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# Strong indications for nonlinear dynamics during Dansgaard-Oeschger events

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Received: 31 May 2009 – Accepted: 11 June 2009 – Published: 1 July 2009

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Published by Copernicus Publications on behalf of the European Geosciences Union.

CPD

5, 1751–1762, 2009

## Nonlinear dynamics during DO events

H. Braun

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



## Abstract

Many climate records show the occurrence of large amplitude (10–15 K), millennial-scale warming events during glacial times, the Dansgaard-Oeschger (DO) events. So far these events have almost exclusively been investigated by means of linear time series analysis. The scope of this paper is to test if the assumption of linearity is fulfilled during DO events. By means of a surrogate-based Monte Carlo method, I here demonstrate that the 60 000-year long  $\delta^{18}\text{O}$ -record from the NGRIP ice core from Greenland allows to reject the null hypothesis of linearity beyond any reasonable level of doubt. Instead, the ice core record supports the interpretation that the events represent regime switches between different states of operation of glacial climate. As a conclusion, future studies on DO events should focus on the development and the application of more adequate (i.e., nonlinear) methods of time series analysis.

## 1 Introduction

Many climate records, e.g. ice cores, deep sea sediments and speleothems, show the existence of large amplitude (10–15 K), millennial-scale oscillations in glacial climate, the Dansgaard-Oeschger (DO) events, cf. Fig. 1a (Dansgaard et al., 1982; Grootes et al., 1993; Bond et al., 1993; Spötl and Mangini, 2002). The common interpretation is that these events represent regime switches (Oeschger et al., 1984; Broecker et al., 1985; Alley et al., 2003), which implies a highly nonlinear dynamical scenario, i.e. a scenario in which the response of the climate system is not proportional to the forcing. Despite to apparent nonlinearity of the events, methods of linear time series analysis have so far almost exclusively been applied by the paleoclimate community to investigate DO events. For example, the 1470-year spectral peak in the GISP2 ice core was estimated as being statistically significant beyond the 99% level, based on the assumption of red noise (Grootes and Stuiver, 1997; Schulz, 2002). However, it is well known in the nonlinear community that the use of simple linear methods to analyse

## Nonlinear dynamics during DO events

H. Braun

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



highly nonlinear processes can lead to fundamentally wrong conclusions. In this light it is of importance to investigate on pure statistic grounds if DO events are nonlinear or are better explained by a simple linear dynamical scenario, which can be confused with true regime shifts.

## 2 Methods and results

Here I use a 60 000-year long bi-decadal  $\delta^{18}\text{O}$ -record from the NGRIP ice core from Greenland, a standard paleo-temperature proxy, to perform a test of nonlinearity. The study focusses on the time interval between 11 000 and 60 000 years before present (BP). During this interval, DO events are a persistent feature of the time series, cf. Fig. 1a (Svensson et al., 2008). In contrast, no events occur in the younger part of the ice core record, during which Greenland climate is much warmer and shows considerably smaller anomalies than in the glacial part between 11 000 and 60 000 years BP. The younger part between 0 and 11 000 years BP is therefore neglected in this study. For the analysis, the  $\delta^{18}\text{O}$ -record is smoothed by applying a 5-point running average, in order to remove the highest frequency variations. The smoothed record is then investigated by means of null-hypothesis testing.

The basic concept of null-hypothesis testing is as follows (Kantz and Schreiber, 1997; Ditlevsen, 2007): (i) A plausible null-hypothesis is stated which might explain a given data set. The standard null-hypothesis is that the data result from a random linear process, e.g. from an autoregressive process. (ii) A test statistic is defined, based on which the null-hypothesis is tested. In principle, the test statistic can be any function which assigns a real number to a time series. (iii) The distribution of the test statistic is calculated or numerically estimated under the assumption that the null-hypothesis is correct. (iv) If the value of the test statistic as calculated from the data set falls into a low-likelihood region of the test statistic distribution (with probability  $p < 0.05$ , for example), the null-hypothesis is rejected with probability  $1-p$ , in favour of an alternative hypothesis. Otherwise it is not possible to reject the null-hypothesis. Of course, be-

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



ing able to reject the null-hypothesis does not necessarily imply that the alternative hypothesis is correct, since more than one alternative hypothesis might exist.

