This manuscript (ms) presents an analysis of pore water $\delta^{18}O$ data from four deep-sea sediment cores (Feni Frift, Bermuda Rise, SW Indian Ridge, and Chatham Rise). Sequential methods of estimation and control theory (a Kalman filter and a related smoother) are used in order to infer, from a quantitative combination of the pore water data with a model describing $\delta^{18}O$ transport in pore fluid, the temporal changes in bottom water $\delta^{18}O$ overlying the cores over the past 100 kyrs. It is concluded that the presence of bottom waters with very high $\delta^{18}O$ during the LGM is possible but not required by the data and the model.

This is a significant contribution. Pore water $\delta^{18}O$ and chlorinity data have traditionally been interpreted to mean that ocean bottom waters during the LGM were both very saline (i.e., exceeding salinities expected from estimated volume of land ice) and very cold (i.e., close to freezing point, a second inference that also involves $\delta^{18}O$ measurements on benthic foraminifera) (e.g., Adkins et al. 2002). The present analysis shows that more data and a better understanding of tracer transport in pore fluids are needed in order to solidify these traditional inferences of glacial oceanography. It is a welcome addition to the study of Miller et al. (2015), who used a different data analysis method that may be considered by some as being more opaque. It echoes results from this previous study as well as from a recent analysis of chlorinity data by the same author (Wunsch 2015). I recommend publication of the present ms, provided that comments (1-4) below can be addressed or are at least discussed in a revised version. More specific points are listed at the end of this report.

MAJOR COMMENTS

1) As in previous work on this topic, the present analysis requires an assumption about the initial profile of $\delta^{18}O$ (here, at t = 100 kyr). In some of the calculations reported in the ms, the initial profile is taken as the measured profile. Whereas this approach appears sensible, it does seem to violate one of the assumptions of the Kalman filter, i.e., the assumption that the errors in the state (here the core $\delta^{18}O$ profile) extrapolated from the model and the errors in the data are uncorrelated at every time (e.g., Bryson and Ho 1975, Applied Optimal Control, Taylor & Francis, 1975; transition from eq. (12.2.10) to eq. (12.2.11) on p. 350 of that textbook). Specifically, if the states extrapolated from the model are calculated from initial conditions that are constrained from terminal data, as done here, then the errors in the extrapolated state for the terminal time and the errors in the terminal data are expected to show some correlation. The author is well aware of the assumption of independence between extrapolated state errors and data errors in the Kalman filter (see, e.g., Wunsch 2006; p. 196). Unless my interpretation of the filter's assumptions is incorrect, I would suggest that the apparent violation of this assumption in the present analysis be addressed or at least discussed in the ms.

2) The present study assumes, again as in previous work, that the $\delta^{18}O$ flux at the core bottom vanishes. The author is quite upfront with this assumption (p. 3, last paragraph). However, as also acknowledged in the ms (p. 3, bottom), the data do not provide evidence for a vanishing vertical gradient of $\delta^{18}O$ and hence of a vanishing vertical $\delta^{18}O$ flux at depth in the cores (fig. 3). As stated in the ms, the problem could be reformulated to determine the $\delta^{18}O$ fluxes at the core bottom, instead of the core top $\delta^{18}O$ values, from the pore water $\delta^{18}O$ data (p. 3, last paragraph). I think that the present study would be even more interesting if it also investigates this other problem, i.e., whether

the pore water $\delta^{18}O$ data could be explained by changes in $\delta^{18}O$ flux (or $\delta^{18}O$ value) at the core bottom rather than by changes in $\delta^{18}O$ value at the core top. In fact, that this could be the case is unclear to this reviewer, since the downward effective velocity induced by the postulated decrease of vertical diffusivity with core depth would compete with vertical diffusion in transmitting upward, along the core, information at the core bottom. It would be useful to test whether this intuition is quantitatively grounded on the 100-kyr time scale in a future version of the manuscript.

3) As mentioned in the above comment, the present analysis assumes that the vertical diffusivity of $\delta^{18}O$ (call it kappa) decreases linearly with depth along the cores. A vertical gradient in kappa induces a vertical effective velocity (p. 2), in this case a movement of $\delta^{18}O$ down-core. This movement tends to propagate downwards the information provided by boundary conditions at the core top. As a consequence, it should exert some influence on the controllability of the system and on the results of this study, although the ms suggests that a uniform kappa would make little difference (p. 4, top). I think that a future version of the ms should clarify the basis for the assumption of a decrease of kappa with depth along the cores. For example, is the assumption based on data of sediment porosity and (or) tortuosity? The paper of Wunsch (2015) does report the measured vertical profiles of porosity for the sediment cores, but whether these measurements truly require kappa to decrease with depth is unclear since kappa also depends on other sediment properties such as tortuosity.

