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Pleistocene glacial variability as a chaotic response to obliquity forcing

P. Huybers

Department of Earth and Planetary Sciences, Harvard University, Cambridge MA, USA

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Correspondence to: P. Huybers (phuybers@fas.harvard.edu)

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Abstract

The mid-Pleistocene transition from 40 ky to ∼100 ky glacial cycles is generally characterized as a singular transition attributable to scouring of continental regolith or a long-term decrease in atmospheric CO_2 concentrations. Here an alternative hypothesis is suggested, that Pleistocene glacial variability is chaotic and that transitions from 40 ky to ∼100 ky modes of variability occur spontaneously. This alternate view is consistent with the presence of ∼80 ky glacial cycles during the early Pleistocene and the lack of evidence for a change in climate forcing during the mid-Pleistocene. A simple model illustrates this chaotic scenario. When forced at a 40 ky period the model chaotically transition between small 40 ky glacial cycles and larger 80 and 120 ky cycles which, on average, give the ∼100 ky variability.

1 Introduction

Early Pleistocene (prior to ∼0.8 Ma BP) glacial cycles occur primarily at 40 ky intervals, and are readily attributed to the 40 ky changes in Earth's obliquity (Raymo and Nisancioglu, 2003; Huybers, 2006). In contrast, late Pleistocene (0.8 Ma-present) glacial cycles have a longer ∼100 ky timescale (Hays et al., 1976; Imbrie et al., 1992). In the absence of substantial change in the external forcing (Pisias and Moore, 1981; Berger et al., 1999), this mid-Pleistocene Transition (MPT) from 40 ky to ∼100 ky glacial cycles is generally attributed to a long term decrease in atmospheric CO_2 or the scouring of continental regolith (Raymo, 1997; Berger et al., 1999; Clark et al., 1999; Raymo et al., 2006; Clark et al., 2006) activating new sources of low-frequency variability (Saltzman and Sutera, 1987; Maasch and Saltzman, 1990; Ghil, 1994; Mudelsee and Schulz, 1997; Berger et al., 1999; Paillard, 1998; Tziperman and Gildor, 2003). Here an alternative possibility is raised: that glacial variability spontaneously switches between long and short period modes of variability independent of changes in atmospheric CO_2, continental regolith, or other external controls. This hypothesis potentially accounts for changes in the mode of glacial variability in the absence of a long-term (>100 ky) trend in CO_2, as observed for the late Pleistocene (Lüthi et al., 2008) and as can be inferred to be the case for the middle Pleistocene (Medina-Elizalde and Lea, 2005). Furthermore, this hypothesis provides an explanation for the multiple switches observed between short and long-period glacial cycles over the Pleistocene (Mudelsee and Schulz, 1997; Heslop et al., 2002; Huybers, 2007).

2 Ice volume and its rate of change

A modification to the well-known simple glacial model of Imbrie and Imbrie (1980) is motivated by examination of the relationship between ice volume and its time rate of change. An average of globally distributed benthic δ^{18}O records (Huybers, 2007) is used as a proxy for ice volume (Fig. 1), where the record is first smoothed using a 7 ky window to focus on longer term variations. (Similar results are obtained if the record of Lisiecki and Raymo (2007) is instead used.) Deglaciation proceeds, on average, at twice the rate of ice growth, but is much more rapid following deep glaciations. Apparently, the rate of deglaciation is sensitive to prior amounts of ice volume (Fig. 2b), agreeing with the previous recognition that glacial terminations tend to follow after a critical build-up of ice volume (Raymo, 1997).

The dependence between ice volume and its rate of change can be quantified by computing the cross-correlation at different lags (Fig. 2c). For the early Pleistocene (2 Ma to 1 Ma), the greatest magnitude cross-correlation occurs when ice volume rates of change are lagged 9 ky behind ice volume magnitude ($r=-0.71$, $p<0.01$, the statistical test is discussed below), but this comes as little surprise because variations are primarily at 40 ky periods (i.e. $d/dt \sin(2\pi t/40\,\text{ky}) = -\sin(2\pi(t-10\,\text{ky})/40\,\text{ky})$). More interestingly, the lagged anti-correlation retains its greatest magnitude at a 9 ky lag and is also highly-significant during the late Pleistocene ($r=-0.55$, $p<0.01$, 1 Ma to the present), when variations are primarily at the longer ~100 ky periods. This consistency in the lagged correlation structure of the δ^{18}O record suggests that the physics controlling glacial variability does not fundamentally change across the mid-Pleistocene, retaining what is fundamentally a 40 ky obliquity-like relationship.