Here the null-hypothesis is that DO events result from a random, linear, time-invariant process. The test statistic is the mean value  $M$  of the absolute difference (in  $\delta^{18}\text{O}$  units) between two consecutive data points in the NGRIP  $\delta^{18}\text{O}$ -record:  $M = 1/N \cdot \sum_n |x_{n+1} - x_n|$ , where the sum is over all data points and  $N=2447$  is the total number of data points in the time interval under investigation. For the NGRIP  $\delta^{18}\text{O}$  record (Fig. 1a), the value  $M_{\text{NGRIP}}=0.226$  of the test statistic is found. To obtain the distribution of the test statistic, a surrogate-based Monte Carlo simulation with a phase-randomization approach is performed (Kantz and Schreiber, 1997): The observed time series of DO events in the ice core data is transformed into the frequency domain by means of a standard Fourier transformation (Fig. 1b). To obtain a surrogate time series, each spectral component of the NGRIP  $\delta^{18}\text{O}$  time series is then assigned a random phase between 0 and  $2\pi$ . The randomized surrogate is transformed back into the time domain (Fig. 2a), in which the value of the test statistic is calculated. For each single surrogate time series, this randomization procedure is repeated, which finally yields the distribution of the test statistic (Fig. 3). The chosen procedure ensures that each surrogate time series has the same amplitude spectrum as the observed time series (compare Fig. 1b and 2b) and that the phase of each spectral component in the surrogate time series is random. This approach is consistent with the considered null-hypothesis, because in a linear time-invariant system each sinusoidal component in the forcing results in an output component of the same frequency, so that a random input (which has a random phase for each spectral component) results in a sum of sinusoidal output cycles with random phases. In the case of non-linear dynamics, in contrast, the phases of the sinusoidal components of a random output signal are not necessarily randomly distributed, because a sinusoidal input does not necessarily result in a sinusoidal output signal of the same frequency.

Figure 3 shows the distribution of the test statistic as obtained with the phase-randomization approach: This distribution shows non-zero values only within the in-

## Nonlinear dynamics during DO events

H. Braun

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



**Nonlinear dynamics  
during DO events**

H. Braun

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



terval between 0.256 and 0.270. The fact that this interval is narrow is explained by the large number of data points (2447) that are available for the analysis. In contrast to the surrogate time series, the test statistic as obtained from the glacial part of the NGRIP  $\delta^{18}\text{O}$ -record ( $M_{\text{NGRIP}}=0.226$ ) is considerably smaller and does not fall within the simulated test statistic distribution. In this sense, the null-hypothesis that DO events represent a linear, random, time-invariant process is almost certainly wrong, because the average difference between two consecutive data points in the NGRIP ice core record is considerably smaller than expected from such a linear process. In contrast, a much more appealing dynamical scenario to explain the NGRIP record is that DO events represent nonlinear oscillations in the form of repeated regime switches between fundamentally different states of operation of the climate system. In such a scenario it is intuitively expected that the absolute difference  $|x_{n+1}-x_n|$  between two consecutive data points is unusually large during the regime shifts and otherwise small. A similar pattern seems to exist in the NGRIP data (Table 1): When compared with the surrogate time series, in the NGRIP data there is an increased tendency of either small ( $<0.3$ ) or large ( $>0.9$ ) values of  $|x_{n+1}-x_n|$ , whereas the intermediate range (0.3–0.9) is considerably less populated. The interpretation of DO events as regime switches is further supported by the fact that the 40 highest values of  $|x_{n+1}-x_n|$  in the ice core time series coincide with the onset or the termination of DO events. If the events were better described by a stochastic linear process, in contrast, it would be expected that high values of  $|x_{n+1}-x_n|$  occurred randomly throughout the entire time series, which is apparently not the case. The interpretation of DO events as nonlinear oscillations is also corroborated by the characteristic saw-tooth shape of the events, which is not necessarily expected to result from a linear process.

### 3 Discussion and conclusions

The aim of this study was to test on pure statistic grounds if DO events are consistent with a random linear process or are better explained by a nonlinear process, e.g. by

