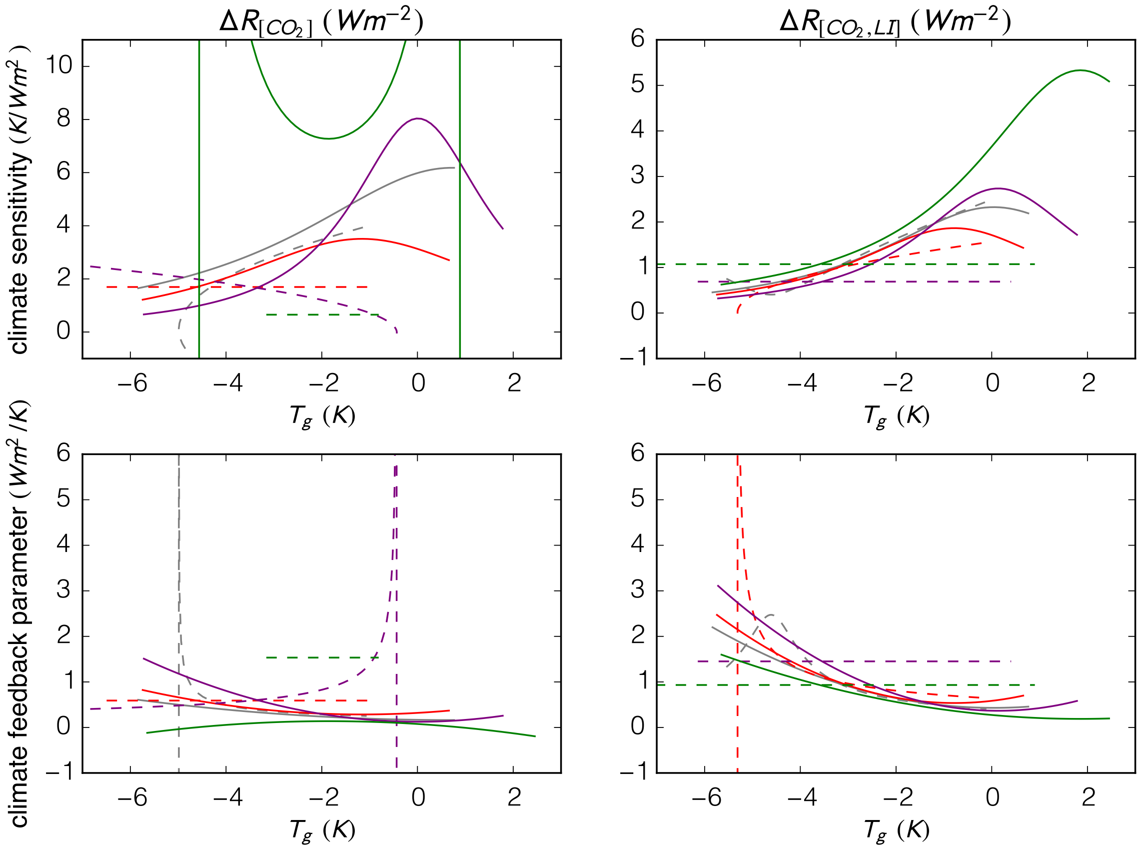

Figure 3: Climate sensitivities ($d\Delta T/d\Delta R$) and climate feedback parameters ($d\delta R/d\Delta T$) for the four datasets (colors from Figure 2) and different approachs (forcing-dependent approach, dashed lines; temperature-dependent approach, solid lines). Aside from the $\Delta R_{[CO_2]}$ Pagani case, for which the fit itself is quite poor, the different temperature-dependent fits roughly agree on the size and change of climate sensitivity and feedback parameter under different temperatures, in contrast with the different forcing-dependent fits.

Köhler et al. (2015) find that the Foster and Pagani datasets disagree with the ice cores and Hönisch datasets as to whether climate sensitivity is state-dependent, and as to the manner of this state-dependence. The temperature-dependent approach instead suggests that these datasets all agree: the climate sensitivity is smaller in climate states colder than present.

3 Conclusions

The authors raise important points about the need to reconcile point-wise and slope-wise estimates of climate sensitivity when that sensitivity is state-dependent. However, I have strong concerns about the fundamental approach taken by the authors with respect to formalizing this phenomenon. The points I raise in my response may seem pedantic, particularly given that for the fits they have included in the present work, the temperature-dependent and forcing-dependent approaches give very similar answers. However, as the examples above show, the distinction is important: the forcing-dependent approach can generate unphysical estimates of the Earth's behavior, and miss important nonlinear behavior. I would argue that their earlier work could have found a coherence between datasets that was missed.

I would either hope that the authors would rethink their use of forcing-dependence, or would justify their approach in a manner that answers the concerns raised in this response. I have a couple of other remarks about the content of this paper: A case study showing the use of the sorts of equations found in Section 2.3 would be useful. It would also be useful to see some of the points saved for the conclusions (4 and 5 in particular) discussed more fully in the body of the paper.

Once more, I feel that this paper is a needed one that builds on important work produced previously by the authors. I look forward to seeing how it develops.

Sincerely, Jonah Bloch-Johnson

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