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Statistical issues about solar-climate relations

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Assessing the relationship between temperature variations and solar activity requires much physical and statistical insight. This paper is devoted to the latter. We focus on the statistical significance of diagnostics to obtain properties of the variability of time series. We illustrate our study by analyses of European temperature datasets and geomagnetic field variations. The goal of the paper is to provide a framework to control the spurious results that statistical tools can generate. We show that some variability diagnostics barely distinguish observed temperatures from auto-regressive random processes. In general, the variability diagnostics between temperature and geomagnetic activity are not significantly correlated, due to a low number of degrees of freedom.

1 Introduction

The detection and attribution of climate change has been extensively investigated with various statistical methods (Hegerl et al., 2007).

The role of solar activity on climate has been examined by many authors with climate models (Haigh, 1994; Shindell et al., 2001; Hansen et al., 2005; Meehl et al., 2009) and by comparing statistically solar and global or regional climate records (e.g. Lean and Rind, 2008, 2009; Benestad and Schmidt, 2009). Although weak in terms of energy balance, it seems to have played a significant role in various episode during the last millennium (Jansen et al., 2007). There is an on-going debate on the exact role of solar activity on 20th century temperature variations. Siscoe (1978) pointed out flaws of the many papers debating on the subject, more than 30 years ago. More recently, Lockwood (2008) provided a critical assessment of the potential mechanisms linking solar variations to climate change, and showed that solar variations are not sufficient to explain the 20th century warming from elementary laws of physics and thermodynamics. Our paper is not aimed at closing this debate. It is motivated by the statistical analyses

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that are often performed to suggest links between geophysical variables. Its ambition is also to shed light on a quite common misuse of the results of statistical analyses, which can suggest links between geophysical variables but, in no case, prove that such mechanisms do – or do not – exist.

A covariance-based transform of time series has recently been applied on climate and geophysical data to assess potential statistical links between temperature and minute variations of a proxy for solar activity (Le Mouél et al., 2008, 2009; Courtillot et al., 2010). This transform is called “Mean Squared Deviation” in these studies which claim that it gives information on the time evolution of the autocorrelation of a random process with slow heteroscedasticity. This paper illustrates some statistical properties of such a transform, and provides tests of its significance from simple random processes. The tests are then applied on temperature and geomagnetic data. The general goal of this paper is to provide a statistical framework to test the significance of mathematical transformations of data. Performing such tests can prevent from “data snooping” (White, 2000) and drawing wrong conclusions from statistical coincidences. Data snooping is sometimes used in fields where the underlying mechanisms remain difficult to grasp and model. It consists of purposefully (or not) exploiting a feature from a data set that turns out to be a statistical artefact. This problem can be circumvented if clear null hypotheses are formulated and proper statistical tests are performed. When mathematical transforms are involved, such tests should explore the sensitivity of the results to the parameters of the transforms. The mathematical transforms we present here essentially capture the variability of second-order moments of a time series. Thus, first order variations (i.e. slow variations of the mean) are not considered in these studies. This caveat should be kept in mind when a physical interpretation is formulated.

Section 2 describes the data sets used in the paper. The methodology is detailed in Sect. 3. Three types of results are provided in Sect. 4.

2 Data

We used the daily mean temperature from the ECA&D database in Europe (Klein-Tank et al., 2002). In particular, we randomly focused on the Paris-Montsouris (France) and De Bilt (Netherlands) stations between 1900 and 2009.

We also used a set of European temperatures starting before 1920 and ending after 2000, and yielding less than 10% of missing or doubtful data. We computed the daily mean of this data set, although its spatial distribution tends to give a lot of weight to Central Europe. The annual means of the three data sets are shown in Fig. 1. They all exhibit a warming over the 20th century. The exceptional warming between 2004 and 2009 in the European mean is due to the fact that a large proportion of “cold” stations (from central to eastern Europe) have systematic missing values after 2004. Thus, when the arithmetic mean of the ensemble is computed after 2004, it tends to be pulled toward higher temperatures. This feature should not be interpreted as an exceptional warming, but is an effect of missing data in ECA&D stations after 2004. We point out that the problem of missing data also appears before 1940. Such a problem could be circumvented by using centered temperature data, because the arithmetic mean of centered data is always centered. Thus, there is no artificial drift of the mean due to missing observations. We computed a mean seasonal cycle from the daily temperature data for the 1960–1990 period, and we removed it from the raw time series to obtain daily anomalies of temperature. We hence used daily temperature anomalies in computations of this paper. For brevity, the term “temperature” now refers to “anomalies of temperature with respect to a seasonal cycle”, unless otherwise specified.

Homogeneity problems have been detected on daily time steps, and to our knowledge they have not been corrected in the ECA&D database (O. Mestre, personal communication). The lack of homogeneity (due to changes in instruments, orientation or simply reporting errors) can generate artificial non-climatic discontinuities. In this paper, we do not question or evaluate the quality of the temperature data, although most stations are indicated as “suspect” on the ECA&D database. This evaluation will be done in a further study.

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We compare those temperature datasets with records of solar activity.

Before the period of precise irradiance measurements by satellites, there are several ways of approximating solar activity by using sunspots observed with telescopes (Hoyt and Schatten, 1998; Solanki, 2002), geomagnetic activity (Mayaud, 1972; Cliver et al., 1998; Lockwood et al., 1999), the frequency of aurora (review by Silverman, 1992) and cosmogenic isotopes such as ^{10}Be and ^{14}C (e.g. Beer et al., 1988; Bard et al., 1997).

The variations of the geomagnetic field are measured in many stations around the earth. The intensity of the geomagnetic field is dominated by dynamo processes of the interior of the earth, varying on secular time scales. The fast variations, on time scales of minutes to years, are mainly influenced by solar activity and galactic cosmic rays. It is then reasonable to assume that the fast variations recorded by stations over the Earth mostly reflect the fluctuations of solar activity. The intensity of the geomagnetic field is measured in two directions (horizontal and vertical). It appears that the fast variability of those components are very similar. Thus we focus on the vertical component variations (Z) of the geomagnetic field.

In this study, we followed the choice of Le Mouél et al. (2009) and used the geomagnetic data from Eskdalemuir (UK) as a proxy for solar activity. The data was obtained from the World Data Center for Geomagnetism (www.wdc.bgs.ac.uk/catalog/master.html). We computed daily averages from the hourly data, from 1911 to 2008. The time series shows a steady increase since 1938 (Fig. 2). This trend reflects secular changes in the geomagnetic field. We thus consider the small variations around this trend.