## Nonlinear dynamics during DO events

H. Braun

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



a scenario of repeated regime switchings. Using a nonlinear measure of regularity it was shown that the glacial part of the  $\delta^{18}\text{O}$ -record from the NGRIP ice core from Greenland almost certainly allows to rule out a random linear scenario. The most direct implication of this finding centres on the misuse of linear time series analysis for the investigation of DO events: Many studies exist in which methods of linear time series analysis have been unthoughtfully used to analyse DO events. For example, the statistical significance of the 1470-year spectral peak in the GISP2 ice core from Greenland has been evaluated on the basis of a first-order autoregressive process (Grootes and Stuiver, 1997; Schulz, 2002), i.e. based on the assumption of a red noise background. However, red noise represents the power spectral density distribution of a linear noise-driven process and is probably not applicable for analysing highly nonlinear oscillations such as DO events, because this approach might yield unrealistically large significance values. Moreover, solar variability as the main century-scale driver of climate has commonly been rejected as a possible trigger of DO events, because of the lack of a pronounced solar 1470-year spectral component. However, in a highly nonlinear system a 1470-year spectral component in the output does not necessarily require a 1470-year input component, because in such a system a pronounced spectral correlation is not necessarily expected to exist between the forcing and the response. In addition to that, the possible solar role in triggering DO events was questioned because of the lack of a stable phase relationship between DO events and solar proxies (Muscheler and Beer, 2006). However, in nonlinear systems, and in particular in regime switching processes, the existence of a stable phase relationship between the forcing and the response of the system is not necessarily expected.

Being able to show that the events are inconsistent with a linear dynamical scenario provides a relevant benchmark for future statistical analyses on DO events. In particular, it highlights the need to develop and to use more advanced methods of nonlinear time series analysis such as Monte-Carlo simulations (Ditlevsen et al., 2007) and recurrence plot methods (Marwan and Kurths, 2002; Marwan et al., 2007), for example, in order to acquire a better understanding of what triggered DO events and other po-

tentially nonlinear paleoclimatic processes.

*Acknowledgements.* H. B. was funded by the German Research Foundation (DFG), project number: BR 3911/1-1.

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## Nonlinear dynamics during DO events

H. Braun

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion



Oeschger, H., Beer, J., Siegenthaler, U., Stauffer, B., Dansgaard, W., and Langway, C. C.: Late glacial climate history from ice cores, in: *Climate Processes and Climate Sensitivity*, Geophys. Monogr. Ser., 5, edited by: Hansen, J. E. and Takahashi, T., 299–306, AGU, Washington, DC, 1984.

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## Nonlinear dynamics during DO events

H. Braun

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

Printer-friendly Version

Interactive Discussion





## Nonlinear dynamics during DO events

H. Braun

**Table 1.** Distribution of the distance  $|x_{n+1}-x_n|$  (in units of  $\delta^{18}\text{O}$ ) between two consecutive data points as obtained from the glacial part of the NGRIP  $\delta^{18}\text{O}$  record (Fig. 1a) respectively from the surrogate time series shown in Fig. 2a. The table shows the number of times that the value of  $|x_{n+1}-x_n|$  falls within certain bins.

$ x_{n+1}-x_n $	NGRIP	Surrogate
0–0.1	746	588
0.1–0.2	600	531
0.2–0.3	473	437
0.3–0.4	270	336
0.4–0.5	168	238
0.5–0.6	61	161
0.6–0.7	42	85
0.7–0.8	28	36
0.8–0.9	18	22
0.9–1.0	9	7
>1.0	32	6

