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Climate of the Past Discussions

Mapping uncertainties through the POM-SAT model of climate reconstruction from borehole data

M. G. Bartlett

Department of Physics, Hollins University, Roanoke, VA, 24020, USA

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Correspondence to: M. G. Bartlett (marshall.bartlett@gmail.com)

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Abstract

Borehole temperature-depth profiles contain information about the ground surface temperature (GST) history of a locale and can be employed in climate reconstruction. The borehole method of climate reconstruction assumes that the dominant heat transport mechanism in the upper few hundred meters of the earth's crust is conduction; mathematically, conduction is a compressive (information losing) mapping from the temperature-time space of GST to the temperature-depth profile (T-z). Because the mapping is compressive, multiple GST histories can map into the same T-z profile; the solution suffers from non-uniqueness. One means of dealing with the non-uniqueness problem is to limit the number of parameters sought in the solution space. However, even when only a single parameter (the pre-observation mean GST, or POM) is sought in the inversion, a certain amount of a priori information must be introduced prior to inverting for a GST history. In the POM-SAT method, a priori information introduced includes the surface-air temperature history (SAT), the thermal diffusivity of the conductive medium, and the reducing parameter used to remove the background (non-climatic) heat flux. I perform a set of Monte Carlo analyses to investigate how uncertainties in these a priori model parameters are mapped into the solution space of the POM-SAT method of climate reconstruction. Results indicate that uncertainties in the thermal diffusivity are generally reduced by an order of magnitude when mapped to the POM. Uncertainties in the SAT time series are approximately equivalent in magnitude to their projection onto the POM. However, uncertainties in the adjustment for the background (non-climatic) thermal regime are magnified by an order of magnitude in the POM solution-space. These results suggest a degree of prudence should be exercised in interpreting surface temperature histories from reduced borehole data.

1 Introduction

Borehole temperature-depth profiles contain significant climatic information and have been employed in climate field reconstructions (Pollack and Huang, 2000; Harris and Chapman, 2001; Beltrami and Bourlon, 2004). They have a number of advantages over "proxy" methods of reconstruction including broader geographic extent, a direct physical signal of the temperature component of past climate, and a comparative simplicity in the processing of their data. The disadvantages of borehole-climate reconstructions are all ultimately related to the fact that the climatic information in boreholes is governed by the physics of diffusion; consequently, climatic signals of various periods are superimposed and attenuated with depth in the ground. The inversion of borehole temperature-depth profiles for surface temperature-time reconstruction is, mathematically, an ill-posed problem: many surface temperature-time fields will result in identical temperature-depth profiles. Addressing the non-uniqueness in the solution space necessitates the introduction of some amount of a priori information to place limits on the charcter of the reconstructed surface temperature field; generally, these a priori limitations take two forms: limits on the number of parameters inverted for, and assumptions about the physical and initial conditions. Mathematically, the purpose of these limitations is to transform the ill-posed, under-determined inverse problem into an over-determined inverse problem with a unique solution.

Of the attempts at a solution to the borehole-climate inverse problem, the POM-SAT inversion scheme is the most restrictive in terms of the degrees of freedom it allows the solution. In the POM-SAT method, a single parameter (the pre-observational mean surface temperature, or POM) forms the solution. Additional a priori information required by the method are a value of the thermal diffusivity of the ground, a surface air temperture (SAT) time series which is employed as a proxy for the surface ground temperature (SGT), and a somewhat subtle assumption that the background geothermal component of the T-z profile can be effectively isolated from the climatic disturbances to the profile. With these three assumptions, the method seeks a single step change

CPD
8, 2503–2518, 2012

in temperature immediately prior to the beginning of the observational SAT record: the pre-observational mean temperature (POM). The produced surface temperature-time series (the POM plus the SAT) is the unique solution to the inversion problem under the a priori constraints of the method. The POM-SAT method has been employed in hemispheric-scale reconstructions of the climate field over the past 1000 yr (Harris and Chapman, 2001; Harris, 2007).

In this paper, I examine the degree to which uncertainty in the three a priori assumptions (thermal parameterization, SAT profile, and our ability to decouple the climatic and geothermal components of the T-z profile) of the method translate into uncertainty in the POM solution space of the method. I employ a Monte Carlo analysis for each of these three, tracing how a proscribed degree of uncertainty in the parameter space translates into uncertainty in the solution space.