Mean daily temperature and geomagnetic times series have no mutual correlation (see Table 1, first column). Hence, the investigation of a potential relationship between the two variables motivates a focus on other statistical diagnostics. The next section gives an example of such analysis.

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Auto-regressive processes of order 1 (AR(1)) are often used in climate studies to define a null hypothesis to describe variations of temperature time series (Allen and Smith, 1994; Ghil et al., 2002; Maraun et al., 2004). By definition, an AR(1) process $R(t)$ is:

$$R(t+1) = aR(t) + b(t), \quad (1)$$

where $0 \leq a < 1$ and $b(t)$ is a centered white noise with unknown but finite variance σ_b^2 . The parameter a gives information on the memory of the process from one time step to the next. We use this simple random process as a pedagogical benchmark for the study of variability properties of time series, because they can be explicitly derived for various kinds of quadratic transforms. This process is commonly denoted "red noise" because its power spectrum decreases with frequency (Priestley, 1981).

Such a model can be refined to include slow time variations of a and σ . Such variations alter the probability distribution of $R(t)$ and an exhaustive list of the properties of such processes lies beyond the scope of this paper (e.g. Embrechts et al., 2000).

For a given centered time series $X(t)$ (of observations, for example), one wants to find an AR(1) process that fits "best" the statistical characteristics of $X(t)$. The classical maximum likelihood estimate (MLE) $\hat{a}$ for a gives (Priestley, 1981):

$$\hat{a} = \frac{C_X(1)}{C_X(0)}, \quad (2)$$

where $C_X(0)$ is the sample variance of X and $C_X(\tau)$ is its sample auto-covariance at lag τ :

$$C_X(\tau) = \frac{1}{N-\tau} \sum_{t=1}^{N-\tau} X(t+\tau)X(t).$$

For the AR(1) process R of Eq. (1), the auto-covariance is (Priestley, 1981):

$$C_R(\tau) = \frac{\sigma_b^2 a^{|\tau|}}{1-a^2}. \quad (3)$$

An alternative approach, on which we focus in this study, is to define the mean squared daily variation: $\zeta_{\Theta}(t)$ for a given time window Θ (Le Mouél et al., 2009):

$$\zeta_{\Theta}(t) = \frac{1}{\Theta} \sum_{\tau=t-\Theta/2}^{t+\Theta/2} (X(\tau+1) - X(\tau))^2. \quad (4)$$

For an AR(1) process ($R(t)$ in Eq. (1)), the mathematical expectation of $\zeta_{\Theta}^R(t)$ for a sufficiently large Θ is:

$$E[\zeta_{\Theta}^R] = \frac{2\sigma_b^2}{1+a}.$$

This can be verified by expanding Eq. (4) and using Eq. (3).

The mean interannual squared variation $Q_{\Theta}(t)$ is defined as:

$$Q_{\Theta}(t) = \frac{1}{\Theta} \sum_{\tau=t-\Theta/2}^{t+\Theta/2} (X(\tau+365) - X(\tau))^2. \quad (5)$$

For an AR(1) process $R(t)$, the mathematical expectation of $Q^R(t)$ for large Θ is:

$$E[Q_{\Theta}^R] = \frac{2\sigma_b^2}{1-a^2}.$$

We note that both $E[\zeta_{\Theta}^R(t)]$ and $E[Q_{\Theta}^R(t)]$ depend on a and σ_b for an AR(1) process, but not on Θ .

The “lifetime” function is defined by Le Mouél et al. (2009) as the normalization of $Q_{\Theta}(t)$ by $\zeta_{\Theta}(t)$:

$$L_{\Theta}(t) = Q_{\Theta}(t) / \zeta_{\Theta}(t). \quad (6)$$

This denomination is not connected with the lifetime notion in statistical survival theory (e.g. Lawless, 2003). For an AR(1) process, it follows that the mathematical expectation of $L_{\Theta}^R(t)$ is:

$$E[L_{\Theta}^R(t)] = \frac{1}{1-a} \equiv \lambda. \quad (7)$$

5 The interesting point of this quotient is that it no longer depends on σ_b for an AR(1) process. Thus for a given Θ , we obtain an estimator $\bar{\lambda}$ for λ :

$$\bar{\lambda} = \text{mean}(L_{\Theta}(t)),$$

where $\text{mean}(\cdot)$ is the arithmetic mean over t . Thus, in principle, the $L_{\Theta}(t)$ transform can be used to estimate a for an AR(1) process ($\bar{a} = 1 - 1/\bar{\lambda}$). However, the properties (bias, variance) are not a priori known but can be assessed by simulation (see next section). As a consequence of this convergence, it is hoped that if a varies slowly in time (over a scale larger than Θ) in Eq. (1), then it should be possible to estimate the variation rate from Eq. (6). This motivates the $L_{\Theta}(t)$ transform.

The MLE estimate of a in Eq. (2) also provides an estimate of λ :

$$\hat{\lambda} = \frac{1}{1-\hat{a}}.$$

15 The second goal of the paper is to obtain the significance of correlations between the transformations $L_{\Theta}(t)$ and $Q_{\Theta}(t)$ of two time series X and Y .
Stat. signif. In general, the non nullity of a correlation r_{XY} between two time series X and Y can be tested with a Student t-test (von Storch and Zwiers, 2001):

$$20 \quad t = |r_{XY}| \left(\frac{n-2}{1-r_{XY}^2} \right)^{1/2}, \quad (8)$$

where n is the number of degrees of freedom in X and Y . From the value of t , one can derive a p-value that is the probability of ~~failure~~ *of correlation confirming with the* when one rejects the null hypothesis
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$r_{XY} = 0$ (von Storch and Zwiers, 2001). The value of n is a priori lower than the number of data points. When transforms such as $Q_{\Theta}(t)$ or $L_{\Theta}(t)$ are applied, the upper bound for n can be estimated by $n \approx N/\Theta$, where N is the number of data points. If the time series cover one century on daily increments and $\Theta = 11 \times 365$, then the number of degrees of freedom n for $Q_{\Theta}(t)$ and $L_{\Theta}(t)$ is $n \approx 10$.

We remind that when a significant correlation is found between two time series there is no proof (or even a suggestion) of causality between the variables because correlation is a symmetric operator. A fortiori, finding a correlation between the L_{Θ} , Q_{Θ} or ζ_{Θ} quadratic transforms of two time series X and Y does not provide any causality relation between X and Y .

