**Climate
of the Past
Discussions**

# *Interactive comment on* "Multi-periodic climate dynamics: spectral analysis of long-term instrumental and proxy temperature records" *by* H.-J. Lüdecke et al.

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Point 1. and 2.
For a better understanding of our point, let us first cite (Lennartz and Bunde , 2011): "Long-term correlated records $\{x_i\}, i = 1, ..., N$ can be characterized by the power spectral density $S(f) = |x(f)|^2$, where $\{x(f)\}, f = 0, ..., N/2$ is the Fourier transform of $\{x_i\}$. With increasing frequency f, S(f) decays by a power law $S(f) \sim f^{-\beta}$, where $\beta > 0$ characterizes the long-term memory. Records with $0 < \beta < 1$ can be also characterized by an autocorrelation function $C(s) = \langle (x_i - \bar{x})(x_{i+s} - \bar{x}) \rangle / \sigma^2$ that decays by a power law $C(s) \sim s^{-\gamma}$."

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Further, $\gamma = 1 - \beta$, and for the Hurst exponent $\alpha = (1 + \beta)/2$ holds. Because M6 and SPA are indeed long-term correlated, the confidence curves in Fig.3 of our paper have to consider this decisive long-term correlation. Contrarily, AR(1) is a model for a time series that has essentially nothing to do with the autocorrelation C(s) of a time record. The only coincidence of AR(1) with autocorrelation is that for an AR(1) process (red noise) the autocorrelation function C(s) corresponds to the C(s) of a long-term correlated record with $\alpha \approx 1$. The appropriate method for the evaluation of the confidence curves in long-term correlated records with $\alpha$ values $\neq 1$, therefore, is Monte-Carlo with surrogate records of the appropriate Hurst-exponents. Only for SPA the confidence curves could have been (roughly) evaluated with an underlying AR(1) process (method of Mudelsee) - roughly because SPA has an $\alpha = 0.9$, not an $\alpha = 1$. For M6 with an $\alpha = 0.58$, AR(1) is not appropriate.

DFA is used in our paper only for the evaluation of Hurst-exponents of M6 and SPA, which are needed for the construction of the DFT confidence curves with Monte-Carlo. We do not call AR(1) generally unrealistic. For the evaluation of the DFA confidence curves of SPA, AR(1) would be roughly realistic, however, for M6 it is not.

It is no problem to include a description of DFA in the paper. However, DFA is only a minor point here. Therefore, we propose to ask E. Zorita and Referee#1 for their opinion in this point. If they also wish a DFA description we would prefer to place it in the appendix not to interrupt the major text.

Point 3.
The log-log plot of the DFA result yields the $\alpha$ value (s. attached Fig. 1). Generally, this plot is straight in the middle of the s interval but bends at the beginning and the end of it. Obviously the gradient ($\alpha$ value) of the log-log-plot depends somewhat where one begins and ends with the s interval. There exits no prescription of the s interval for the gradient evaluation. As a consequence, one can estimate for M6 an error of

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$\alpha = 0.58 \pm 0.2$.

Point 4.

We addressed this point in our revised version CPR but we do know whether Referee#2 has noticed our corrections. If "yes" and if no agreement we ask Referee#2 for an appropriate further comment.

Point "Additional remark of Referee#2"

We agree completely. Furthermore, we like to stress that the distinction of the Lennartz/Bunde method between "natural" and "unnatural" is statistical - by observing the appropriate significance limits. Perhaps our answer to Mudelsee was too short and needs an expansion. We wrote: "Natural means that the record inhibits only natural trends, unnatural that it inhibits an external trend". This is correct. As an example, a long-term correlated record without an external trend is by this definition "natural". Clearly, the method of Lennartz/Bunde can not decide whether a real record is natural or unnatural. Only probabilities can be given and it is even a little bit more complicated. In an (improbable) event, it could happen that a strong (= very improbable) negative NATURAL trend cancels a positive external trend - with the result, that for this record a strong probability is evaluated that the record is natural. Therefore, the answer "natural" of the Lennartz/Bunde method is more moderate like this: If "natural" is evaluated by the method one is only NOT FORCED to assume an external trend. On the other hand, if the probability for an external trend is very strong (or the other way around, if the probability for "natural" is very low), then one MUST assume an external trend within the significance limits.

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## References

Lennartz, S. and Bunde, A. Distribution of natural trends in long-term correlated records: A scaling approach. Phys. Rev. **E84**, 021129 (2011)

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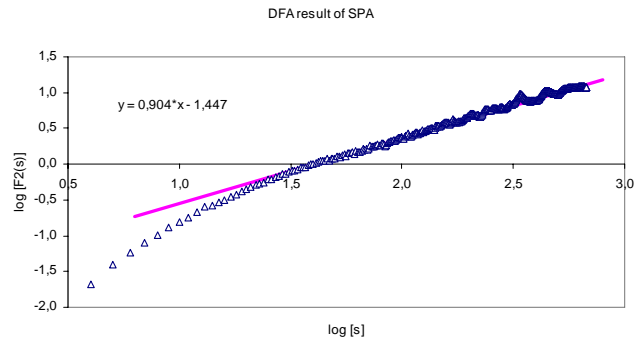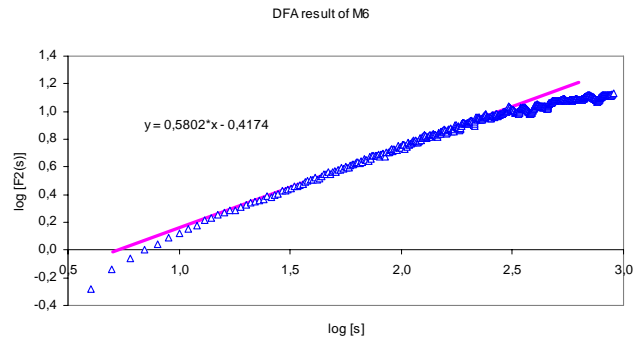DFA result of M6

$y = 0.5802*x - 0.4174$

DFA result of SPA

$y = 0.904*x - 1.447$

**Fig. 1.**

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