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# Limitations of red noise in analysing Dansgaard-Oeschger events

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## Abstract

During the last glacial period, climate records from the North Atlantic region exhibit a pronounced spectral component corresponding to a period of about 1470 years, which has attracted much attention. This spectral peak is closely related to the recurrence pattern of Dansgaard-Oeschger (DO) events. A red noise random process was used to evaluate the statistical significance of this peak, with a reported significance of more than 99%. Here we use a simple two-state model of DO events, which itself was derived from a much more sophisticated ocean-atmosphere model of intermediate complexity, to numerically evaluate the spectral properties of random (i.e., solely noise-driven) events. This way we find that the power spectral density of random DO events differs fundamentally from a simple red noise random process. These results question the applicability of linear spectral analysis for estimating the statistical significance of highly non-linear processes such as DO events.

## 1 Introduction

Many climate records from the North Atlantic region exhibit an outstanding spectral peak corresponding to a period of about 1470 years during the last glacial period (Grootes and Stuiver, 1997; Schulz 2002). This spectral component, which has attracted much interest recently, is apparently non-stationary and particularly pronounced in the GISP2 ice core record during the time interval between 31 000 and 36 000 years before present (Schulz, 2002). A red noise random process (i.e., a first-order autoregressive (AR1) process) was used to estimate the statistically significance of this 1470-year spectral peak (Grootes and Stuiver, 1997; Schulz, 2002). The non-normalized power spectral density distribution PSD($f$) of a discrete time series of an AR1 process (with spacing unity and innovation standard deviation $\sigma$) is given by the expression:

$$\mathrm{PSD}(f)=2\cdot\sigma^2/[1-2a\cdot\cos(2\pi f)+a^2], \quad (1)$$

where a is the autocorrelation parameter and f denotes the frequency (Priestley, 1981). This approach resulted in a reported significance of more than 99%. In the time domain the 1470-year spectral peak is closely linked with the occurrence of Dansgaard-Oeschger (DO) events (Schulz, 2002), which show some tendency to recur in near-multiples of about 1470 years during the last glacial period (Alley, 2001; Schulz, 2002; Rahmstorf, 2003), cf. Fig. 1. The statistical significance of this tendency, however, is still a matter of debate (Ditlevsen et al., 2007).

The standard interpretation is that DO events represent regime shifts between two different modes of the ocean/atmosphere system (Dansgaard et al., 1982; Oeschger et al., 1984; Broecker et al. 1985; Sarnthein et al., 1994; Alley and Clark, 1999), as has been concluded from climate records (e.g. Steffensen et al., 2008) and ocean-atmosphere models (e.g. Ganopolski and Rahmstorf, 2001). Transitions between both modes apparently happened very quickly, i.e. on the annual to decadal time scale (Taylor et al., 1997; Severinghaus and Brook, 1999; Steffensen et al., 2008), which is commonly regarded as observational support for the existence of threshold-crossing processes during DO events (Alley et al., 2003; Steffensen et al., 2008). Such a threshold could be provided e.g. by the process of buoyancy deep convection in the ocean ("deep water formation"), which occurs when surface water gets denser than the deeper ocean water (Ganopolski and Rahmstorf, 2001). From a theoretical point of view, DO events could thus be regarded as repeated oscillations in a system with two possible states of operation and with a threshold (Braun et al., 2007).

In dynamical system theory it is well known that highly non-linear systems, e.g. systems with thresholds, can respond at a preferred time scale, the stochastic time scale, even when driven by a random input (i.e., "noise") only (Pikovski and Kurths, 1997; Gammaitoni et al., 1998). The stochastic time scale corresponds to the average spacing between successive noise-induced events and is closely related with the magnitude of the noise in the forcing. In other words, noise-induced DO events are expected to evolve on a very distinctive time scale, more precisely on the millennial to multi-millennial scale (Ganopolski and Rahmstorf 2002; Braun et al., 2007). The red noise




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random process, in contrast, describes the spectral properties of a linear, noise-driven system with damping. In contrast to a system with a threshold, such a system responds on all time scales when driven by noise, with maximum variance on the very long time scale and decreasing variance on shorter time scales. In other words, the red noise random process might not be applicable to estimate the spectral properties of random DO events and more sophisticated approaches might be needed, based e.g. on Monte-Carlo simulations with models that are able to mimic the dynamics of DO events.