An autoregressive model (Wunsch, 2004) is adopted as a null-hypothesis to test the significance of the delayed cross-correlation structure between ice volume and its rate of change. In particular, normally distributed and uncorrelated noise inovations are used along with a second order auto-regressive relationship with coefficients of 1.181 and –0.1984. Individual realizations are made from this auto-correlation model, each a million years long, and a record is kept of the maximum and minimum lagged cross-correlation between the time-series and its rate of change over a range of zero to 200 lags. This process is repeated 10 000 times, and p-values are derived from the ensemble of realizations. As reported above, the lagged cross-correlation observed between ice volume and its rate of change during both the early and late Pleistocene are found to have a $p<0.01$, indicating that the lagged relationship is highly statistically significant. (Note that if no delay were included, the observed cross-correlation is very nearly zero and statistically insignificant.) Experiments with a wide range of alternate auto-correlation models yield consistent results.

3 An adaptation of Imbrie and Imbrie's simple model

The statistical significance of the delayed relationship between the magnitude of ice volume and its rate of change prompts modification of the simple glacial model proposed by Imbrie and Imbrie (1980), who posited a zero-lag relationship between ice volume and its rate of change. Two modification are made which serve to make rates of ice volume change sensitive to prior ice volume, but only during deglaciation,

$$V_t = V_{t-1} + V_{t-L}^{E} \times \frac{a-\theta}{T} \qquad E = \begin{cases} 0 \text{ if } a > \theta, \\ e \text{ if } a < \theta, \end{cases} \tag{1}$$

Here V is ice volume, t is time in ky, L is the time lag, T is a time constant, a is an accumulation rate, and E is an exponent whose value depends on whether the model

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is accumulating or ablating. The model is linearly dependent on the forcing when ice is accumulating, but when ice is ablating, it sensitively depends on the ice volume L ky ago, raised to a power e. External forcing is introduced through θ. Deglaciations tend to occur during times of increased orbital obliquity (Huybers and Wunsch, 2005), and here the model forcing, θ, is parameterized using a simple 40 ky sinusoid, thus avoiding obscuration of model transitions by the amplitude and frequency modulations associated with obliquity or the full insolation forcing (Berger et al., 1998; Tziperman et al., 2006). Note that Eq. (1) is discrete and is thus technically a map rather than a differential equation. As is common to all such simple models (Maasch and Saltzman, 1990; Ghil, 1994; Paillard, 1998; Tziperman and Gildor, 2003), Eq. (1) should be interpreted as a schematic, and serves to illustrate dynamical scenarios which the climate system may be capable of. One advantage of the simplicity of this model is that it facilitates thorough analysis of its rich behavior.

Parameterizations are chosen in keeping with observations during the late-Pleistocene: a timescale of T=90 ky, accumulation rate a=0.9, lag L=9 ky, and an exponent e=9. (These parameters are also selected for their simplicity, but many choices for the four adjustable parameters in the model yield qualitatively similar behavior, some of which reproduce the timing and amplitude of late Pleistocene glacial variability.) The exponent, e=9, may seem large, but note that the time rate of change in the surface height of an ice sheet is proportional to its height to the fifth power and its surface slope to the third power, assuming the standard shallow-ice approximation and a Glen's flow law with an exponent of three (van der Veen, 1999). Instabilities within the ice may be represented by even higher power-law relationships. The forcing variance is set to one, and is not considered an adjustable parameter because any change can equivalently be achieved by adjusting a and T.