4 Results

4.1 Bias

The first step of this study is to verify that the $L_{\Theta}(t)$ transform is indeed a practical estimator of λ , i.e. with satisfactory properties of convergence and bias. We designed an ensemble of numerical experiments with AR(1) processes, by sampling the (0,20) interval of λ values with increments of 0.4. From those samples of λ , we take $a (= 1 - 1/\lambda)$, and simulate AR(1) processes, with a unit variance σ_b^2 for $b(t)$ (in Eq. 1). The experiments are carried on $N = 30000$ increments. For each realization, we computed $L_{\Theta}(t)$ with $\Theta = 11 \times 365$ and its average $\bar{\lambda}$. We also determined the variance and autocovariance, to estimate $\hat{a}$ in a direct way from Eq. (2). We then plotted the values of $\hat{\lambda}$ for both estimates, as functions of the “real” λ from which the processes were constructed. The choice of $\Theta = 11 \times 365$ is somewhat arbitrary. Its heuristic justification is that it covers a sunspot cycle of ≈ 11 years. The exact length of Θ should not alter the L_{Θ} estimates if an interpretation in term of memory needs to be done.

The results (Fig. 3) show that the MLE estimate $\hat{\lambda}$ generally performs well. The “lifetime” estimate $\bar{\lambda}$ does not yield an apparent bias. The confidence intervals for $\bar{\lambda}$

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seem to increase with a , indicating a higher variability of $L_{\Theta}(t)$ when a (or λ) increases. This is explained by the singularity near $a = 1$.

This exercise can be performed on the Paris or De Bilt temperature data. We obtain respectively $\hat{a} \approx 0.8$ and $\bar{a} \approx 0.7$. If the same computation is done on daily mean European temperature, we obtain $\hat{a} > 0.93$. Thus, assuming temporarily that local temperatures can be represented by AR(1) processes, this shows that their memories are rather high ($a > 0.8$). However, we point out that the average of independent AR(1) processes is *almost never* an AR(1) process (Embrechts et al., 2000). This implies that the interpretation of $\bar{\lambda}$ of the European mean temperature in term of process memory is a priori not possible.

4.2 Variability of $Q_{\Theta}(t)$ and $L_{\Theta}(t)$ for temperature

The second step of the study is to investigate the range of variations of $Q_{\Theta}(t)$ (the mean squared daily variations) and $L_{\Theta}(t)$ for a random process (AR(1)). From Eq. (2), we determined $\hat{a}$ and $\hat{\sigma}$ for the De Bilt and Paris temperature anomalies. This allows us to simulate AR(1) processes having the same variance and autocovariance (and length) as those temperature series. For each random realization and the temperature data, we computed $Q_{\Theta}(t)$, $\zeta_{\Theta}(t)$ and $L_{\Theta}(t)$. We determined the 5th, 50th and 95th quantiles of $Q_{\Theta}(t)$ and $L_{\Theta}(t)$.

The variations of $Q_{\Theta}(t)$ for De Bilt and Paris temperature anomalies and the 90% bounds for red noise are shown in Fig. 4a,c. Overall, the large deviations of $Q_{\Theta}(t)$ are significant, with respect to an AR(1) process. Hence the large variations of $Q_{\Theta}(t)$ are meaningful for temperature and could be interpreted as such.

The variations of $L_{\Theta}(t)$ for temperature anomalies (Fig. 4b,d) show generally similar shapes as those of $Q_{\Theta}(t)$ for De Bilt and Paris. Their significance with respect to an AR(1) process has the same feature as $Q_{\Theta}(t)$. We note that when the window Θ increases from 11 to 22 years, some of the maxima of $L_{\Theta}(t)$ lose their significance with respect to an AR(1) process (Fig. 4b,d). Curiously, the variations of $Q_{\Theta}(t)$ and $L_{\Theta}(t)$ are different for the mean European temperature. This can be explained by the fact

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that the statistical properties of the mean of time series are not the same as individual series, in particular in term of persistence.

We note that the $L_{\Theta}(t)$ transform for raw temperature (i.e. without removing the seasonal cycle) has the same general behavior and order of magnitude as for the temperature anomalies. This feature motivates the use of $L_{\Theta}(t)$ to investigate the variability of time series with periodic components that are potentially complex.

We checked the dependence of the analysis on Θ in the $L_{\Theta}(t)$ estimate. Indeed, the interpretation of a single outstanding “event” in $L_{\Theta}(t)$ should be robust to the choice of the window Θ in Eq. (6). We hence computed $L_{\Theta}(t)$ with $\Theta = 7, 11$ and 22 years (Fig. 4b) for the De Bilt and Paris daily temperature. This illustrates that an alteration in the window size changes important details of the transform $L_{\Theta}(t)$. This provides an additional (albeit more heuristic) way of verifying the robustness of the features of $L_{\Theta}(t)$. If De Bilt or Paris temperature series are considered, the major peaks or troughs of $Q_{\Theta}(t)$ and $L_{\Theta}(t)$ found for $\Theta = 11$ years seem unstable to this parameter, and it is not reasonable to load them with a physical interpretation.

We have also applied the analysis to the ensemble mean of temperature from the ECA&D database. The auto-correlation function $C_X(1)$ of the ensemble daily mean is larger than for each station. The time variation for the ensemble mean are coherent with the De Bilt or Paris data, but with a higher baseline (Fig. 4e,f). The conclusions of the significance of the variations of $L_{\Theta}(t)$ remain for ensemble averages of temperature.

4.3 Significance of correlations

Should one persist in using $L_{\Theta}(t)$ as an estimator of variability for a time series, the next question that arises is the significance of correlations between two transformed data sets. The $L_{\Theta}(t)$ transform intrinsically reduces the number of degrees of freedom of a time series by dividing it by Θ . Further filters, like moving averages or splines, also contribute to the reduction of the number of degrees of freedom in a time series.

The goal here is to compare the time variations of the $Q_{\Theta}(t)$ and $L_{\Theta}(t)$ functions of two time series X and Y through their correlation. The idea of this approach is to

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compare the variability features of X and Y , although both time series might be uncorrelated. If X and Y are correlated, there is no real point in comparing their variability because they are trivially linked. In any case, we repeat that the correlation between $L_{\odot}(t)$ transforms does not allow for an inference of a mutual relation between the original time series, unless a specific model of covariation is provided.