4) Most of the calculations reported in the ms seem to assume that the data error variance (**R**) is "about 10 times larger than the value in Adkins and Schrag (2001)" (p. 9). Could this assumption be justified? If data error variance is poorly understood, I would recommend that calculations with different **R** (i.e., with different data errors for all terminal data, not only for data near the measured $\delta^{18}O$ maximum) be conducted in order to further test the robustness of the results.

SPECIFIC POINTS

Abstract and everywhere in the manuscript: replace "salinity/chlorinity" with "salinity (chlorinity)".

Line 4: "… by them."

p1, line 12: "… that the deep ocean …"

p1, line 16: "Recently, Miller (2014) and Miller et al. (2015) have …"

p1, line 20: "… Adkins and Schrag (2003), Miller (2014), and Miller et al. …"

p2, line 4: "… and the model …"

p2, eq. (1b): There should be a minus (not plus) sign in front of the last term on the left-hand side.

p3, line 1: I think that symbols for chemical elements (here oxygen) are generally not italicized.

p3, line 9: "… is an estimate of their uncertainty".

p3, 1st full paragraph: the last sentence may need to be rephrased (a verb seems to be lacking).

p3, line 24: “… based upon measurements in corals …” (sea level curves are not based on $\delta^{18}O$ values in corals).

p4, line 4: “… to the results (see the Appendix).”

p4, line 19: “… matrices **A**, **B**, …” (drop comma after “matrices”).

p5, line 16: “Lagrange multipliers, or adjoint, methods and the Rauch…”

p5, line 24: I think that the observability matrix for the present system would be more conventionally defined as the partitioned matrix (e.g., Gelb et al. 1974):

$$O = [0|0| \ldots |(A^T)^{t_f - 1} I_{2M}],$$

which has a similar form as the controllability matrix (5).

p6, line 9: “… 100,000 yr …”

p6, line 24: “…$\delta^{18}O_w$ distribution …” (drop comma).

p7, “… likely connected to the extreme volatility of dynamical properties in the equatorial Pacific Ocean and is not further discussed here”: this could be elaborated or dropped.

p7, line 17, and everywhere in the manuscript: replace “… and/or …” with “… and (or) …”.

p8, line 18: please define the matrices $\mathbf{P}_0$, **Q**, and **I**.

p8, line 20: “… k decreases linearly with core depth from … at z = h to … at z = 0 m.”

p9, line 3: “… conditions, figure 8 shows …”

p9, line 11: “… and the terminal data uncertainty was strongly reduced in the vicinity …”

p9, lines 22-23: “… (about 10 times .. in Adkins and Schrag [2001] and now meant … error), $\mathbf{P}_0$ = …”.

p9, last sentence: is it meant “It would appear that this record is not consistent with the prior d18O profile at 100 kyr within its stipulated error bars”? Some clarification would be useful.

p10, section 3.3: Please also explore cases with a sea-level prior and zero initial conditions, and briefly describe the results in the ms.

p10, section 3.4: Please also explore the case with zero initial conditions and briefly describe the results in the ms.

p10, line 23: “… rough summary would include: …”

p10, line 4: “(1) Physical transport of $\delta^{18}O$ is one-dimensional (vertical)”. Although this seems to be common in the literature, I would suggest not to use the division sign in non-mathematical expressions, such as “Physics/chemistry”, “diffusivity/porosity”, “Advection/diffusion”, etc.

p11, line 23 (in Appendix): “… from the bottom of the core at z = 0 m to the top of the core at z = h”. In the development following (A1), I would suggest first to introduce the change in variable ($\zeta =

…”$) and then to set $c = \hat{c}(\zeta) …”. The ordinary differential equation between (A1) and (A2) should have ζ as the sole independent variable.

p14, line 16 (Reference list): have “Olver …” starting on a separate line.

Fig. 1 could be enlarged.

Caption of Table 2: “… $\boldmath{x}_t$ …”

Figs. 7-16 could all be enlarged.

Caption of fig. 7: “… with k linearly increasing from … at the core bottom (z = 0 m) to … at the core top (z = h).” In panel (c), I interpret the solid line as the difference between the filter estimate of d18O and the measured δ^{18}O, but I am not sure. Panel (e) could be zoomed in, perhaps on the last kyr, to better see the changing control and its estimated error near t = 0 kyr.

Caption of fig. 10: Please define **R** in the main text.

Fig. 17: the legend indicates that the solution with constant kappa is a numerical one, whereas the caption indicates that it is an analytical one. Please clarify. The initial conditions and the time for which the solutions are displayed could be specified.

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