4 Chaotic 40 ky and ∼100 ky glacial cycles

For significant ablation to occur in the model, the obliquity forcing must be greater than a and V_{t-L} must exceed one. This state-dependent sensitivity to external forcing is a common feature of what are broadly referred to as *excitable* systems (FitzHugh, 1961; Pikovsky et al., 2001). (A pioneering study of excitable systems by *Van der Pol* in 1927 focused on cardiac pacemaking by periodic electrical stimuli, in good analogy with insolation being described as the pacemaker of the ice ages (Hays et al., 1976).) As with other excitable systems (Strain and Greenside, 1998; Othmer and Xie, 1999), Eq. (1) exhibits chaotic behavior. In particular, the amplitude of the model ice volume becomes chaotic under certain amplitude of the forcing, whereas the phase is invariably synchronized with the forcing (Rosenblum and Pikovsky, 2003), in this case because some ablation always occurs near obliquity maxima. (Note that the chaotic variability in Eq. (1) is distinct from the chaotic motion of the solar system (Laskar, 1989).)

The period of the model can be determined by counting the successive number of maxima which occur before returning to a previously visited state. As the amplitude of the forcing increases, the model exhibits a period-doubling route to chaos (Fig. 2a), in qualitative agreement with that of the logistic map and the Rossler system (Strogatz, 1994). The model undergoes periodic 40 ky variations at a very small forcing amplitude, but as the amplitude increases, period doubling ensues, first giving 80 ky glacial cycles, and eventually the infinite repeat times associated with chaos. At yet higher amplitudes of the forcing, periodic solutions reappear, generally with a 120 ky repeat time. Thus the timescales inherent to Pleistocene glacial cycles – short 40 ky cycle and longer 80 ky to 120 ky cycles – are present within this simple model.

Another feature of chaotic systems is exponential divergence of model trajectories subject to small perturbations. Perturbations to V in Eq. (1) grow exponentially with a timescale of ∼300 ky. Thus, in the face of imperfect observations, the model trajectory will only be predictable for a few glacial cycles.

The model exhibits chaotic variability when it is forced by a 40 ky sinusoid with unit variance. The chaos is characterized by alternating strings of 40 ky and ∼100 ky cycles, independent of changing any model parameters. It is useful to divide ice volume maxima into three states: interglacial (i), mild glacial (g), and full glacial (G); and the origins of these distinct 40 ky and ∼100 ky modes of glacial variability can be understood by examining successive ice volume maxima against one another (Fig. 2b). The model tends toward greater ice volume over most of a glacial cycle, so that if one ice volume maximum is in state i, the next is in g and then G. But the transition out of the G state, representing a deglaciation, can be into either an i or g state. Note these transitions between glacial states emerge as an intrinsic features of the model, perhaps explaining the analogous three climate states specified in the Paillard model (Paillard, 1998).

The cycling between G-g states gives an 80 ky glacial cycles, and cycling between G-i-g gives a 120 ky glacial cycle, each occuring with roughly equal probability and, on average, giving a ∼100 ky time-scale. Thus this purely obliquity paced model generates ∼100 ky variability, in keeping with observations (Huybers and Wunsch, 2005).

The model also spontaneously generates strings of 40 ky glacial cycles. The 40 ky mode of glacial cycling results from the presence of an unstable fixed point at the boundary between the g and G states (Fig. 2b). If the model ever landed precisely on the g-G boundary it would be trapped, forever returning to the exact same state. While the probability of becoming permanently trapped is vanishingly small, the model intermittently happens to land near the unstable fixed point and then requires many g-G cycles to escape the fixed point's influence. This string of temporarily-trapped g-G cycles are of nearly the same amplitude, similar to the glacial cycles observed during the Pliocene and Early-Pleistocene. Note that even during these 40 ky oscillation the model generates an asymmetry where ablation is more rapid than accumulation, in agreement with observations of the Pliocene and early Pleistocene glacial cycles (Hagelberg et al., 1991; Ashkenazy and Tziperman, 2004).

The model episodically becomes trapped near the unstable fixed point, giving a train of 40 ky glacial cycles, and then transitions into the full *g*-*G* and *i*-*g*-*G* cycles, giving larger amplitude and longer-period ∼100 ky glacial variability. The suggestion is then that the realization of glacial cycles which occured during the Plio-Pleistocene was one in which the climate was initially trapped near a 40 ky fixed point; briefly escaped a few times near 1.8, 1.6, and 1.2 Ma; and then fully escaped during the mid-Pleistocene into ∼100 ky variations. All runs of the model produce episodic trains of 40 ky and 100 ky glacial cycles and, given a long run of the model, a sequence similar to the Plio-Pleistocene is inevitably generated (Fig. 2c).