## 2 A simple two-state model of DO events

In this paper we use a very simple two-state model for the dynamics of DO events (Fig. 2) to estimate the spectral properties of random (i.e., solely noise-induced) DO events. Our model is identical to the one described and used in the publications of Braun et al. (2007) and Braun et al. (2008). A comprehensive description of this model was presented in the work of Braun et al. (2007), including a detailed discussion of its physical motivation, its applicability and its limitations. This model has been derived from the dynamical principles of DO events as simulated with a much more sophisticated ocean-atmosphere model of intermediate complexity (CLIMBER-2), which itself is too slow for most statistical analyses. The ability of the simple two-state model to reproduce the waiting time statistics of the events in that ocean-atmosphere model was already demonstrated in a few simple forcing scenarios (Braun et al., 2007).

The dynamics of DO events in the simple two-state model is depicted in Fig. 2. It is assumed that the events represent transitions between two states of operation ("stadial" = cold state, "interstadial" = warm state) in a system with a threshold. Transition between these states occur when a given forcing $f$ (in freshwater flux units, i.e. in mSv, 1 mSv = 1 milli-Sverdrup = $10^3$ $m^3/s$) crosses a certain threshold function $T$. More precisely, a switch from the cold state to the warm state happens when $f<T$. The opposite switch occurs when $f>T$. In the model these switches are regarded as the onset and





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the termination of a DO event, respectively. During the switches a discontinuity in the threshold function is assumed, i.e., $T$ overshoots and afterwards approaches its respective equilibrium value following a relaxation process with a millennial time scale (Fig. 2).

Note that the dynamical principles and the transition rules in the two-state model are a first order approximation of the dynamics of DO events in the ocean-atmosphere model CLIMBER-2 (Petoukhov et al., 2000; Ganopolski and Rahmstorf, 2001). In that model DO events also represent threshold-crossing events in a system with two possible states of operation (corresponding to two fundamentally different modes of buoyancy deep convection in the northern North Atlantic) and with an overshooting in the stability of the system during these shifts (Ganopolski and Rahmstorf, 2001; Braun et al., 2007). Analogous to the simple two-state model, switches from the stadial mode to the interstadial one are triggered by sufficiently large negative forcing anomalies (more precisely, by a reduction in the surface freshwater flux to the northern North Atlantic that exceeds a certain threshold value), whereas the opposite shifts are triggered by sufficiently large positive forcing anomalies (that is, by an increase in the freshwater flux that exceeds a certain threshold value). The simple two-state model has six independent parameters (Table 1), which have also been estimated from the ocean-atmosphere model CLIMBER-2, as demonstrated in the supporting online material in the publications of Braun et al. (2005) and Braun et al. (2007).

To illustrate the agreement between the simple two-state model and the ocean-atmosphere model CLIMBER-2 we here present one example (Fig. 3). A detailed comparison, which also includes some more examples, can be found in the supplementary material of the publication of Braun et al. (2007). Figure 3 shows the response of both models to a periodic, bi-sinusoidal forcing in cycles of about 210 and 86.5 years, respectively. The amplitude of the forcing is chosen to be supra-threshold, because a sub-threshold forcing is not able to trigger repeated DO events in either of the models. As can be seen from Fig. 3, to a reasonable approximation the conceptual model is able to reproduce several aspects of the DO events as simulated with the ocean-




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atmosphere model, e.g. the timing and the duration of the events, the overshooting during the transitions between both model states, the subsequent millennial relaxation process and the decrease of the inter-event waiting times when the forcing amplitude increases. In this sense, the conceptual model apparently has the ability to mimic the main principles that govern the dynamics of DO events in the ocean-atmosphere model CLIMBER-2.