It should be noted that correlating $\zeta_{\odot}$ variations of one variable with $L_{\odot}$ variations of another variable is very difficult to justify. Indeed, $\zeta_{\odot}$ measures mean fluctuations of one day to the next, and $L_{\odot}$ measures fluctuations of days separated by one year. Nevertheless, in order to have an extensive view of the correlations among transforms of temperature and geomagnetic data, we compute the correlations between the $L(t)$ transforms of temperature data sets, and $\zeta(t)$, $Q(t)$ and $L(t)$ transforms for geomagnetic data.

From the behavior of the geomagnetic data (Fig. 6a), it is reasonable to subtract the low frequency part of the time series, which is connected to slow internal dynamo processes, and not relevant to solar activity. We hence smoothed the data with a spline function with 20 degrees of freedom (Green and Silverman, 1994), and retained the anomalies with respect to this spline function. We verified that those anomalies do not have a trend. The $L_{\odot}(t)$ and $Q_{\odot}(t)$ transform of the geomagnetic data are actually insensitive to the removal of the trend. This removal is done to estimate the variance and auto-covariance of the solar influence on the geomagnetic field, and hence simulate an AR(1) with the same parameters for Monte Carlo tests.

The first step is to estimate the probability of spurious correlation of $L_{\odot}(t)$, $Q_{\odot}(t)$ and $\zeta_{\odot}(t)$ for two independent AR(1) processes X and Y . For the sake of the exercise, we computed the variance and autocovariance of mean daily temperature over Europe and the geomagnetic field intensity anomalies. We then generated 100 independent realizations of each AR(1) process over 30 000 time steps (approximating the number of days during the 20th century), and computed $L_{\odot}(t)$, $\zeta_{\odot}(t)$ and $Q_{\odot}(t)$ for each realization. We took $\Theta = 11$ years for those experiments.

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The correlation distributions show that it is relatively frequent (with probability $p > 0.1$) that the correlations between $Q_{\Theta}(t)$ of two independent processes exceed 0.5 in absolute value (Fig. 5). The p-values of correlations for $n \approx 10$ degrees of freedom are below 0.07 only when $r > 0.6$, see Fig. 5 (right axis). out of order

This implies that $L_{\Theta}(t)$ or $Q_{\Theta}(t)$ transforms of two independent AR(1) processes with the same auto-covariance features as temperature and geomagnetic field are likely to yield large absolute values of correlation. Conversely, this means that large absolute values of correlation for $L_{\Theta}(t)$ or $Q_{\Theta}(t)$ transforms do not necessarily imply statistical significance. This emphasizes the importance of correlation testing, especially when the number of degrees of freedom is low.

In a second step, we computed $\zeta_{\Theta}(t)$, $Q_{\Theta}(t)$ and $L_{\Theta}(t)$ for the geomagnetic daily anomaly data anomalies, with the same parameters as Le Mouél et al. (2009).

From the geomagnetic anomalies, we determined the AR(1) process with the same variance and covariance, to estimate confidence intervals. The $\zeta_{\Theta}(t)$ and $Q_{\Theta}(t)$ variations are overall significant with respect to an AR(1). The variations of $L_{\Theta}(t)$ are generally not significant after 1940 because their amplitude is smaller than for an AR(1) process. The $L_{\Theta}(t)$ and $Q_{\Theta}(t)$ variations present a major decrease near 1940, and $L_{\Theta}(t)$ yields relatively weak variations after this date. The reasons for this change are undocumented, but could come from changes of instrument, or measurement frequency.

The (linear) correlations between the $L_{\Theta}(t)$ of daily mean temperature (resp. Paris, De Bilt and European average) and $L_{\Theta}(t)$ of Z are generally weak and not significant, as summarized in Table 1. The correlation even changes signs when European mean temperature is considered. The correlations between $L_{\Theta}(t)$ of temperatures and $Q_{\Theta}(t)$ of Z are also weak and not significant. The correlations between $L_{\Theta}(t)$ of temperatures and $\zeta_{\Theta}(t)$ of Z is higher, and can reach $r = 0.61$. But the p-value of the correlation is $p > 0.06$, which still make it insignificant by usual standards, because of the low number of degrees of freedom.

Overall, this shows that no conclusion about a covariation diagnostic can be derived from the comparison of the $L_{\Theta}(t)$, $Q_{\Theta}(t)$ or $\zeta_{\Theta}(t)$ transforms of temperature and geomagnetic activity. We also point out that higher correlation values (although not significant) are obtained with a special choice of transforms, which have no a priori justification. Thus, choosing different kinds of data transforms (i.e. $L_{\Theta}(t)$ vs. $\zeta_{\Theta}(t)$) to compare two data sets, in order to maximize a correlation can potentially lead to “data snooping”.

4.4 Identification of causality in $L_{\Theta}(t)$

Although none of the transforms outlined above provide tools to assess any causal link between two variables, one may *assume* that such a relationship exists, and determine the amplitude of the link. For example, consider two generalized AR(1) variables X and Y such that the memory parameter a^Y is controlled by a function $F^X(t)$, i.e. $a^Y(t) = F^X(t)$. The question we want to address is whether $L^Y(t)$ is also connected to the function $F^X(t)$.

In general, the answer to such a question is negative because Eq. (7) is not true when a varies continuously (even slowly) with time. We illustrate this point by creating a generalized AR(1) process:

$$R(t+1) = a(t)R(t) + b(t), \quad (9)$$

where $0 \leq a(t) < 1$ slowly with time and $b(t)$ is a centered white noise. In this example, we set X to be the mean squared daily variation of the Z component of the geomagnetic data after 1940 and Y is a mean daily temperature in Paris modeled by the generalized AR(1) process of Eq. (9).

We generate a function $F^X(t)$ by scaling $\zeta_{\Theta}^{(Z)}(t)$ (the Z component of the geomagnetic data) to vary between the bounds of $L_{\Theta}^{(\text{Paris})}(t)$ (the mean daily Paris temperature). This arbitrary choice is motivated by Le Mouél et al. (2009) who computed the correlation between $\zeta_{\Theta}^{(Z)}(t)$ and $L_{\Theta}^{(\text{Paris})}(t)$ of temperature. We then determine $a^Y(t)$ from $F^X(t)$. Hence, all the variability of $a^Y(t)$ is controlled by $\zeta_{\Theta}^{(Z)}(t)$. We simulated 100 such

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generalized AR(1) processes and computed the correlation between $F^X(t) \equiv \zeta_{\Theta}^{(Z)}(t)$ and $L_{\Theta}^Y(t)$, with $\Theta = 11$ years. In those realizations, the variance of $b(t)$ is the same as the sample variance of temperature anomalies in Paris.