2 Methodology

The POM-SAT methodology is well described by Harris and Chapman (2001) and in Harris (2007). A solution to the inverse problem (solving for the optimal POM given the borehole temperature-depth profile, a thermal diffusivity, and a SAT time-series) is outlined in these papers; that solution is implemented here for use in the Monte Carlo analysis.

Synthetic borehole temperature profiles are constructed in the following manner. I begin by constructing a surface temperature-time series consisting of a synthetic SAT field and a prescribed POM step. This is diffused into the ground, using a prescribed value of thermal diffusivity. A chosen background geothermal gradient is then added to the results to produce a synthetic borehole temperature-depth profile.

For each of the three a priori assumptions to the method, I produce a Gaussian profile of 10 000 realizations of the parameter centered on the chosen value used to create the synthetic borehole profile. Standard deviations are chosen in terms of a percentage variation from the prescribed value and are selected in a range that is representative

CPD
8, 2503–2518, 2012

of our uncertainty of these values for real data. For diffusivity, the standard deviation is 10 % of the assumed value; this is representative of diffusivity variations within a given rock type. For the SAT profile, uncertainty is derived from the uncertainty in individual annual values of SAT. Assuming these are 10 %, 10 000 random realizations of a 150-yr SAT profile produce uncertainties in the mean slope of the SAT profile of 3 %. The background gradient uncertainty can be estimated by examining the data in the University of Michigan's Borehole Temperatures and Climate Reconstruction Database (Huang and Pollack, 2009). For each borehole, the raw observations and an estimate of the mean thermal gradient (called the "reducing parameter" – arrived at from a linear fit to the data below 160 m) is available within the database. Under the assumption that there is no long-term climatic information below 160 m in the boreholes, residuals below this level should be Gaussian distributed about zero. While this does not prove to be the case (see Sect. 4), examination of all of the residuals do provide us with an estimate of the degree of uncertainty associated with the reducing parameter: it appears to be known to within a standard deviation of 1 % for most of the boreholes.

These estimates of the uncertainty levels of the three a priori assumptions are used to produce synthetic realizations which are then employed in the inversion. Each of the three assumptions is analyzed individually for its impact on the POM produced by the inversion. Since the inversion method is linear with respect to these three a priori parameters, the cumulative uncertainty from varying all three parameters will be the summation of the uncertainties from the individual parameter assessments.

3 Results

Figure 2 illustrates the compression of the uncertainty range that occurs for the diffusivity. An initial 10 % deviation in the diffusivity values maps into a 2.8 % uncertainty in the POM solution space. This compression of uncertainty results from the conductive nature of the medium: altering the diffusivity value simply compresses the changes into a narrower depth range for a given time interval. This narrowing is least severe at the

CPD
8, 2503–2518, 2012

Title Page
Abstract | Introduction
Conclusions | References
Tables | Figures
◂◂ | ▸▸
◂ | ▸
Back | Close
Full Screen / Esc
Printer-friendly Version
Interactive Discussion

top of the hole, growing as the square-root of depth as the signal moves down the hole. Since the entire climatic signal is still contained within the hole, the POM is left relatively undisturbed. This result is consistent with recent work done on the POM-SAT method (Harris, 2007).

The result of mapping the 3 % uncertainty in the SAT values through the inversion is less benign. The initial 3 % uncertainty in the mean slope of the SAT time series produces an equivalent 3 % uncertainty in the POM results. This result makes sense when one considers that the borehole data is most "climatically disturbed" near the top of the hole, the same area the SAT is meant to explain. Any discrepancy in the SAT time-series' ability to explain this climatic signal will be pushed into the lone degree of freedom available to the inversion, the POM.

Uncertainty in the reducing parameter of 1 % produces a marked increase in the uncertainty of the POM of 10 %. This order of magnitude increase in uncertainty is readily explained by the fact that a small deviation in the reducing parameter (which is, effectively, a slope), magnifies the adjustment that is being made to each data point as one proceeds from the bottom of the hole (where the reducing parameter is estimated) to the top (where the climatic information is located). Small deviations in the reducing parameter can produce large changes in the offset in the upper portion of the profile, radically adjusting the climatic disturbance found there. This can lead to the reduced borehole's climatic disturbance having a markedly different slope than the SAT time series which produced it; an effect which the inversion compensates for via the POM.

