

## ***Interactive comment on “Glacial cycles: exogenous orbital changes vs. endogenous climate dynamics” by R. K. Kaufmann and K. Juselius***

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Response to Reviewer #3

C40 An autocorrelation of order one is assumed for all the time series. This is not really correct. We assume a vector autocorrelation process of order 2 conditional on the exogenous drivers. This implies that the model associates current temperatures, say, with once and twice lagged values of it self and of all the other endogenous variables as well as the exogenous drivers (and similarly for all the other endogenous variables). When a VAR model of order 2 is converted to a CVAR model, equation (1) contains This allows for a very rich and complex dynamic adjustment structure. As

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climate variables (similarly as economic variables) exhibit pronounced persistence (i.e. are strongly autocorrelated) a reformulation into differences and linear cointegration relations is a useful device to eliminate as much as possible of the multicollinearity that would otherwise produce less reliable results. Differences and cointegrated relations are often close to being orthogonal while the levels of variables often are strongly correlated. The latter leads to a problem of multicollinearity and can lead to spurious effects as well difficulty to see real effects when they are there. This discussion is added to the section that describes the statistical model (section 2.2) after equation (1).

C410 First, I am not convince that the CVAR method alone is able to solve the complex issues addressed by the authors, at least not in the manner the authors conducted their experiments. In my opinion, a single equilibrium state between climatic variables that would be valid on such a long time period (400kyr) is questionable. This comment indicates that we have not explained the nature of the cointegrating relation clearly. There would be no single equilibrium state. Rather, the cointegrating relation gives the equilibrium state for a given set of values for climate variables. In this case, the values for the set of exogenous variables for solar insolation imply a single equilibrium state for the set of endogenous climate variables. So, given the change in the set of solar insolation variables over the 390Kyr, there would be 390 different equilibria. We explain this by adding the following sentence to the description of the statistical model: “These cointegrating relations then define a unique equilibrium for each endogenous climate variable as a function of the exogenous variables. Under this condition, the rank of  $\Pi$  is equal to the number of endogenous variables.”

C411 CVAR method as a pure stochastic method (no physics) has few chances to lead to a physical equilibrium state. Although the CVAR model is not based on physical first principles, it contains variables that represent the physical state of various aspects of the climate system and these endogenous variables do come into a physical equilibrium state, and is a unique solution for a given set of exogenous variables for solar insolation To illustrate, we run Model 4 to equilibrium using the values for the solar

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forcings at the LGM. As indicated in the figure, this suite of solar variables generates an equilibrium value of about  $-14^{\circ}\text{C}$ . Then we raise the concentration of  $\text{CO}_2$  by 180 PPM. The model comes to a new equilibrium at a higher Antarctic surface temperature (about  $-3^{\circ}\text{C}$ ). This change implies  $\Delta T_{2x} = 5.5^{\circ}\text{C}$  when changes in Antarctic temperature are corrected for global temperature (Masson-Delmotte, 2006:2010). This estimate for  $\Delta T_{2x}$  is consistent with previous estimates and includes many feedbacks, including changes in land ice, sea ice, sea level, etc. Indeed, we may be able to manipulate the model to determine the size of these various feedbacks. As described in the original Conclusion, we plan to write a paper that uses the fully identified CVAR model to evaluate  $\Delta T_{2x}$ .

C411 In particular, autoregression order 1 for all the data series is questionable on long time periods, regarding the non-linearities expected in the climate system dynamic and the wide range of characteristic time scales for the feedback mechanisms. We are confused by these comments. First of all we do not assume an AR(1) for all variables as explained above. What we assume (after testing) is that the vector process is unit root nonstationary which is what the pronounced persistence typical of climate data suggests. To postulate that climate time series are stationary seems inconsistent with these features. For example, the simplest data generating process for a nonstationary variable is  $Y_t = Y_{t-1} + \mu_t$  where  $Y$  is a climate variable (e.g. ice volume,  $\text{CO}_2$ ) and  $\mu$  is a normally distributed random error term. Most people would agree that the best forecast (guess) of temperature, say, for the next period ( $t+1$ ) would be the temperature value of that variable in the previous period ( $t$ ) as opposed to the average value of the variable ( $\bar{Y}$ ). The former would be an optimal forecast for a random walk variable the latter for a stationary variable. The time series plots in figures 1-3 show how temperature,  $\text{CO}_2$ , and ice drift away from their sample average for extended periods of time, which suggests that the random walk hypothesis is consistent with the data.