We find that the correlation between $L_{\Theta}^Y(t)$ and $\zeta_{\Theta}^{(Z)}(t)$ is generally positive (in 90% of cases), although the median correlation is $r = 0.53$ and is hence not significant because of the low number of degrees of freedom ($n = 7$). This synthetic example shows that when the causality between memory parameters $a(t)$ is assumed, then the $L(t)$ transforms is barely sufficient to retrieve the original forcing function. This poor score, even in an idealized case, is due to the fact that the variance of $b(t)$ plays a role in $L(t)$ when $a(t)$ varies with time in Eq. (9), although it does not appear in Eq. (7). This further illustrates that an interpretation of $L(t)$ for a generalized AR(1) process with varying $a(t)$ cannot be obtained in a satisfactory manner. In general, $L(t)$ does not allow for an estimate of the time variations of the parameters of a generalized AR(1) process, unless those variations follow stepwise constant functions (as outlined by Le Mouél et al. (2009)). If the underlying process is more complicated than an AR(1) (for example if the variance of $b(t)$ also varies with time), then the analysis and interpretation of $L(t)$ are potentially irrelevant.

5 Conclusion

We analyzed the potential of $L_{\Theta}(t)$, $\zeta_{\Theta}(t)$ and $Q_{\Theta}(t)$ transforms of time series to characterize the variability of a time series. Using elementary statistical techniques of hypothesis testing, based on Monte Carlo simulations, we have provided a framework to check the statistical significance of such transforms, and hence help with their physical interpretation.

Following Le Mouél et al. (2009), we applied those procedures to temperature and geomagnetic activity time series. We found that the $\zeta_{\Theta}(t)$, $Q_{\Theta}(t)$ and $L_{\Theta}(t)$ variations of those variables are generally significant with respect to an AR(1) process. Moreover, a rigorous test between both variables shows that no significant correlation exists

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between them. Of course, we do not exclude that such transforms would not give significant results with other data sets.

This study can be extended to other transforms and statistical diagnostics. We emphasize the importance of testing statistical estimates with respect to reasonably chosen null hypotheses in order to avoid data snooping. In the case of $L_{\Theta}(t)$, the most reasonable and simple null hypothesis is an AR(1) process, for which $L_{\Theta}(t)$ has a direct interpretation. But temperature variations generally cannot be modelled by an elementary AR(1) process (Huong Hoang et al., 2009; Yiou et al., 2009). If time variations in a and σ_b are introduced in Eq. (1), their contribution to the $\zeta_{\Theta}(t)$ and $Q_{\Theta}(t)$ transforms cannot be separated as simply as for an AR(1). This strongly limits the use and interpretation of those transforms to diagnose the variability of a time series.

Finally, we point out that all the results presented in this paper are based on second order statistics of time series. The L_{Θ} diagnostic omits variations of first order, i.e. variations of the mean. From a quick inspection of Fig. 1, it is clear that the mean temperature evolves with time on multi-annual time scales. None of the presented analyses allow to explain such a temperature variation with a geomagnetic series. The increase of temperature (or temperature anomalies) after 1940 is still unexplained by the variations of the geomagnetic field anomalies.

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Table 1. Correlations between $\zeta_{\odot}$, $Q_{\odot}$ and $L_{\odot}$ transforms of daily temperature series and the Z component of the geomagnetic field, between 1940 and 2009 ($\Theta = 11 \times 365$). p -values for $n = 10$ degrees of freedom are indicated in parentheses when the correlation coefficient exceeds 0.4. Correlations under 0.4 are not considered significant.

Variables	Z	ζ^Z	Q^Z	L^Z
T_{Paris}	0.07	0.52 (0.12)	0.14	0.08
L_{Paris}				
T_{DeBilt}	0.06	0.53 (0.11)	0.33	0.26
L_{DeBilt}				
T_{Europe}	0.04	0.61 (0.06)	0.30	−0.43 (0.21)
L_{Europe}				

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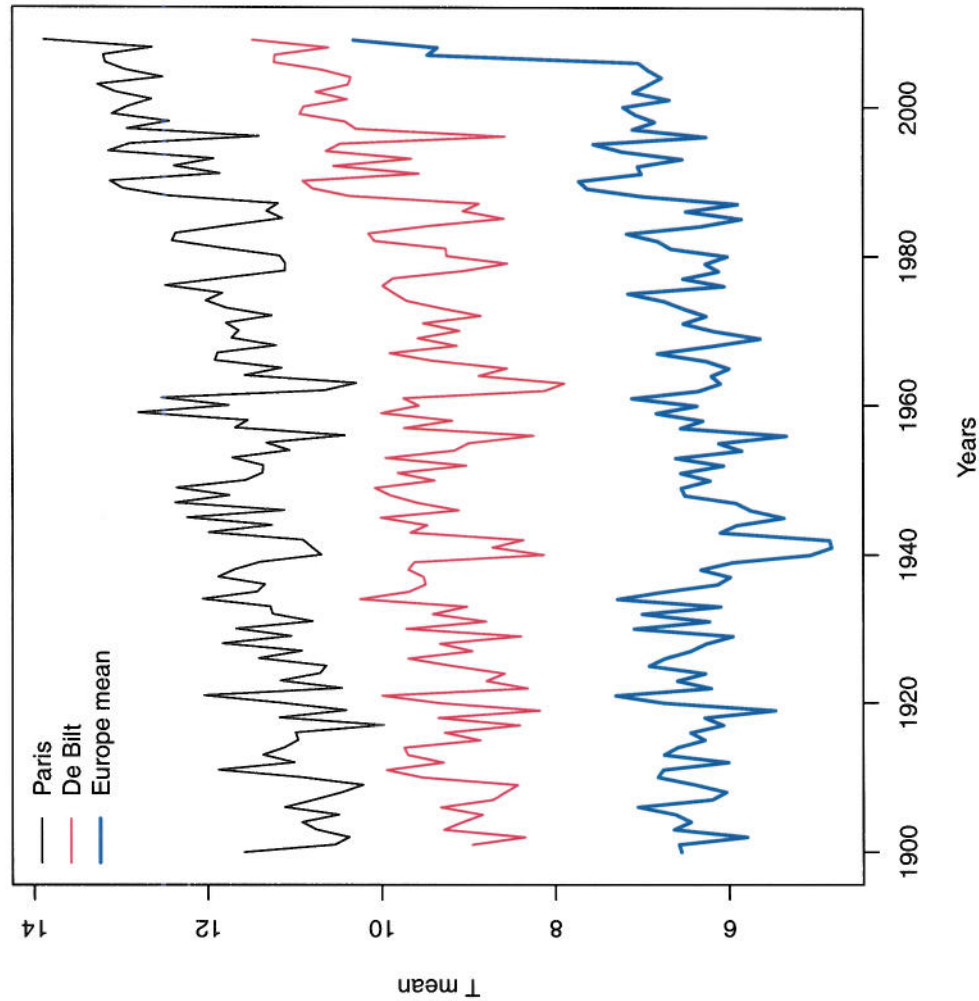


Fig. 1. Variations of the annual averages of daily mean temperature in Paris, De Bilt and a European mean, between 1900 and 2009. The data are taken from the ECA&D database (Klein-Tank et al., 2002).