## 3 Spectral properties of random DO events

In the following we use this simple two-state model to evaluate the spectral properties of random DO events. This is done in the following: We force the model by a random, Gaussian-distributed input with white-noise power signature within a certain sprectral interval. Let $\sigma$ be the standard deviation of the noise. The cut-off frequency of the noise is 1/50 years$^{-1}$. In other words, for spectral components with frequency higher than 1/50 years$^{-1}$ the amplitude of the noise-term is zero. For lower frequencies a uniform amplitude distribution ("white noise") is used. The cut-off is applied following the publication of Braun et al. (2007) to account for the fact that the simple two-state model shows an unrealistic large sensitivity to decadal-scale or faster forcing. Note that the magnitude and the spectral composition of the noise in the freshwater flux to the North Atlantic is of course unknown during the last Glacial. Finally we calculate the spectral properties of the model response (i.e. of the threshold function $T$, which resembles the saw-tooth shape of DO events) to the forcing, following standard Fourier spectral analysis. Our focus is the question whether or not red noise is a realistic assumption for the power spectral density distribution of noise-driven DO events.

Figure 4 shows the response of the two-state model for three forcing scenarios with different noise magnitude $\sigma$. As expected, the average spacing between successive events decreases with increasing magnitude of the noise, since a larger forcing can trigger more threshold crossings. The model can show surprisingly regular oscillations on the millennial time scale even when driven by noise only, as depicted in the Fig. 4. In the spectral domain the output of the model can thus show outstanding spectral peaks on the millennial time scale, which are clearly inconsistent with red noise (i.e., with a first-order autoregressive (AR1) process, cf. Fig. 4), despite the fact that the model is driven by a random input with a uniform power spectral density distribution. From Figs. 4 and 5 it is also evident that the events in the output of the system occur on a characteristic time scale, which is the millennial to multi-millennial scale. As expected, this time scale is determined by the magnitude $\sigma$ of the noise (cf. Fig. 5). The fact that the model output occurs on a distinct time scale is also evident from the power spectral density distribution of the simulated events (right column in Fig. 4), which exhibits a prominent maximum on the millennial time scale, corresponding approximately to the inverse of the average spacing between successive DO events in the simulation (Figs. 4 and 5). Leaving this maximum aside, the power spectral density distribution of the simulated events fits a red noise random process fairly well. However, the maximum in the simulated power spectral distribution is considerably larger than expected from a red noise process (Fig. 4). This clearly demonstrates that red noise is not applicable to estimate the statistical significance of the 1470-year spectral peak of DO events, since even in our simple model the use of red noise would typically lead to a strong overestimation of the significance of spectral components on the millennial time scale. In other words, it is not possible the exclude the idea that the pronounced glacial 1470-year peak of DO events is just random and that the reported 99% significance of the peak in the GISP2 ice core record results solely from the inappropriate use of linear methods for analysing highly non-linear processes such as DO events.

As a final comment it should be stressed that our results do not in any way exclude the possibility that DO events exhibit characteristics in their recurrence properties which are indeed inconsistent with a random occurrence. However, this has so far not been shown in a rigorous statistical approach (Braun, 2009; Ditlevsen and Ditlevsen, 2009) and thus needs to be tested in the future. We would recommend methods that allow to estimate the statistical significance without making use of linear theories, e.g. Monte Carlo simulations with models of DO events (Ditlevsen et al., 2007; Braun et al., 2008;





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Ditlevsen and Ditlevsen, 2009) or non-linear methods such as recurrence plots (Marwan and Kurths, 2002; Marwan et al., 2007; Schinkel et al., 2009).