The cumulative uncertainty of the method (assuming uncertainty levels in the three a priori parameterizations as above) is on the order of 16 %. For a POM step change on the order of 1 K, this translates into a two-sigma error bound of ± 0.3 K. This is a three-fold increase in the estimate of uncertainty in the reconstruction technique over that based on a doubling of the RMS misfit indicated in Harris and Chapman (2001). Importantly, the majority of this increase in uncertainty in the POM is transferred from the small degree of uncertainty in the reduction parameter, a source not considered in the later work on the POM by Harris (2007).

4 Discussion and conclusions

Transferring the borehole temperature-depth profiles to "reduced-temperature space" via the subtraction of a linear trend below some depth level (160 m) clearly has an adverse effect on the level of uncertainty in the POM solution. Figure 2 illustrates the data reduction process by which the geothermal gradient below 160 m is removed from observed profiles. Figure 3 shows the typical residual structure for one of the boreholes in the University of Michigan's Borehole Temperatures and Climate Reconstruction Database after removal of the geotherm. If there were no climate signal below this level, the expected residual structure would be Gaussian, showing no spatial correlation and only reflecting random noise process (instrumentation, etc.). For the particular borehole illustrated in Fig. 3 (and for most in the Michigan database), the residuals are clearly non-Gaussian; climatic perturbations below the 160 m level lead to a high degree of spatial correlation in the residuals. For the case illustrated, the linear trend removed via the "reducing parameter" is effectively an intermingling of both the background geothermal gradient and some (but not all) of the long-period climatic signal in the temperature-depth profile.

That this is the case generally across the boreholes used in the Harris and Chapman (2001) Northern Hemisphere climate field reconstruction is apparent when one examines the composite residual distribution for points below 160 m across all of the reduced temperature profiles used in that study. If each borehole's residual structure were random, the expected composite residual distribution should be Gaussian, a result guaranteed by Cramér's theorem (Cramér, 1936). Figure 4 illustrates the composite residual structure across all of these boreholes, together with a peak-matched Gaussian distribution function for comparison. The clear departure of the residuals from the Gaussian (witnessed by the larger than Gaussian residual population far from the mean) is indicative of the fact that there remains significant climatic information in the region from which the reducing parameter is being derived in each profile. As a result, the reduced temperature data from which the POM is derived can best be interpreted

CPD
8, 2503–2518, 2012

as residuals relative to some long-term, linear climatic trend; unfortunately, it is precisely the inference of these long-term trends that are the goal of borehole climate reconstructions.

What climatic information can be gleaned then from the borehole data? The central problem is data quality, which, for climatic information retrieval from a diffusive medium, means depth. At some depth, the long-term climatic signatures diffused into the ground will be below the level of the instrumental uncertainty of the measurements (typically on the order of 10 mK). At this level, any residual climate information will become swamped by the truly random sources of noise in the data. However, for most of the boreholes in the University of Michigan's Borehole Temperatures and Climate Reconstruction Database, the maximum depths are less than 500 m and this criterion is not met; the boreholes are simply too shallow to permit a robust estimate of the background, non-climatic geothermal gradient for the purposes of climate signal isolation. Moving these boreholes into "reduced-temperature space" involves the removal (or addition) of an unknown amount of climatic "signal" in addition to the adjustment being made for the background geothermal "noise". Consequently, a relatively large amount of uncertainty (compared to the uncertainty of the fit parameter) is introduced into the climatic inverse problem's solution through the pre-inversion transformation of the data.

While this work indicates that the uncertainty for reconstructions of individual temperature-depth profiles are on the order of 15–20 %, doesn't statistics come to the rescue of multi-borehole reconstructions? While it is true that for a random distribution of uncertainties in multiple measurements, the overall uncertainty is inversely proportional to the square of the number of measurements (or boreholes, in this case), this result only holds if the distribution of the uncertainties in the ensemble of measurements is Gaussian. If there is a consistent bias in which all of the boreholes reconstructions are skewed in the same direction, the uncertainty of the mean is no longer "hammered-down" by taking more measurements into the average. For the ensembles of boreholes used in regional or hemispheric climate reconstructions, the diffusivity uncertainties are likely to be Gaussian, showing no overall correlation between boreholes.