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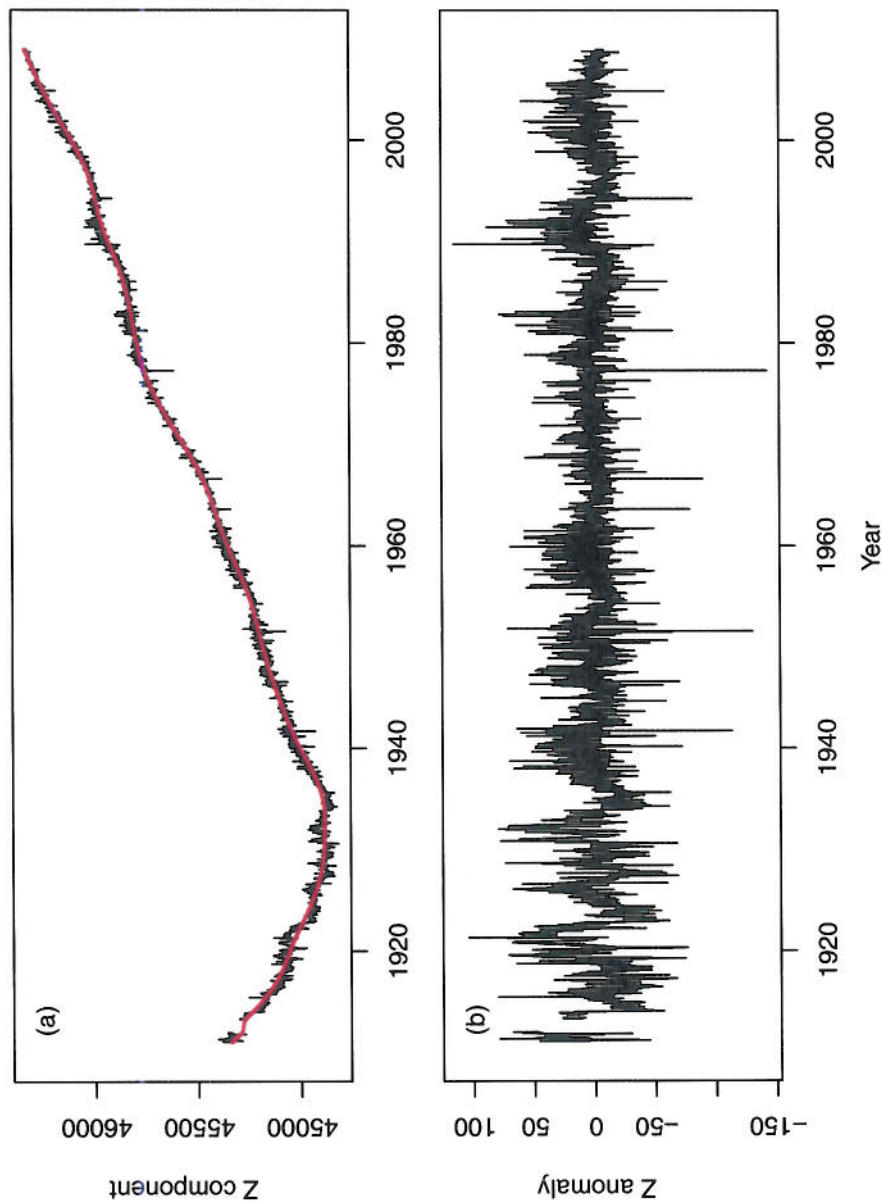


Fig. 2. (a) Daily variations of the Z component of the geomagnetic field measure at Eskdalemuir between 1911 and 2008. The red line represents the smoothed data with a spline function with 20 degrees of freedom. (b) Anomaly of the Z component with respect to the spline smoothing function.

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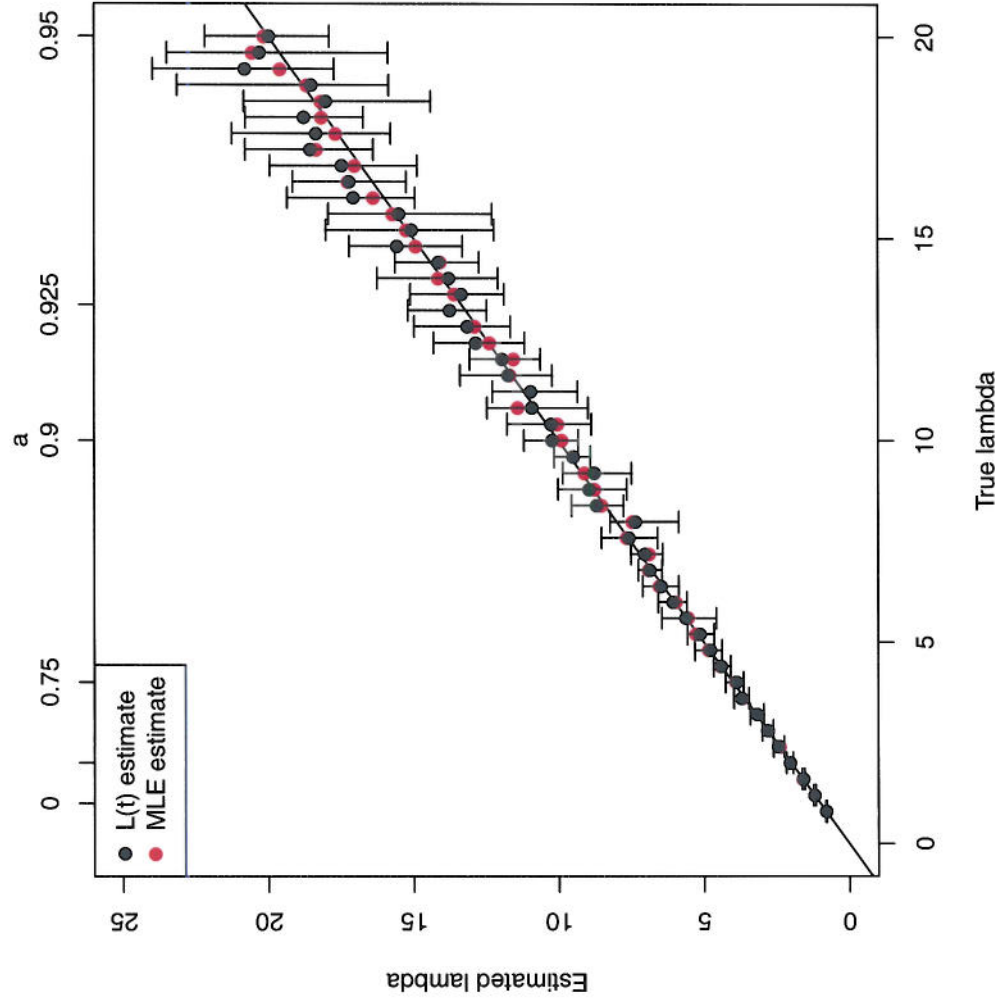


