On the verification of climate reconstructions

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Received: 30 November 2006 – Accepted: 25 January 2007 – Published: 31 January 2007
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Abstract

The skill of proxy-based reconstructions of Northern hemisphere temperature is reassessed. Using an almost complete set of proxy and instrumental data of the past 130 years a multi-crossvalidation is conducted of a number of statistical methods, producing a distribution of verification skill scores. The scores show considerable variation for all methods, but previous estimates, such as a 50% reduction of error ($RE$), appear as outliers and more realistic estimates vary about 25%. It is shown that the overestimation of skill is possible in the presence of strong persistence (trends). In that case, the classical “early” or “late” calibration sets are not representative for the intended (instrumental, millennial) domain. As a consequence, $RE$ scores are generally inflated, and the proxy predictions are easily outperformed by random-based, a priori skill-less predictions.

To obtain robust significance levels the multi-crossvalidation is repeated using predictors based on red noise. Comparing both distributions, it turns out that the proxies perform significantly better for almost all methods. The nonsense predictor scores do not vanish, nonetheless, with an estimated 10% of spurious skill based on representative samples. I argue that this residual score is due to the limited sample size of 130 years, where the memory of the processes degrades the independence of calibration and validation sets. It is likely that proxy prediction scores are inflated correspondingly, and have to be adjusted further.

The consequences of the limited verification skill for millennial reconstructions is briefly discussed.

1 Introduction

Several attempts have been made to reconstruct the millennial history of global or Northern hemisphere temperature (NHT) by way of proxy information (Overpeck et al., 1997; Jones et al., 1998), (Mann et al., 1998, henceforth MBH98), (Mann et al., 1999;
Crowley and Lowery, 2000; Briffa, 2000; Briffa et al., 2001; Esper et al., 2002; Moberg et al., 2005). Since past variability is essential for the understanding of, and attributing forcing factors to the present climate some of these reconstructions have played a prominent role in the last report of the IPCC (IPCC, 2001). This was followed by an intense debate about the used data and methods (McIntyre and McKitrick, 2003; von Storch et al., 2004), (McIntyre and McKitrick, 2005a, henceforth MM05). (Rutherford et al., 2005b; Mann et al., 2005; Bürger and Cubasch, 2005; Huybers, 2005; McIntyre and McKitrick, 2005b; Bürger et al., 2006; Wahl et al., 2006; Wahl and Ammann, 2006). While that debate mostly turned on the variability and actual shape of the reconstructions (the "hockey stick") the aspect of verification has not found a comparable assessment.

In the above models (that term used informally here to mean any empirical scheme), a limited number of proxies – usually in the order of several dozens – serve as predictors, either for the local temperature itself or for some typical global pattern of it. The models are defined/calibrated in the overlapping period of instrumental data, and predicted back to those years of the past millennium where proxies are available but temperature observations are not. Calibrating is done, in one way or another, by optimizing the model skill for a selected sample (the calibration set) and is almost certainly affected by the presence of "sampling noise". This renders the model imperfect, and its "true" skill is bound to shrink. But it is this skill that is relevant when independent data are to be predicted (cf. Cooley and Lohnes, 1971).

Instrumental temperatures are available only back until about 1850. Therefore, the period of overlap is just a small fraction of the intended millennial domain. It is evident that empirical models calibrated in that relatively short time span (or even portions of it) must be taken with great care and deserve thorough validation. This applies even more since proxy and temperature records in that period are strongly trended or persistent, which considerably reduces the effective size of independent samples that are available to fit and verify a model.

It is therefore essential to find robust estimates of the predictive model skill, as a ba-

sis for model selection as well as for the general assessment of the resulting temperature reconstructions. Besides analytical approaches to estimate the true predictive skill from the shrinkage of the calibration skill (Cattin, 1980; Raju et al., 1997) various forms of cross validation are utilized, where skill is accordingly being referred to as cross-validity (see below). Simple cross validation (Mann et al., 1999; Cook et al., 2000; Luterbacher et al., 2002; Guiot et al., 2005, MBH98) proceeds as follows: From the period of overlapping data with both proxy and temperature information a calibrating set is selected to define the model. This model is applied to the remaining independent set of proxy data (as a guard against overfitting), and modeled and observed temperature data are compared. A more thorough estimate, called double cross validation, is obtained by additionally swapping calibrating and validating sets (Briffa et al., 1988, 1990, 1992; Rutherford et al., 2003, 2005). Multiple cross validation ("multi-crossvalidation") using random calibration sets (Krus and Fuller, 1982) is a form of bootstrapping (Efron, 1979; Efron and Gong, 1983) that, to my knowledge, has been applied only once (Fritts and Guiot, 1990), in the context of a single site study with rather moderate trends. In this study, that approach will be applied to the NHT.