CPD
8, 2503–2518, 2012

The SAT uncertainties are likely to be roughly correlated: fewer SAT stations globally in the past will lead to larger uncertainties in the 19th and early 20th century for almost all borehole locations in the ensemble. The largest uncertainty correlations, however, are likely to be found in the reducing parameter estimates for each borehole in the ensemble. Since the long-term climate influences on the borehole temperature-depth profiles are more likely to show spatial scale correlation, the residual curvature in holes over large regions are likely to all bend in the same direction within the depth range over which data reduction is being performed. Consequently, hemispheric-scale averaging such as that of Harris and Chapman (2001) is unlikely to result in the inverse square-root benefit that would be had if the boreholes truely exibihit Gaussian noise in their deeper sections. The question of exactly what the error bounds are on such a reconstruction will require a deeper insight into the degree of correlation between the uncertainties in the a priori inputs between the boreholes used in the reconstruction. Such a reconstruction with a robust handling of uncertainties remains a taks for the future.

Recent work by Beltrami et al. (2011) using a different inversion scheme (singular value decomposition for a suite of 20 step changes in temperature over the past 1000 yr) have reached a similar conclusion regarding the role of uncertainty in the reducing parameter – robust estimates of the GST history are only achieved by using boreholes of signficant depth. What the present study adds to Beltrami et al.'s work is a recognition that limiting the degrees of freedom for the solution space does not mitigate the problem of poor constraint on the reducing parameter. The uncertainty associated with the separation of deep hole background gradient from long-time climatic influence is persistent. Indeed, even Beltrami et al.'s suggestion that what are needed are holes in the 500–600 m depth range will be insufficient to guarantee an accurate separation of geotherm from climate signal if millenial- to glacial-scale climatic changes have imprinted curvature in the holes at depths of >500 m whose magnitudes surpasses the instrumental noise level.

CPD
8, 2503–2518, 2012

In recent work, Rath et al. (2012) suggest a work around to the reducing parameter issue by adding an additional a priori piece of inversion to the inversion: an estimate of the long term climatic changes the hole has experienced in the past. This estimate could then be used to created a diffused signal in the ground representing the millenial- to glacial-scale changes; a signal which could be removed from the borehole data before inversion in order to isolate the recent climatic changes. This idea merits further investigation; in light of what is presented here, particular effort should be devoted to investigating how uncertainty in our estimates of the past glacial-scale climatic changes a site has experienced would translate into uncertainty in the reconstructed GST history for the recent past based on the "long-term adjusted" profile. I therefore conclude the following:

- The POM-SAT method of geothermal climate reconstruction is relatively insensitive to variations in the thermal diffusivity of the ground and is moderately sensitive to uncertainties in the SAT record.
- The method is very sensitive to the value of the reducing parameter used to isolate the climate signal from the background geothermal gradient. The reducing parameter amounts to the method's discrimination between climate signal and background noise.
- The published Northern Hemisphere reconstructions based on the POM-SAT method estimates the uncertainty in the POM based on the statistical distribution of the areally weighted solutions for the holes in the Northern Hemisphere. This does not account for any of the inherent, data-related uncertainties detailed here and is likely an underestimate. Based on this work, an additional uncertainty of 15–20 % of the value of the POM is present in these reconstructions.
- The uncertainty in reconstructions based on geothermal data is ultimately a data quality issue. Methodological changes are unlikely to further tighten the reconstructed climate field from boreholes forany reconstruction methodology that

CPD
8, 2503–2518, 2012

works in the reduced-temperature space (which includes all of the major reconstruction methodologies to date). What is needed is a clearer differentiation in the data between climate signal and the background geothermal noise. Without access to deeper measurements, this differentiation is difficult to achieve reliably, though the introduction of additional a priori information (estimates of the long-term climatic changes) merit further study.