Fig. 3. Bias estimates of λ from an MLE computation $\hat{\lambda}$ (red circles) and mean “lifetime” computation $\bar{\lambda}$ (black circles). The bars around the $\bar{\lambda}$ estimates indicate the 5th and 95th quantiles of $L_{\Theta}(t)$ variations of AR(1) realizations. See text for formulas. The first diagonal is indicated for reference. The upper axis indicates variations of a (as in Eq. 1).

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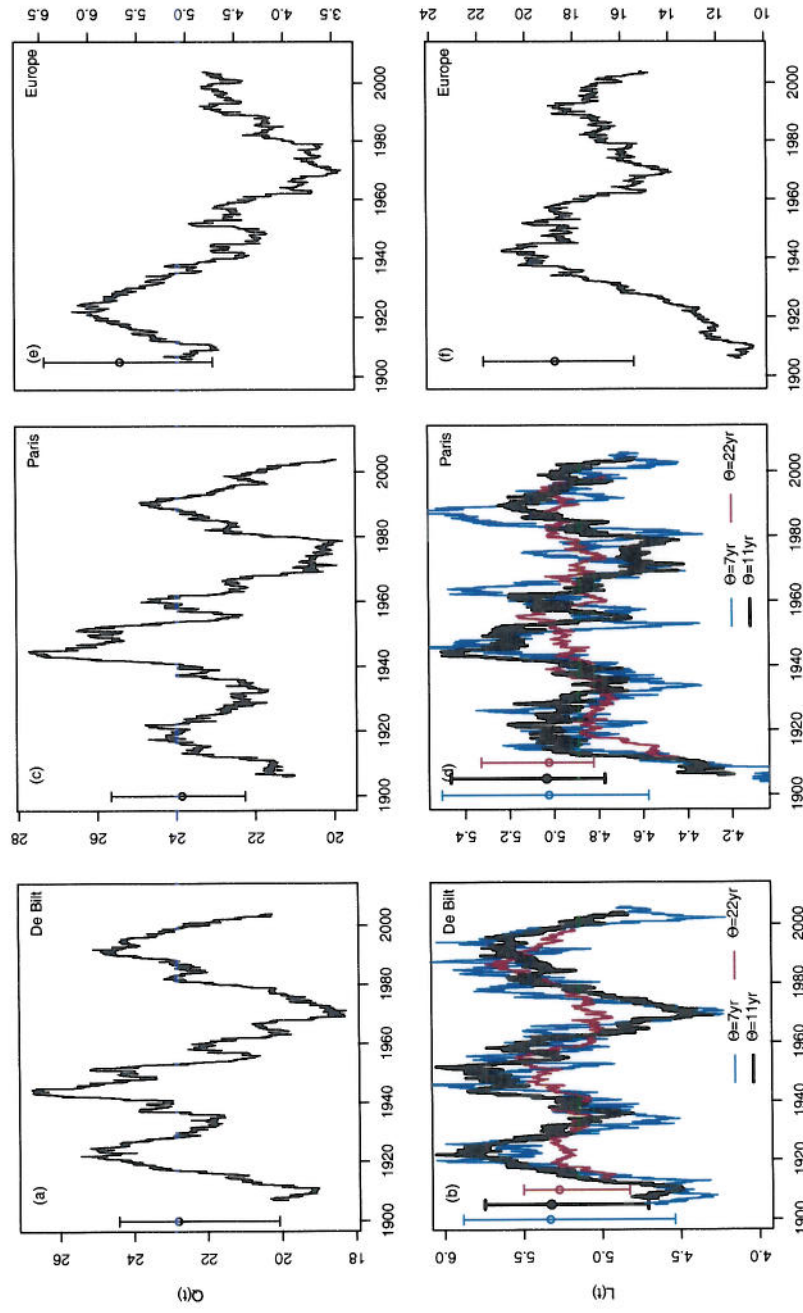


Fig. 4. (a) Variations of daily $Q_{\Theta}(t)$ for De Bilt temperature between 1900 and 2008, with $\Theta = 11$ years. (b) Same convention as (a) with $L_{\Theta}(t)$. The colored continuous lines show the variations of $L_{\Theta}(t)$ when the parameter Θ is varied from 7 years to 22 years. (c) Same as (a) for the mean daily Paris temperature. (d) same as (b) for the mean daily Paris temperature. (e) Same as (a) for the mean daily European temperature. (f) same as (b) for the mean daily European temperature. The vertical confidence intervals indicate the median, 5th and 95th quantiles for the variations of $Q_{\Theta}(t)$ (or $L_{\Theta}(t)$) of an AR(1) process with the same variance and auto-covariance.