Multi-crossvalidation makes explicit a basic principle of statistical practise: that skill estimates are always affected by random properties of the sample from which they were derived. In other words: that the skill of a model, be it calibration or validation skill, is a random variable. Accordingly, picking out of several alternatives the best scoring version as the "true" model is bound to introduce a sampling bias and, moreover, as has been pointed out elsewhere (cf. Bürger et al., 2006), basically renders the model unverified. This equally applies to any other possible variation in the model setting, as long as there is no a priori argument against its use.

Like any bootstrapping, multi-crossvalidation is blind to any predefined (temporal) structure on contiguous calibration or validation periods, such as the 20th century warming trend, and will pick its sets purely by chance. This appears to entirely conflict with a dynamical approach, since any "physical process" that one attempts to reflect (cf. Wahl et al., 2006) is destroyed that way. However, empirical models of this kind do
in no way contain or reflect dynamical processes beyond properties that can be sampled in instantaneous covariations between the variables. The trend may be an integral part of such a model, but only as long as it represents these covariations.

To estimate whether a verification score represents a significantly skillful prediction it must be viewed relative to score levels attained by skill-less, or "nonsense", predictions. This is necessary because such predictions, in fact, may attain nonzero values for some of the scores. Inferences based on nonsense ("spurious", "illusory", "misleading") correlations turn up since the first statistical measures of association came to light (Pearson, 1897; Yule, 1926), and are a typical byproduct of small samples; see also Aldrich (1995).

There is some analogy to classical weather forecasting where climatology and persistence serve as skill-less predictions whose scores are, especially in the case of persistence, not so easy to beat. While the notion of a skill-less prediction is common sense in weather forecasting, it is the subject of considerable confusion and discussion in the field of climate reconstruction. To give an example: for the reduction of error ($RE$, see below) in NHT reconstructions, MBH98 and MM05 report the level of no skill to be as different as 0% and 59%, respectively. On this background, the usefulness of millennial climate reconstructions, such as MBH98 with a reported $RE$ of 51%, depends on the very notion of a nonsense predictor. This confusion evidently requires a clarification of terms. Towards that goal, the study begins by analyzing and discussing a very basic example of a nonsense prediction with remarkable $RE$ scores. This is followed by a more refined bootstrapping and significance analysis, with models that are currently in use for proxy reconstructions. Having obtained levels of skill and significance the consequences for millennial applications are reflected.

2 Skill calculations, and shrinkage

The study is based on proxy and temperature data that were used in the MBH98 reconstruction of the 15th century. Specifically, the multiproxy dataset, $P$, consists of the 22 proxies as described in detail in the MBH98 supplement. To meet the bootstrapping conditions of a fixed set of model parameters, the 219 temperature grid points, $T$, are used that are almost complete between 1854 and 1980, and which were used by MBH98 for verification (see their Fig. 1). This gives 127 years of common proxy and temperature data. Note that the proxies represent a typical portion of what is available back to AD 1400, showing a large overlap with comparable studies (cf. Briffa et al., 1992; Overpeck et al., 1997; Jones et al., 1998; Crowley and Lowery, 2000; Rutherford et al., 2005). Other studies, such as Briffa et al. (2001) or Esper et al. (2002), relied on these proxies as well but processed them differently.

The verification measures in the above studies are usually borrowed from the verification of classical weather forecasting, such as $RE$ or simple correlation. $RE$ relates the squared model error to the squared anomalies from the climatological mean, and thus equals the skill score relative to the climatology forecast (Lorenz, 1956; Wilks, 1995). Note that in this stationary context climatology is usually taken as a constant, equal for calibration and validation. This changes in Fritts (1976) and Briffa et al. (1988); see also Cook et al. (1994) where reference is explicitly made to the calibration (dependent) period mean. For this case, Briffa et al. (1988) note that instationarities, such as systematic differences between calibration and validation period mean, can artificially inflate $RE$ scores (see below). To account for this deficiency they suggest a measure, the "coefficient of efficiency", $CE$, that relates the model error explicitly to anomalies from the validation period mean, and attribute that measure to Nash and Sutcliffe (1970). While $CE$ is a useful measure that is now frequently applied, it is not the "efficiency of a model" as defined by Nash and Sutcliffe (1970), as that happens to be nothing else than $RE$, with no explicit mention of a validation period. (Note also that Cook et al. (1994) incorrectly refer to Nash and Sutcliffe (1970) as a multiple regression study.)