## 4 Conclusions

In this paper it was shown that the spectral properties of highly non-linear processes such as DO events can be fundamentally different from a red noise random process. In this sense, red noise is not applicable to estimate the statistical significance of the 1470-year glacial peak of DO events, since the use of red noise can lead to a strong overinterpretation of the reported 99% significance of that 1470-year peak. More sophisticated, non-linear methods should be used to analyse DO events in future studies.

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**Table 1.** Parameters of the simple two-state model. Values of $A_0$, $A_1$, $B_0$ and $B_1$ are im mSv (1 mSv = 1 milli-Sverdrup = $10^3\ m^3/s$), that it is freshwater flux units, since the model was originally designed to mimic the response of an ocean-atmosphere model to a given freshwater anomaly in the northern North Atlantic. Note that these parameter values are identical to the values used in the original version of the two-state model, cf. supplementary material in the publication of Braun et al. (2005). For these values it was shown that the two-state model is able to mimic the dynamical principles of DO events as simulated with a much more comprehensive ocean-atmosphere model of "intermediate complexity" (Braun et al., 2007).

| Parameter | Value |
|---|---|
| $A_0$ | −27 mSv |
| $A_1$ | 27 mSv |
| $B_0$ | −9.7 mSv |
| $B_1$ | 11.2 mSv |
| $\tau_0$ | 1200 years |
| $\tau_1$ | 800 years |




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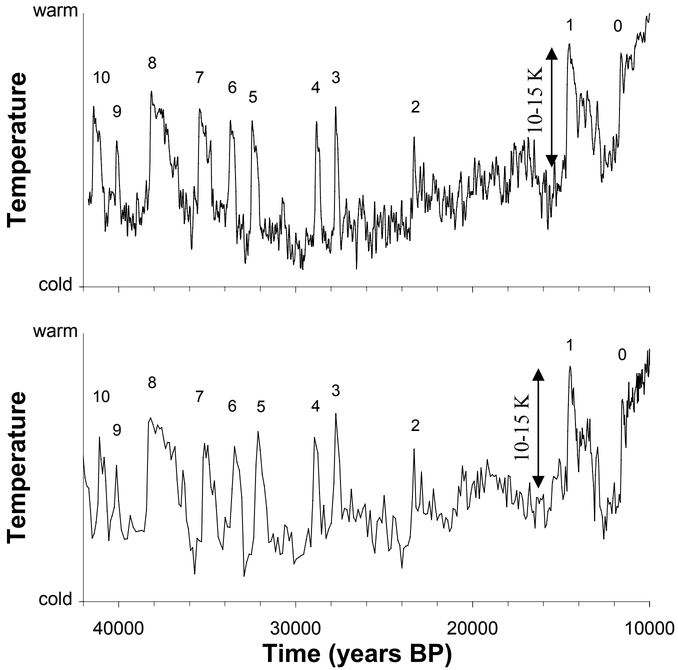


**Fig. 1.** Dansgaard-Oeschger (DO) events 0–10 as seen in two ice core records from Greenland (top: NGRIP, bottom: GISP2) during the time interval between 10 000 to 42 000 years before present, for which the dating is most accurate. Despite the fact that both ice core records were obtained from different locations in Greenland they show a very similar sequence of events, which rules out the possibility that DO events are merely artifacts in the data.