It is instructive to analyze a particular realization of the model, selected because of its qualitative agreement with the marine δ^{18}O record over the last 2 Ma. Spectral analysis of the first (2 to 1.1 Ma) and second parts (1.1 to 0 Ma) of the model realization (Fig. 2c) indicate that energy concentrated at the obliquity period (taken as bands between $1/41\pm1/150$ ky) decreases from 63% to 35% of the total, while energy at ∼100 ky period (between $1/100\pm1/150$ ky) increases from 23% to 51% (Fig. 2d). Note that while individual glacial cycles have 80 ky and 120 ky periods, these periods do not appear in spectral estimates because the concatenation of glacial cycles of alternating lengths, unlike superposition, tends to concentrate spectral energy at the average period. By comparison, the analogous time intervals in the δ^{18}O record show a decrease from 51% to 19% at the obliquity period, and an increase from 35% to 71% at the ∼100 ky period (Fig. 2a). Thus, the redistribution of spectral energy from 40 ky to ∼100 ky periods generated spontaneously by this realization of the model is similar, if slightly weaker than, the redistribution observed in the observations.

5 Discussion and conclusions

The model behavior resembles a simple chaotic model proposed for El Niño (Tziperman et al., 1994) – both models are driven by periodic forcing, incorporate a time delay, and exhibit a period doubling route to chaos. It has been suggested that obliquity period variability in the Tropical Pacific is responsible for glacial variability (Liu and Herbert,

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2004; Fedorov et al., 2006), but establishing such a connection requires further theoretical and observational study. Furthermore, currently available methods (Gottwald and Melbourne, 2004) are not able to distinguish whether El Niño or the glacial cycles are truly chaotic, given the finite and noisy data which is available.

A more realistic representation of the forcing term in Eq. (1) would include some stochastic parameterization of the myriad climatic variations not explicitly represented in the model, including weather and longer-period variations. Trials including stochastic terms decrease the fraction of energy at the obliquity periods but does not alter the basic structure of the variability.

A weaker version of this hypothesis is that a small shift in the background conditions governing glacial variability could be sufficient to shift the model's trajectory from one of primarily 40 ky to ~100 ky glacial cycles. If the underlying dynamics of the glacial system are chaotic, or in a regime near chaos, the qualitative behavior of the glacial system could be exquisitely sensitive to small changes in atmospheric CO_2, conditions at the base of an ice sheet, or other features of the climate (Raymo, 1997; Clark et al., 1999; Raymo et al., 2006; Clark et al., 2006).

Implications are that the climate system may spontaneously transition from 40 to ~100 ky modes of glacial variability. Changes in background climate conditions, to the extent they occur, are expected to influence the glacial cycle trajectory but, in this view, are not strictly needed to explain the transitions from 40 to ~100 ky modes of glacial variability, or earlier episodes of long-period glacial cycles. Furthermore, if the glacial cycles are chaotic, the implied sensitivity to initial conditions and likely stochastic influences makes prediction of future glacial cycles highly uncertain.

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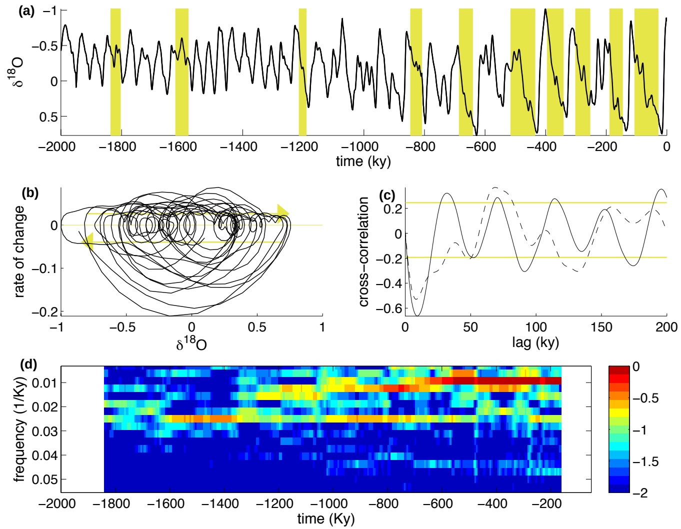