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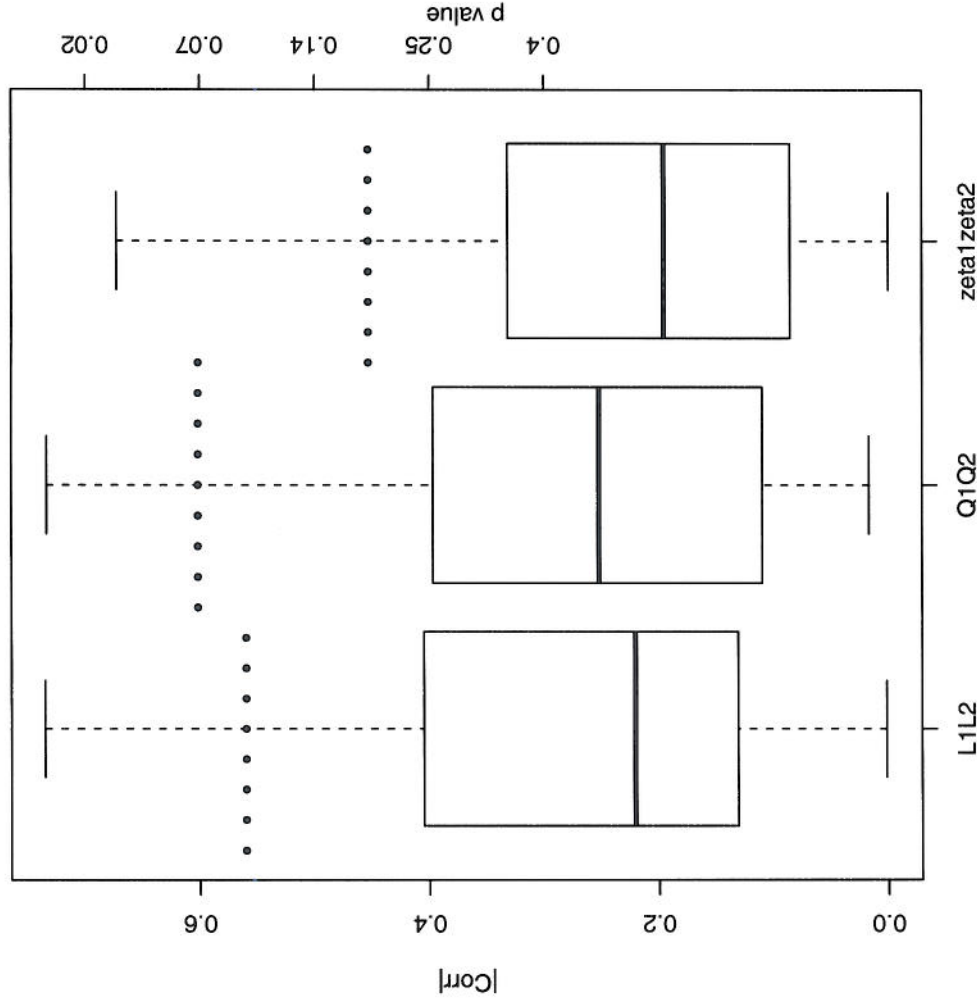


Fig. 5. Distribution of absolute values of correlation $|r|$ between $L_O(t)$, $Q_O(t)$ and $\zeta_O(t)$ trans- forms of two random AR(1) processes with the respective variance and autocovariance of ECA&D European temperatures and geomagnetic field from Eskdalemuir. The box and whisker plots indicate the 25, 50 and 75th quantiles q (boxes). The upper whisker corresponds to: $\min(\max(|r|), q_{50} + 1.5 \times (q_{75} - q_{25}))$. The right hand vertical axis shows the p values for $r = 0.3$ to 0.7, with increments of 0.1, corresponding to a null hypothesis of no correlation and $n = 10$ degrees of freedom. The horizontal thick dotted lines indicate the 90th quantile of $|r|$ values.

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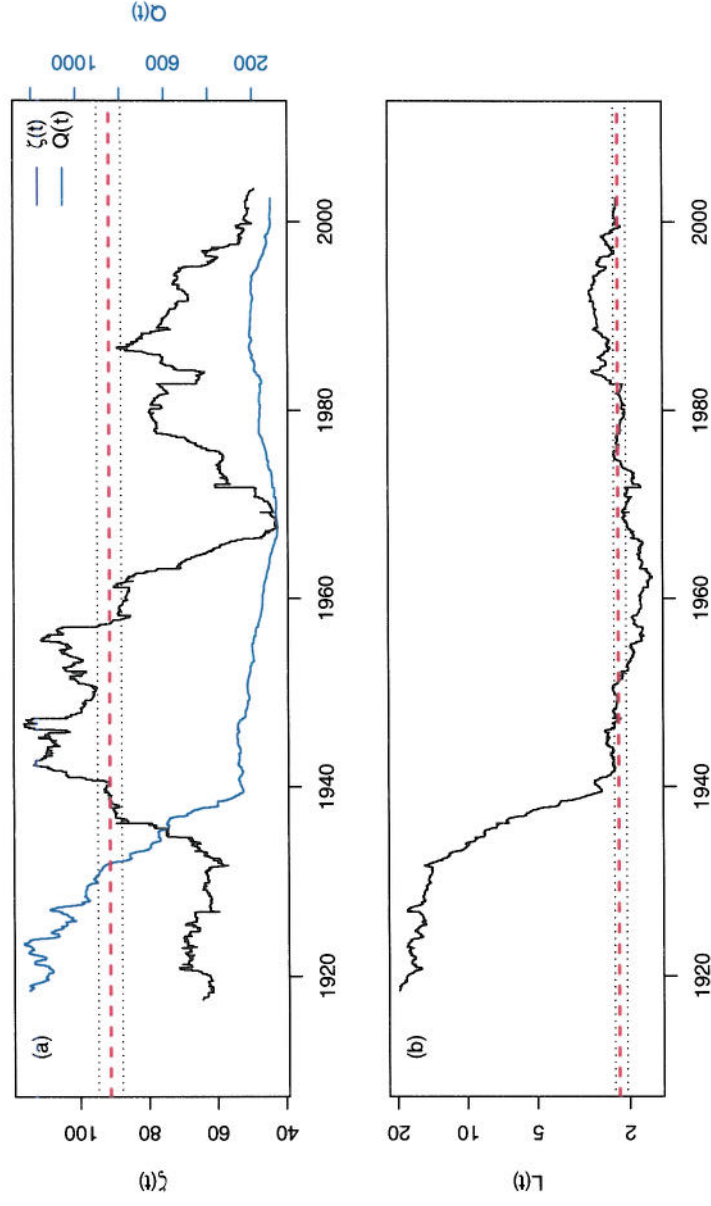


Fig. 6. (a) $Q_{\Theta}(t)$ of the geomagnetic data (blue line). Mean squared daily deviation ($Z_{\Theta}(t)$) of Z (black line). The confidence intervals are computed for the mean squared daily deviation $Z_{\Theta}(t)$ between 1940 and 2008. (b) $L_{\Theta}(t)$ of the geomagnetic data with $\Theta = 11$ years. The confidence intervals are determined for this parameter between 1940 and 2008 (vertical line).

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