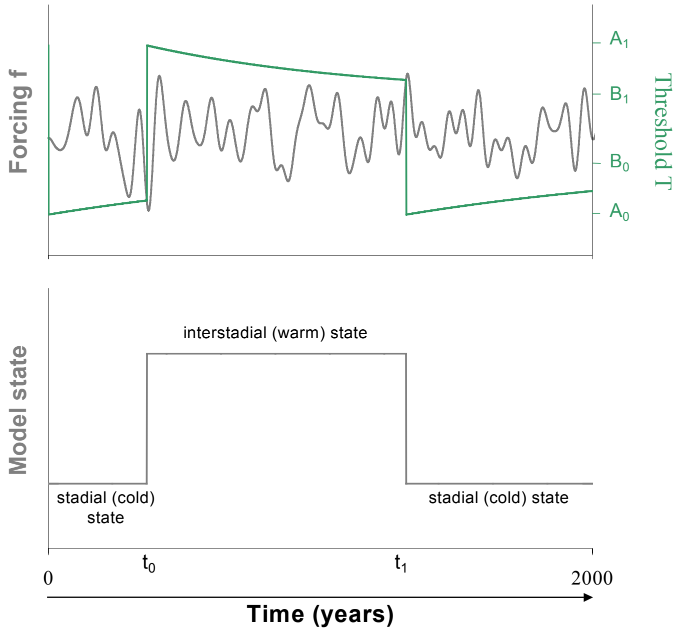


**Fig. 2.** Dynamics of DO events in the two-state model. Top: Forcing (grey) and threshold function (green). Bottom: Model state. A switch from the stadial to the interstadial state is triggered when the forcing falls below the threshold function (at time $t_0$ in the figure). During this switch, which is interpreted as the beginning of a DO event in the model, the threshold function takes a non-equilibrium value ($A_1$) and afterwards approaches its new equilibrium $B_1$ following a millennial scale relaxation process with relaxation time $\tau_1$. The opposite switch, which terminates a DO event in the model, takes place when the forcing exceeds the threshold function (at time $t_1$ in the figure). Again, the threshold function takes a non-equilibrium value ($A_0$) and approaches its new equilibrium value $B_0$ following another millennial scale relaxation process with relaxation time $\tau_0$. All model parameters are given in Table 1.





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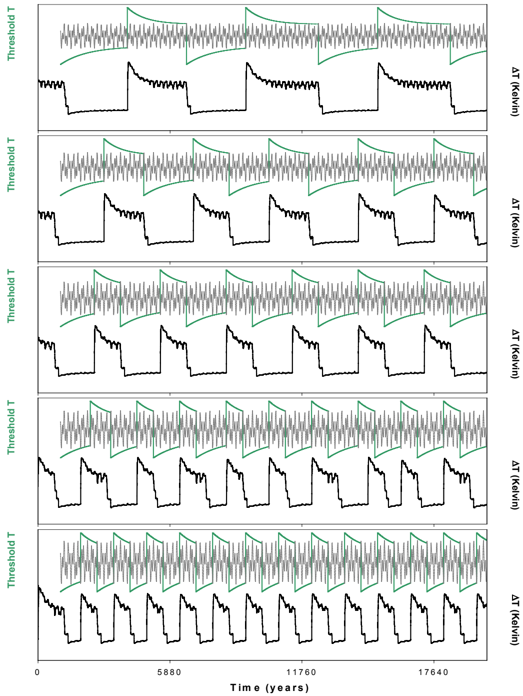


**Fig. 3.** Comparision between the simple two-state model and an ocean-atmosphere model of intermediate complexity. The figure shows the output of the simple two state model (green) and of the ocean-atmosphere model (black), in response to a simple periodic forcing function consisting of two century-scale spectral components of equal amplitude (grey). Note that the forcing amplitude increases from the top to the bottom (from 6 mSv in the top panel to 10 mSv in the bottom panel) whereas the waiting time between successive events decreases. In particular, the two-state model reproduces the onset and the termination of the events in the ocean-atmosphere model fairly well. A more detailed comparison between both models exists in the work of Braun et al. (2007) and in the supplementary material of that publication.