Title Page

Abstract

Introduction

Conclusions

References

Tables

Figures

◀

▶

◀

▶

Back

Close

Full Screen / Esc

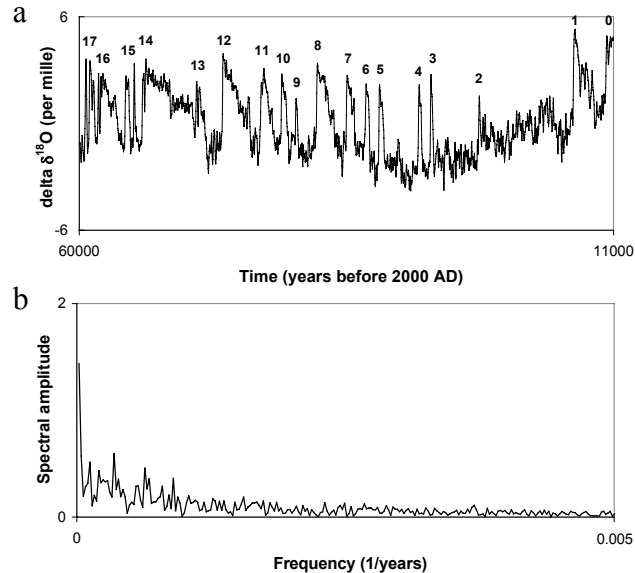
Printer-friendly Version

Interactive Discussion



Nonlinear dynamics  
during DO events

H. Braun

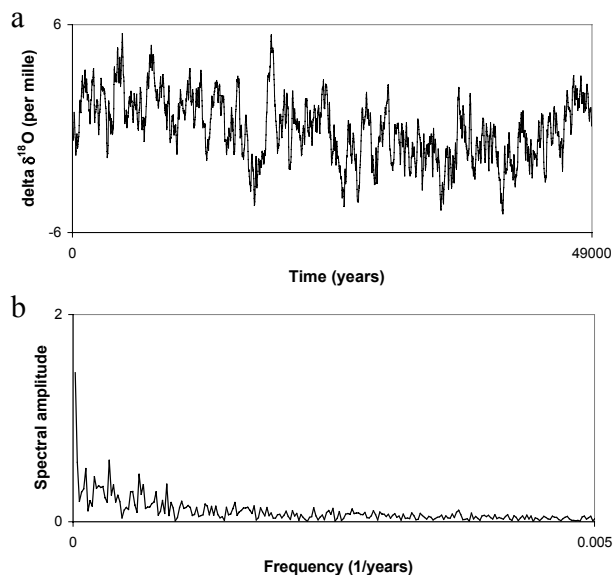


**Fig. 1.** Dansgaard-Oeschger (DO) events 0–17 as seen in the NGRIP ice core record from Greenland during the time interval between 11 000 to 60 000 years before present (Svensson et al., 2008). Top: Time domain. The expression “delta  $\delta^{18}\text{O}$ ” indicates that the average value is removed. In addition to that, a 5-point running mean is applied to the raw data to remove the highest frequency oscillations. Higher  $\delta^{18}\text{O}$  values correspond to warmer conditions in Greenland. DO events manifest themselves as outstanding saw-tooth shaped spikes in the  $\delta^{18}\text{O}$  time series. Bottom: Spectral domain.

[Title Page](#)[Abstract](#)[Introduction](#)[Conclusions](#)[References](#)[Tables](#)[Figures](#)[◀](#)[▶](#)[◀](#)[▶](#)[Back](#)[Close](#)[Full Screen / Esc](#)[Printer-friendly Version](#)[Interactive Discussion](#)

Nonlinear dynamics  
during DO events

H. Braun

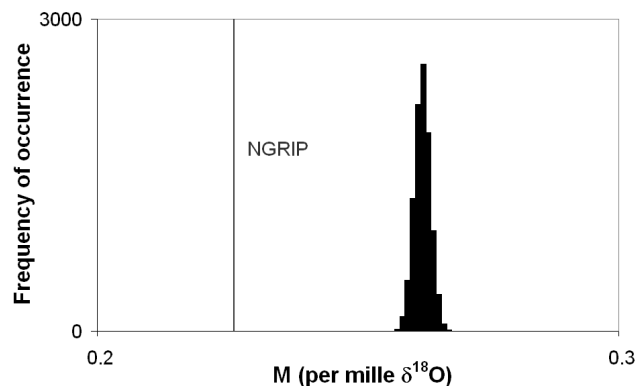


**Fig. 2.** Surrogate time series. The figure shows one example of the surrogate time series as obtained with the phase-randomization procedure described in the text. Top: Time domain. Bottom: Spectral domain. Note that the amplitude spectrum of each randomized time series is identical to the amplitude spectrum of the NGRIP  $\delta^{18}\text{O}$  record (Fig. 1b).

[Title Page](#)[Abstract](#)[Introduction](#)[Conclusions](#)[References](#)[Tables](#)[Figures](#)[◀](#)[▶](#)[◀](#)[▶](#)[Back](#)[Close](#)[Full Screen / Esc](#)[Printer-friendly Version](#)[Interactive Discussion](#)

Nonlinear dynamics  
during DO events

H. Braun



**Fig. 3.** Monte Carlo simulation. The figure shows the test statistic distribution, i.e. the distribution of the measure  $M$  as obtained from the surrogate time series (ensemble size: 10 000 members), together with the value obtained from the NGRIP ice core (depicted by the grey line). Note that the x-axis starts at  $M=0.2$ . Beyond any reasonable level of doubt, the value of the test statistic as obtained from the NGRIP  $\delta^{18}\text{O}$  record is inconsistent with the null-hypothesis of a random, linear, time-invariant process.

[Title Page](#)[Abstract](#)[Introduction](#)[Conclusions](#)[References](#)[Tables](#)[Figures](#)[◀](#)[▶](#)[◀](#)[▶](#)[Back](#)[Close](#)[Full Screen / Esc](#)[Printer-friendly Version](#)[Interactive Discussion](#)