Suppose now that we have formulated a statistical model and estimated its parameters from some calibration set. Now true (observed) and modeled NHT, denoted here by $x$ and $\hat{x}$, respectively, are to be compared. For ease of notation I assume in this
section that validation is done using the entire population, with a population mean of zero. Denoting the calibration mean by $\bar{x}_c$, $RE$ and $CE$ (in the sense of Briffa et al., 1988) are given as

$$RE = 1 - \frac{\langle (\hat{x} - x)^2 \rangle}{\langle (x - \bar{x}_c)^2 \rangle} \quad ; \quad CE = 1 - \frac{\langle (\hat{x} - x)^2 \rangle}{\langle x^2 \rangle}.$$  

(1)

with brackets indicating expectation. These two scores, along with the correlation between true and modeled values, $R_c$, are now very simply related. Using the following three forms of relative bias: the calibration mean bias, $\alpha = x_c/\sqrt{\langle x^2 \rangle}$, and the two biases in mean, $\beta = \langle \hat{x} \rangle/\sqrt{\langle x^2 \rangle}$, and amplitude, $\gamma = \sqrt{\langle (\hat{x} - \langle \hat{x} \rangle)^2 \rangle/\langle x^2 \rangle}$, one gets (see Appendix):

$$CE = \gamma (2R_c - \gamma) - \beta^2,$$

(2)

$$RE = \frac{CE + \alpha^2}{1 + \alpha^2}.$$

(3)

For example, applying a multiple regression for the complete population gives $R^2 = R^2_c$ as the squared multiple correlation (coefficient of determination) and, since in that case $\alpha = \beta = 0$ and $\gamma = R$, Eq. (2) gives the classical result $CE = RE = R^2$. From Eqs. (2) and (3) it follows generally $CE \leq R^2_c$ and $CE \leq RE$. That $CE \leq R^2_c$ has the important consequence that skill-less predictions, for which $R_c = 0$, must have $CE \leq 0$. Equation (3) illustrates the dependence of $RE$ on the calibration mean bias, $\alpha$, and how large $\alpha$ values inflate that score. For example, if $\alpha = 1$, that is, one standard deviation, a score of $CE = 0$% would yield $RE = 50$%. This applies, e.g., to time series that exhibit long memory, such as a trend, be it deterministic or stochastic. For example, Wahl and Ammann (2006) report for their MBH98 emulation $RE$ and $CE$ validation scores of 48% and ~22%, respectively. That discrepancy is solely caused, as calculated from Eq. (3), by a calibration mean bias of $\alpha = 1.2$. Similar sensitivities are reported by Rutherford et al. (2005) and Mann et al. (2005).

For an impression of what skills, and in particular what shrinkage thereof, might generally be expected let us consider, as the most straightforward statistical model, a multiple regression of average NHT on $p$ proxies, using $N$ years of calibration. In view of the intended domain, the full millennium, these $N$ years will always be small and our estimate imperfect. With increasing sample size, $N$, and with decreasing number of required predictors, $p$, the model would generally improve. This dependency has been approximated analytically for the first time by Lorenz (1956). A refined estimate can be described as follows: Let $R^2$ denote the squared multiple correlation of a model with $p$ predictors estimated from a sample of $N$ years, adjusted for $p$ (cf. Seber and Lee, 2003). From that, an unbiased estimate of the squared correlation to be expected from a prediction is calculated as (Nicholson, 1960; Cattin, 1980):

$$\hat{R}^2_c = \frac{(N - 1)\hat{R}^4 + \hat{R}^2}{(N - p)\hat{R}^2 + p}.$$  

(4)

This estimate of $R^2_c$, also called the cross validity, thus describes the shrinkage in skill that is to be expected for predictions of a model estimated from $N$ years and $p$ predictors, having an adjusted calibration skill of $\hat{R}^2$. The dependence of $\hat{R}^2_c$ on $\hat{R}^2$ is shown in Fig. 1 for the $P$ and $T$ setting with $N = 127$ and $p = 22$. Even with very large multiple correlations the cross-validity remains quite moderate, so that, for example, to achieve $\hat{R}^2_c = 50$% one already needs $\hat{R}^2 = 80$%. Conversely, a score of $\hat{R}^2 = 36$%, as for regressing NHT from the full instrumental period, dramatically shrinks to a cross validity of only 6%.