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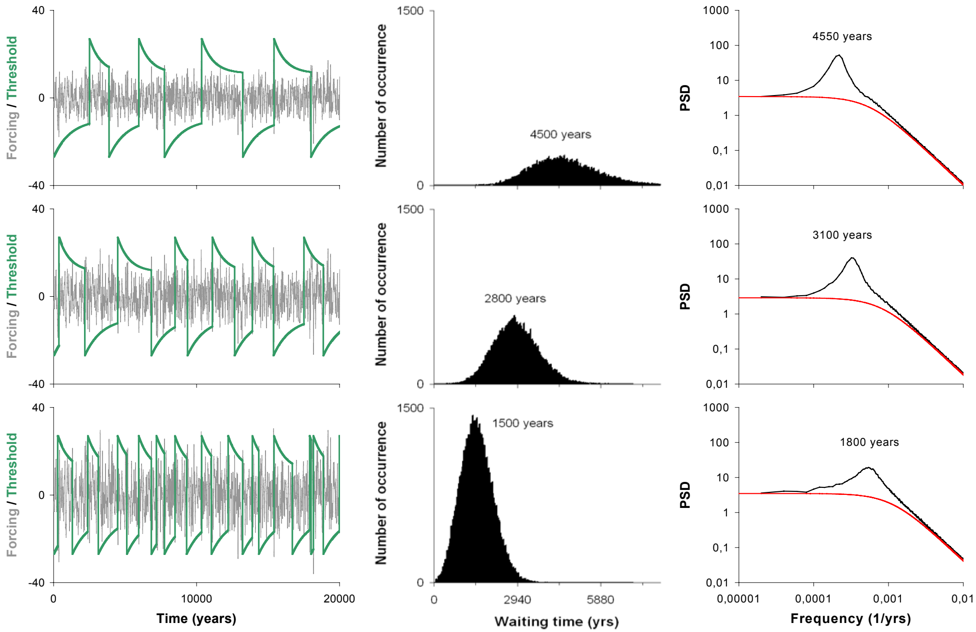


**Fig. 4.** Output of the two-state model. The figure shows the input (grey) and the response of the model (green). The magnitude of the noise is 5.5 (top), 7 (middle) and 9.5 (bottom). Units are mSv = milli-Sverdrup = $10^3$ $m^3/s$. The green curves show the model response in the time domain, i.e. the time evolution of the threshold function (left). The waiting time distribution between successive events is depicted in the middle column. The distributions are obtained from 100 000 000-year runs with the simple two-state model. The right column shows the power spectral density distribution of the simulated events. The power spectral density distributions are obtained from 50 000-year runs, averaged over 1000 different realisations with the same noise magnitude. The red curve represents a theoretical red noise (AR1) random process, cf. Eq. (1). Note that the simulated power spectral density distributions show a pronounced peak at the millennial time scale, with a magnitude that is of the order of 10 times larger than expected from a red noise process.





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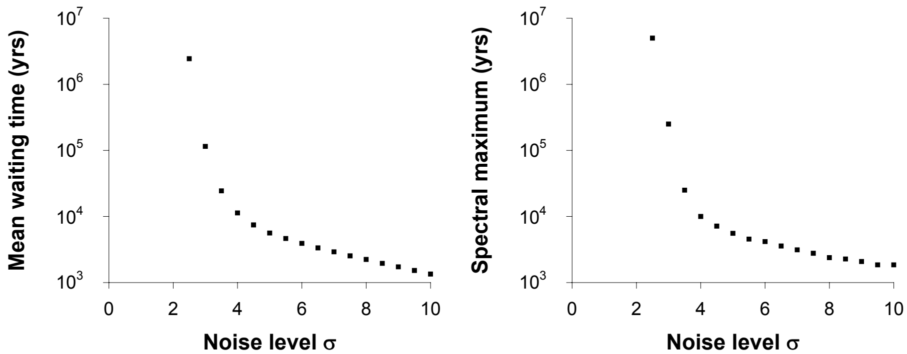


**Fig. 5.** Time scale of the model output as a function of the noise level. The left figure shows the mean waiting time between successive events in the two-state model as a function of the noise magnitude. The right figure shows the period of the leading spectral component (defined by the maximum of the spectral power) in the model output as a function of the noise magnitude. Units of the noise magnitude are mSv = milli-Sverdrup = $10^3\ m^3/s$. No events occur for a noise level of 2 mSv or smaller.