Fig. 1. Glacial variability over the last 2 Ma. **(a)** Marine δ^{18}O averaged across many sediment cores (Huybers, 2007). Glacial cycles lasting more than 60 ky are indicated by vertical bars. **(b)** δ^{18}O in ‰ plotted against the time rate of change in ‰ ky^{-1}. Arrows indicate the direction and average rate of change. Note that deglaciation (decreasing δ^{18}O) is more rapid than glaciation, and that the most rapid deglaciations follow after the most positive excursions in δ^{18}O. **(c)** The lagged cross-correlation between δ^{18}O and its rate of change for the early Pleistocene (solid line) and the late Pleistocene (dashed line). The largest magnitude cross-correlation occurs when rates of change are lagged by 10 ky for both the early ($r=-0.6$, $p<0.01$) and late ($r=-0.5$, $p<0.01$) Pleistocene. Note that the lagged auto-correlation relationship reaches extremum (and tends to exceed the 99% confidence interval, indicated by the yellow lines) at integer multiples of the obliquity period – e.g. 9, 50, 90, 130 ky for negative excursions and 30, 70, 110, 150 ky for positive excursion during the early Pleistocene, and lags of 9 and 130 ky as well as 70 and 150 ky for the late Pleistocene. **(d)** Spectrogram of the marine δ^{18}O record over the last 2 Ma.

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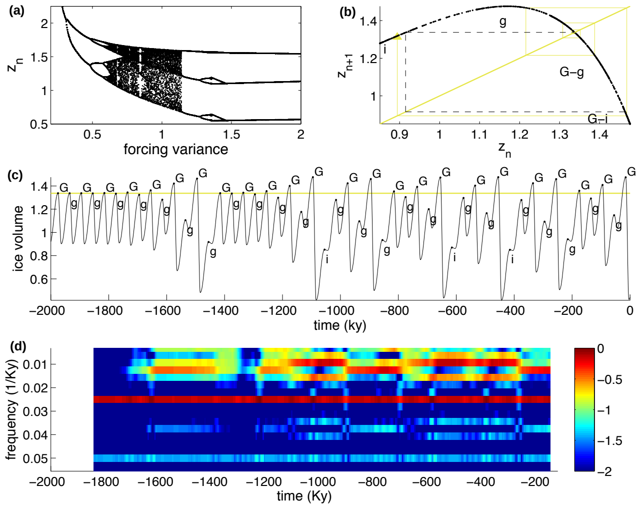

Fig. 2. A chaotic glacial model. **(a)** ice volume maxima occur every 40 ky. When these maxima are plotted as a function of the forcing variance, a structure emerges indicative of a period-doubling route to chaos. All other panels indicate results obtained with a forcing variance of one, yielding chaotic behavior. **(b)** Successive ice volume maxima (z_n, z_{n+1}) plotted against one another (i.e. a Lorenz Map, dots) indicate the trajectory of the model. The x-axis can be divided into three model states: deep glacial G (z_n>1.34), mild glacial g (0.92<z_n<1.34), and interglacial i ($z_n \le 0.92$). The model *fixed-point* lies at $z_n = z_{n+1} = 1.34$ and is indicated by the intersection with the diagonal line. G-g indicates that the model trajectory is from state G to state g; likewise for G-i. Yellow lines indicate the trajectory between 1.4 Ma and 1.0 Ma, where the model is initially trapped near the fixed point and then escapes into a 120 ky cycle. **(c)** A model realization with successive glacial maxima labeled according to model state. Initially the model is trapped near the fixed point and undergoes successive 40 ky glacial cycles ($z_n \approx z_{n+1}$), but then escapes, undergoing larger 80 ky g-G ($z_n < z_{n+1}$) and 120 ky i-g-G ($z_n < z_{n+1} < z_{n+2}$) glacial cycles, giving ∼100 ky glacial cycles. **(d)** A spectrogram of the model output, comparable with Fig. 1d.