Equation (4) applies to models estimated by ordinary least squares and thus to all reconstructions ("predictions") that are based on some form of multiple regression. It should illustrate the order of magnitude that is to be expected from shrinking, given a ratio of predictors and sample size that is typical for millennial climate reconstructions. Estimates based on multi-crossvalidation shall be provided in §5.
3 The trivial NHT predictor

Having studied the close relation between RE and CE via the calibration mean bias, \( \alpha \), let us turn our attention now to the possible causes of such bias, using a very basic example. Figure 2 shows the average NHT as estimated from the set of 219 temperature grid points, \( T \). There is an obvious imbalance between the early and the late half of the period: while colder, even cooling conditions prevail in the early portion, much warmer conditions, initiated by a strong positive trend between 1920 and 1940, dominate the second half. Along with NHT, the linear model is plotted that results from regressing the late portion (1917–1980) against a very simple predictor: the series of calendar years. I will call this the trivial model or trivial predictor. This is in effect nothing more than fitting a linear trend to that portion. And as a positive trend, the trivial model predicts colder conditions for the past earlier portion. While this does not seem to be an overwhelming performance, the model attains for that part (1854–1916) a verification RE score of 56%! Recalling that RE measures the relative improvement to the climatology forecast, \( \bar{x}_c \), indicated by the zero line, the trivial model outperforms that forecast easily by simply predicting colder conditions.

On the other hand, the trivial prediction attains a CE of \(-70\%\). According to Eq. (3), this large discrepancy is caused by the enormous bias in the calibration mean of \( \alpha = 1.7 \) standard deviations (recall that \( \alpha = 1.2 \) from the last section is based on a 1902–1980 calibration period). At this point it is important to understand what – besides the presence of the overall trend – leads to that bias. The trend is obviously only effective because of the clean temporal separation of calibration and validation sets. Large values of \( \alpha \), and thus high RE scores, are obtained because of a) a positive trend in the late calibration and b) negative anomalies in the early validation. In general, it needs a calibration trend of the same sign as the mean difference between late and the early portion.

To clarify the interplay between trend and the degree of temporal separation the following Monte Carlo exercise is performed. Starting from the original partition with the 1917–1980 (1854–1916) late calibration (early validation) period, single calibration and validation years are iteratively swapped, the latter being picked randomly, and a regression model is calibrated. After a certain amount of swappings, here 100, the initial separation is lost and calibration and validation years are equally distributed. Each of the generated configurations is now once more “mirrored” by exchanging calibration and validation sets. For each step, the individual degree of separation can be measured, for example, by the relative difference

\[
\text{degree of separation} = \frac{\bar{T}_c - \bar{T}_v}{\bar{T}_{\text{late}} - \bar{T}_{\text{early}}}
\]

where \( \bar{T} \) indicates the mean of the respective calendar years (with subscripts c and v indicating calibration and validation, respectively). In each step the verification scores RE and CE of the corresponding model are calculated, relating it to the degree of separation. Each single relation is, however, bound to be noisy due to the random selections of calendar years. The above is therefore repeated 500 times to study the average behavior, as shown in Fig. 3. It shows a smooth dependence of the average RE and CE values on the degree of temporal separation. Both scores show opposite behavior, with RE preferring positive and CE negative values. RE values rise from about 30\% for the full mixture to almost 60\% for the full separation of the late calibration, while the early calibration shows much lower scores due to the missing, or negative, trend there. CE is more symmetric about the full mixture. There, CE nearly equals RE, while it strongly decreases to about \(-50\%\) at both ends of the full separation. It is thus found that a trend creates enormous RE scores, but at least half of it is due to the particular selection of calibration and validation sets.