C411 No statistical test is reported (Augmented Dickey-Fuller statistic) to test this assumption. That we have not discussed the testing of stationarity is a point that needs to

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be clarified. We have added a section that describes that the trace test (a multivariate Dickey-Fuller type of test) has been used and why the univariate augmented Dickey-Fuller test, such as the augmented statistic, is not appropriate in this case.

The number of common driving trends is equal to  $p-r$ . The rank is determined based on the so-called trace test, which can be thought of as a multivariate Dickey-Fuller test. Note, however, that a univariate Dickey-Fuller test of each variable is an inefficient procedure that frequently leads to misleading and internally inconsistent results. It should, therefore, not be used in a multivariate context.

Later in the text, we present evidence that the CVAR models contain characteristic roots which are close to the unit circle and that the trace statistic indicates that the long-run matrix has reduced rank, both of them suggesting that the time series are nonstationary.

That paleoclimate variables seem to evolve in a nonstationary manner over time as can be verified both by a visual inspection and by statistical tests. The characteristic polynomial of the estimated CVAR models contained roots which were very close to the unit circle (e.g. Model 4 Rank =10, root = 0.98). In this case, inference based on the assumption of stationarity would be misleading and more precise results will be obtained by assuming that our paleoclimatic variables are unit root nonstationary (Johansen, 2006).

C411 In my opinion, the CVAR method might be relevant only if applied to very specific climatic data and on short time intervals. We strongly disagree with this comment—the ability to evaluate statistical model as good or bad improves as the sample period increases. For example, Kaufmann and Stern (2002) in an earlier application of the CVAR model used it to estimate the relation between surface temperature and radiative forcing over the last 150 year using annual data. Ironically, one of the comments received from physical scientists is that the model's reliability would have been improved by extending the sample period (which, unfortunately, was not possible). Also,

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we disagree that the model does not include 'very specific climate data.' But to avoid any misunderstanding, we add the following explanation of how our data can be used to test various hypotheses about the drivers of climate cycles (starting after line 16 on page 589 in italics):

Surface temperature, atmospheric CO<sub>2</sub>, and ice volume are among the most commonly used proxies for glacial cycles and are thought to be directly related to one another, and so their inclusion requires little justification. Methane is included because changes in its concentration are responsible for the second largest change in radiative forcing due to greenhouse gases (Kohler et al 2010). Sea surface temperature is included to proxy changes in stratification that affect convection and buoyancy flux (e.g. Watson and Garabato, 2006), which may affect the exchange of CO<sub>2</sub> between the atmosphere and ocean. Finally, sea level is included because it affects the Earth's albedo and there is strong interest in understanding its long-term determinants (vonStorch et al, 2008).

Indirect linkages among the six climate variables are proxied by four variables; iron (Fe), sea salt sodium (Na), non sea-salt calcium (Ca), and non sea-salt sulfate (SO<sub>4</sub>). Fe is derived almost entirely from terrestrial sources and is used as a proxy or a so-called iron fertilization effect that may enhance biotic uptake of CO<sub>2</sub> (Martin, 1990). Sulfate (SO<sub>4</sub>) originates mainly from marine biogenic emissions of dimethylsulphide (after removing sea-salt sources using the Na data), and so is a proxy for marine biogenic emissions (Wolff et al., 2008). It is included to test whether the increased levels of iron-containing dust lead to an increase in biological activity. Sea salt sodium is derived from the sea-ice surface and proxies winter sea-ice extent (Wolff et al 2006). Sea salt sodium is included to represent the possible effect of sea ice on the flow of CO<sub>2</sub> from the ocean to the atmosphere (Stephens and Keeling, 2001). Non sea-salt calcium has a terrestrial origin (mainly Patagonia) and may represent changes in temperature, moisture, vegetation, wind strength, glacial coverage, or changes in sea level in and around Patagonia (Basile et al 1997). It is included to represent local climate

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conditions and also has been used to proxy the iron fertilization effect (Rothlisberger et al., 2004).