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CPD
8, 2503–2518, 2012

Mapping uncertainties through the POM-SAT model

M. G. Bartlett

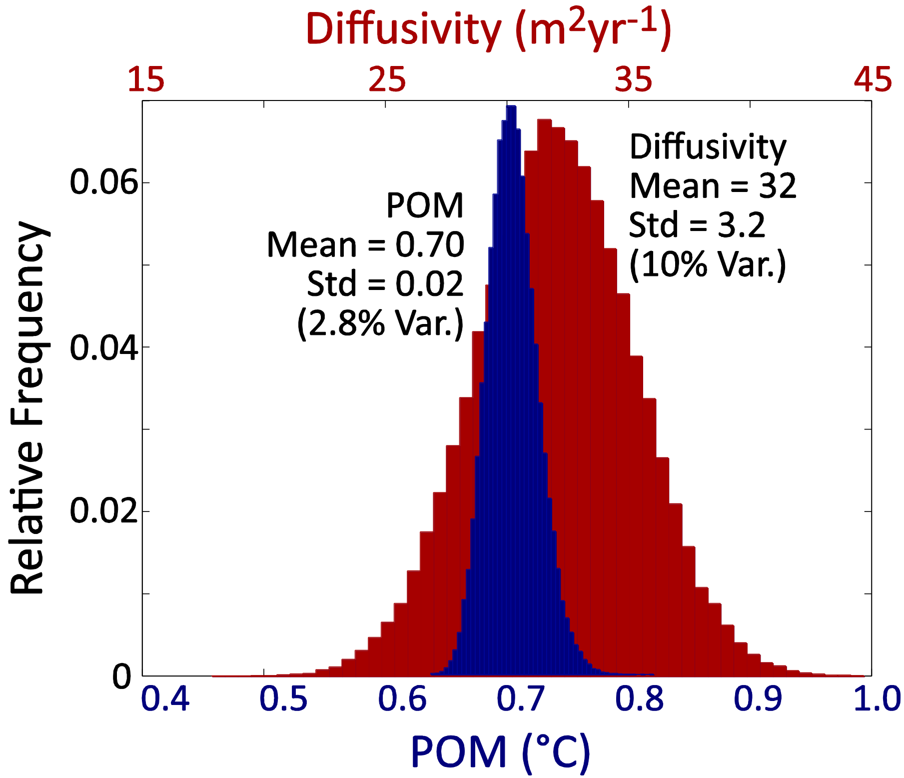

Fig. 1. Mapping of diffusivity uncertainty distribution into uncertainty distribution for the POM. A Gaussian distribution with standard deviation of $3.2\,m^2\,yr^{-1}$ about the mean diffusivity of $32\,m^2\,yr^{-1}$ (STD is 10 % of mean) is compressed into an uncertainty in the POM result of 2.8 % of the mean.

CPD
8, 2503–2518, 2012

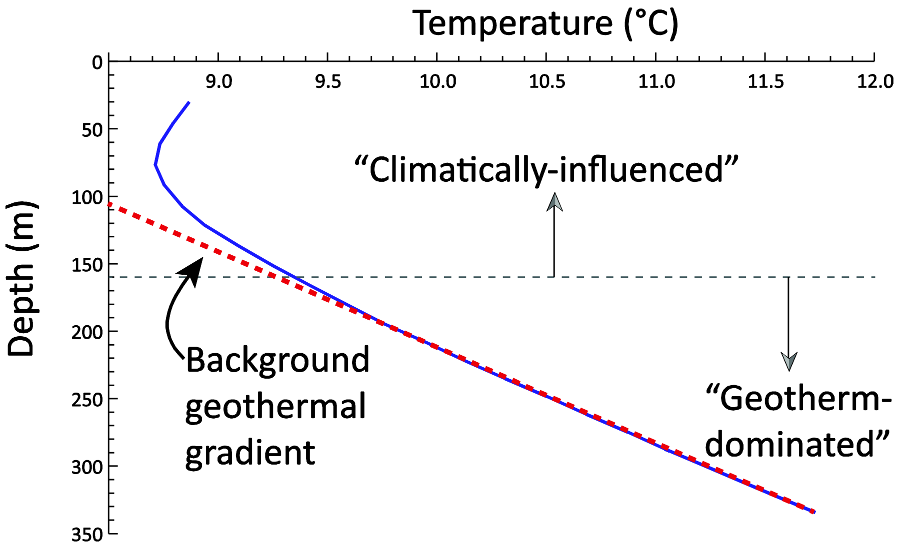

Fig. 2. Example of transforming a borehole into reduced-temperature space prior to inversion for the POM via the removal of the background geothermal gradient. The depth over which the background geothermal gradient is calculated (> 160 m) effectively divides the borehole into "climatically-influenced" and "geotherm-dominated" portions.