The statistics of each single calibration set is now, with varying degree, representative of the full set (population). As a simple measure of that representativeness one can, for example, test the hypothesis that the NHT values from the calibration and those of the full set are equally distributed, using the Mann-Whitney (ranksum) test, and take the resulting \( p \)-value. Accordingly averaged over the 500 realizations one
finds, not surprisingly, a strong dependence of that index on the degree of separation (see Fig. 3). It is symmetric about zero separation, i.e. full mixture, with a maximum attained there and calibration sets that are representative. At both ends, under full separation, the values are practically zero and the calibration sets not representative. It is at these minima where both scores, $RE$ and $CE$, happen to show the most extreme values.

Note that this representativeness is closely related to the missing-at-random (MAR) criterion that is important for the imputation of missing data and algorithms such as EM and RegEM (see below; cf. Rubin, 1976; Little and Rubin, 1987). It is also relevant for the extrapolation argument given by Bürger and Cubasch (2005).

One could have used other predictors, such as, for example, the number of reporting stations for the temperature grid points (which scores 35% $RE$ for the late calibration). They will give similar results as long as the predictor contains a trend. In view of the intended time span – the full millennium – however, such simple, trend-based predictors are obviously not useful as they would just extrapolate the trend backwards into the millennium and produce unrealistic cooling. Hence, at least in this simple case not much useful information is to be expected from the $RE$ scores.

I will now turn to “real” predictors, that is, proxy information made up of tree-rings, corals, ice cores, etc., and the more sophisticated empirical models that make use of them.

4 Reconstruction flavors

Several statistical methods exist or have extra been developed to derive millennial NHT from proxy information. They are distinguished by using or not using a number of independent options in the derivation of the final temperature from the proxies. These options mainly pertain to the specific choice of the preprocessing, the statistical model, and the postprocessing.

The methods basically fall into two categories: those which employ a transfer function and those which employ direct infilling of the missing data. In the first approach, the heterogeneous proxy information is transformed to a temperature series by means of a transfer function that is estimated from the period of overlapping data. In the second approach, data are successively infilled to give a completed dataset that is most consistent (see below) with the original data. The transfer function approach uses either some a priori weighting of the proxies, based on, e.g., areal representation, or a weighting directly fitted from the data, that is, multiple regression. To reduce the number of weights in favor of significance, several filtering techniques can be applied, such as averaging or EOF truncation on both the predictor (Briffa et al., 1988, 1992) and the predictand side (MBH98; Evans et al., 2002; Luterbacher et al., 2002).

4.1 Preprocessing (PRE)

Besides using

1) NHT directly as a target, that is, calibrating the empirical model with the NH mean of the $T$ series, so that no spatial detail is modeled at all,

intermediate targets can be defined, as follows:

2) PC truncation. Here a model is calibrated from the dominant principal components (PCs) of $T$, and a hemispheric mean is calculated from their reconstruction. This is applied by MBH98, who have used a single PC. To be compatible with that study I also used only one PC (explaining about 20%–30% depending on the calibration set).

3) full set. The third possibility, applied by Mann and Rutherford (2002); Rutherford et al. (2003, 2005); Mann et al. (2005), does not apply any reduction at all to the target quantity, treating the entire set of temperature grid points (more than 1000 in those studies) as missing. In our emulation, the full set $T$ of 219 temperature grid points is set to missing. From the reconstructed series the NH mean is calculated.
4.2 Statistical method (METH)

The reconstruction of temperatures from proxies can be viewed in the broader context of infilling missing values. The infilling is done by using either a transfer function between knowns and unknowns that is fitted in the calibration (1–4 below), or in a direct way using iterative techniques (5, 6):

1) Classical (forward) regression. Between the known $P$ and unknown $T$ quantities, a linear relation $R$ is assumed, as follows:

$$T = RP + \varepsilon,$$

where $\varepsilon$ represents unresolved noise. The matrix $R = \Sigma_P^{-1}\Sigma_{PT}$, with $\Sigma_{xy}$ denoting the cross covariance matrix between $x$ and $y$ (taking $\Sigma_x = \Sigma_{xx}$), is determined by least squares (LS) regression, with $T$ assumed to be noisy.

2) Inverse (backward) regression. This method is applied by MBH98. It also uses a linear model as in 1), but now $P$ is assumed noisy, leading to the LS estimate $R^I = \Sigma_T^{+}\Sigma_{TP}$ ($^{+}$ denoting pseudo inverse).