C411 The results and discussion seem confuse to me. Some statistical tests are questionable (R<sup>2</sup> coefficient). Although the R<sup>2</sup> statistic is questionable in a multiple regression model when data are nonstationary, it is a valid measure in the CVAR formulation. To clarify this issue, we add the following material to the Introduction:

A significant association between two variables in the CVAR model is measured by cointegration rather than by correlation. The difference between the two is that two nonstationary variables can exhibit a high empirical correlation in spite of them being unrelated, whereas cointegration would be statistically rejected in this case. For example, R<sup>2</sup> from a linear regression model with nonstationary variables is prone to be spuriously high also when no causal relationship exists, whereas the R<sup>2</sup> for the corresponding CVAR equation would be close to zero in such a case. The condition for two time series to be cointegrated is that they have been affected by the same stochastic trend. This is a much stronger criterion for a causal relationship between two variables than a correlation coefficient, however high it may be. Therefore, if a physically meaningful relationship exists among variables for climate and solar insolation that contain a stochastic trend, cointegration analysis is an efficient tool for estimating the unknown parameters characterizing the relationship.

C411 and very few elements (No tables for the  $\alpha$  matrix component) are given to justify the main point of the manuscript: how authors conclude that the equilibrium state equations are valid or not (see "Specific comments" 2.3) are given to justify the main point of the manuscript. This comment shows that we have not explained the CVAR methodology being followed in our paper sufficiently well. Even though the reduced rank of the  $\Pi$  matrix ( $\Pi = \alpha\beta'$ ) is crucial for the CVAR methodology as well as the identification of the elements of  $\alpha$  and  $\beta$ , such identifying restrictions are not necessary for the main purpose of the article (which is to evaluate the importance of exogenous changes in Earth orbit relative to endogenous climate dynamics). This purpose can be

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achieved solely by knowing  $\Pi$  without knowing the elements of the  $\alpha$  and  $\beta$  matrix as the equality ( $\Pi = \alpha\beta'$ ) shows, i.e. imposing identifying restrictions on  $\alpha$  and  $\beta$  will have no effect on the simulations used to evaluate the drivers of glacial cycles. This is why there is no table for the  $\alpha$  or  $\beta$  matrices and we should have explained this more clearly in our manuscript. While we agree with the reviewer that the elements of  $\alpha$  and  $\beta$  are critical for the physical interpretation of the cointegrating relations, we found that such an analysis was comprehensive enough to warrant a new article. To test hypotheses about the drivers of glacial cycles by imposing a set of identifying restrictions on  $\alpha$  and  $\beta$  is the focus of a second paper which we are currently writing.

To explain this issue, have rewritten the description of the statistical model (section 2.2 and a new section 2.3) so that it focuses on how it is used in this manuscript to identify the endogenous and exogenous drivers of glacial cycles.

C412 The improved fit from model 1 to model 2b (discussed section 4.4) was easy to anticipate: Here we disagree with the reviewer. First, the model is able to show just how much an improvement is associated with each component of solar insolation. (The importance of this results is recognized by reviewer #2). For example, the relatively small improvement from Model 1 to Model 1b, which uses the Milankovitch forcing (insolation June 21 at 65oN) is surprising—much of the literature talks about how this is the critical driver to climate cycles. Conversely, the author is not surprised that Insol275 and Insol550 have relatively little power. But this result would come as a surprise the readers of Huybers and Denton (2008), who argue in Nature Geosciences that these measures of insolation are critical to glacial cycles. So, our results confirm some of the notions and dispel others. As such, these should be important results.

### 2.3 Statistical tests

surprisingly, very few explanations are provided concerning the specific result brought by a CVAR analysis which is to say the "long term equilibrium state". As described above, describing the long-run equilibrium state" is not the point of this analysis. It is

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the focus of a future paper on estimates for  $\Delta T_{2x}$

What is the trace statistic? As described above, we have now changed the text explaining that the trace test is a multivariate Dickey-Fuller type of test. For the interested reader, the mathematics of this statistic is described in (Juselius, 2006), which is referenced in the manuscript.