Discussion Paper | Discussion Paper | Discussion Paper | Discussion Paper |
CPD
8, 2503–2518, 2012
Mapping uncertainties through the POM-SAT model
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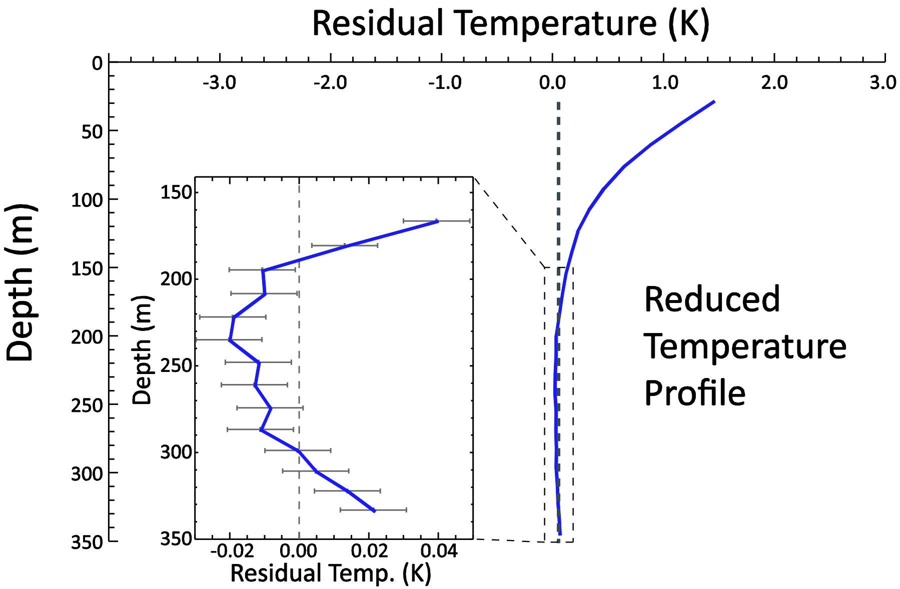

Fig. 3. Details of the post-reduction temperature profile, including the high-degree of spatial structure in the geotherm-dominated portion of the borehole below 160 m. This spatial structure is indicative of long-period climatic perturbations. Their influence has been folded into the reducing parameter estimate.

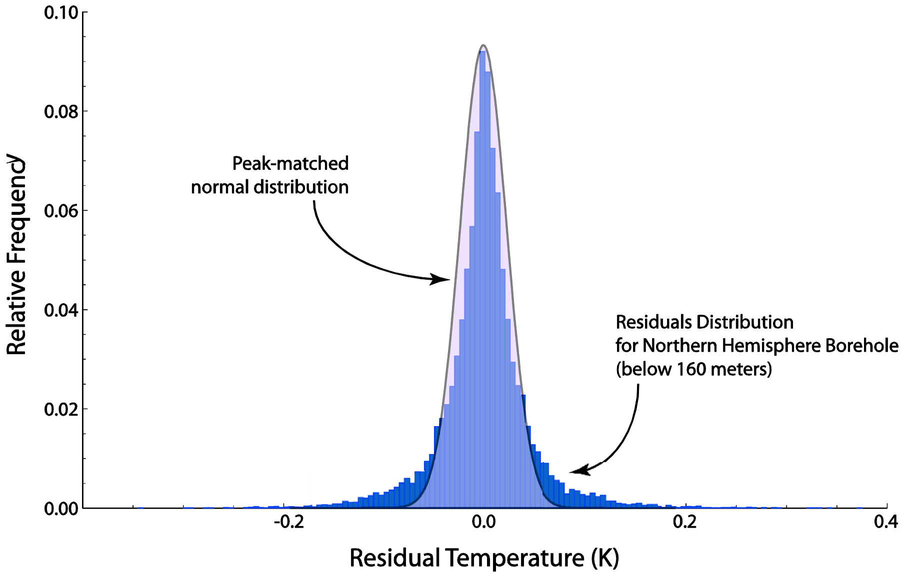

Fig. 4. Comparison between a peak-match Gaussian distribution to that of residual temperatures for all Northern Hemisphere borehole observations below 160 m following data reduction to remove the geothermal gradient in each profile. The significant number of residuals that lie outside the normal distribution is indicative of spatial correlation in the reduced profiles below 160 m.

CPD
8, 2503–2518, 2012