The  $\Pi$  is not defined in manuscript. References are at least necessary. We agree with the reviewer that this was an important omission in the original manuscript. Consistent with this we have rewritten the section on the statistical model. In the revised version we add a new section focusing on the use of the CVAR to identify the endogenous and exogenous drivers of glacial cycles. The revised version has included an extensive discussion of the  $\Pi$  matrix.

Moreover, regarding long discussions and tables connected to coefficient of determination  $R^2$  and  $S^2$  a statistic test, more precisions are expected to justify their conclusions about the cointegrated relationships. Obviously, an adjusted coefficient of determination would me more appropriate to discriminate the explanatory power of the models, since they have growing number of degrees of freedom. The reported coefficient of determination is the adjusted  $R^2$ . There might be some misunderstandings regarding the calculation of the degrees of freedom: as we add more variables, the degrees of freedom decline, not increase. Thus the diagnostic statistics has a built-in penalty for using additional explanatory variables. An increase in the number of explanatory variables reduces the degrees of freedom, making it more difficult to reject the null hypothesis that the results are statistically meaningful. For example, each equation in Model 1 (the model with the fewest explanatory variables) has 380 degrees of freedom whereas Model 4, (the model with the largest number of explanatory variables) has 339 degrees of freedom. Because the degrees of freedom is large in both cases, the reduction in degrees of freedom has a tiny effect (third decimal point) on the critical value of t and chi square distributions that are used to evaluate diagnostic statistics. For example, the significance level for a t statistic of 2.0 based on 380 degrees of free-

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dom is 0.04621155, compared to 0.04629769 for 339 degrees of freedom. We add this notion to the manuscript:

Differences in the number of explanatory variables among models affect the degrees of freedom that are used to assess the statistical significance of the results. Because the large number of observations is large (391 observations), the penalty for adding additional explanatory variables is small. For example, each equation in Model 1 (the model with the fewest explanatory variables) has 380 degrees of freedom. Model 4, (the model with the greatest number of explanatory variables) has 339 degrees of freedom. This reduction in the degrees of freedom has a tiny effect on the critical value of  $t$  and chi square distributions that are used to evaluate diagnostic statistics. For example, the significance level for a  $t$  statistic with a value of 2.0 and has 380 degrees of freedom is 0.0462, the significance level is 0.0463 for 339 degrees of freedom. Consistent with this small effect, increases in the number of explanatory variables does not automatically increase explanatory power. Section 4 describes several comparisons in which increasing the number of explanatory variables does not increase the explanatory power of the model.

The issue of adding additional variables is illustrated by changes in the performance of Model 1-3. For example, Model 2 adds nine variables to Model 1 and generates a statistically measurable increase in the performance of Model 2 relative to Model 1. Conversely, Model 3 adds seven variables to Model 1, but this increase does not generate a noticeable change in the performance of Model 3 relative to Model 1. This issue is addressed directly by modifying the discussion on lines 24-28 on page 600:

While adding variables does not diminish a statistical model's ability to simulate in-sample (additional variables that do not have a statistically measurable effect will be "zeroed out" by the estimation procedure), adding more variables does not automatically improve a statistical model's skill, as indicated by the performance of Model 3 and Model 1. Model 3 has seven more endogenous variables than Model 1, which implies an additional 14 parameters. Despite this increase, the in-sample simulations for Ice

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generated by Model 3 are not 'more accurate than those generated by Model 1 (Table 4 Figure 1c).

#### 2.4 Figures

C413 Too many experiments are compared on the same figure. Consistent with the comment by Reviewer #2, who states that the CVAR's model ability to identify the components of solar insolation that have the greatest explanatory power, we expand section 4.4 of the manuscript by using the S2a statistic to compare the accuracy of in-sample forecasts generated by various versions of model 2. These results are shown in a new Table 5. This is an explicit test and so addresses the difficulty associated with the fact that the "9 curves on top of each other." We alleviate this difficulty by creating a new set of figures, Figures 4a-c that show results for Models 2e-g and Model 3a. The results contained in the new table and Figures are discussed on detail in the revised section 4.4.

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Interactive comment on Clim. Past Discuss., 6, 585, 2010.

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