3) Truncated total least squares (TTLS). This form of regression, in combining 1) and 2), assumes errors in both quantities $P$ and $T$ (cf. Golub and Loan, 1996). The 10 major singular values were retained.

4) Ridge regression. As 1), but with an extra offset given to the diagonal elements of the (possibly ill-conditioned) matrix $\Sigma_p$ used as regularization parameters (Hoerl, 1962).

5) EM. Unlike using a fixed transfer function defined from a calibration set, there are methods that exploit all available information when infilling data, including those from a validation predictor set. A very popular method uses the Expectation-Maximization (EM) algorithm, which provides maximum-likelihood estimates of statistical parameters in the presence of missing data (Dempster et al., 1977). EM is applied using the more specialized regularized EM algorithm, RegEM (see below), with a vanishing regularization parameter.

6) RegEM. RegEM has been invented to utilize the EM algorithm for the estimation of mean and covariance in ill-posed problems with fewer cases than unknowns (cf. Schneider, 2001). It was intended for, and first applied to, the interpolation/completion of large climatic data sets, such as gridded temperature observations, with a limited number of missing values (3% in Schneider, 2001). The technique was then extended to proxy-based climate reconstructions (with a rate of missing values easily approaching 50%) and seen as a successor of the MBH98 method (Mann and Rutherford, 2002; Rutherford et al., 2003, 2005; Mann et al., 2005). Note, however, that millennial applications utilize rather few proxies, so that the infilling problem is no longer ill-posed and the much simpler EM could have been used. Moreover, the reported millennial verification $RE$ of RegEM is less than that of the original MBH98 (cf. Rutherford et al., 2005). The performance of EM and RegEM are here compared for the first time. Details on RegEM are given in the Appendix.

4.3 Postprocessing (POST)

In applications (e.g. verifications) the output of the statistical model is either taken 1) as is or 2) rescaled to match the calibration variance (cf. Esper et al., 2005; Bürger et al., 2006). Note that this operation increases the expected model error.

As all of PRE, METH, and POST represent independent groups of options, they can be combined to form a possible reconstruction “flavor” (cf. Bürger et al., 2006). As a reference, each such flavor receives a code $\varphi$ in the form of a triple from the set $(1,2,3)\times(1,2,3,4,5,6)\times(1,2)$, indicating which options were selected from the 3 groups above. This defines a set of $3\times6\times2 = 36$ flavors. For example, the MBH98 method corresponds to flavor $\varphi = 222$ and Rutherford et al. (2005) to $\varphi = 161$. Table 1 illustrates the various settings.
5 Multi-crossvalidation of NHT reconstructions

I consider 300 random partitions \( \pi \) of the set \( I = \{1854, \ldots, 1980\} \) of calendar years,
\[
I = C_\pi \cup V_\pi ,
\]
into calibration and validation sets \( C_\pi \) and \( V_\pi \), where both sets are roughly of equal size (\(|C_\pi| = 64 \) and \(|V_\pi| = 63\)). For any of the 36 flavors, \( \pi \), it is now possible to calibrate an empirical model, with corresponding scores \( RE_\pi(\pi) \) and \( CE_\pi(\pi) \). \( RE_\pi(\pi) \) and \( CE_\pi(\pi) \) thus appear as realizations of random variables \( RE_\pi \) and \( CE_\pi \), with corresponding distributions. Along with the 300 random partitions I also consider the two complementary partitions with full temporal separation.

The distributions of \( RE_\pi \) and \( CE_\pi \) are depicted in Fig. 4 as a boxplot. For most flavors the distributions show a remarkable spread, with minimum and maximum (low and high 10%-quantiles) easily departed by more than 50% (20%) of skill. Moreover, between the flavors the distributions are quite different. For example, the flavors \( \pi = 161 \) and \( \pi = 182 \) are merely distinguished by the use of rescaling. Their performance, however, is grossly different. This applies likewise to the flavors \( \pi = 141/2 \) and \( \pi = 151/2 \), so that at least in these cases skill is strongly degraded by rescaling (note, however, \( \pi = 261/2 \)). While there is so much spread in skill within and between the flavors the distributions themselves are quite similar for both scores \( RE_\pi \) and \( CE_\pi \). This indicates that, in fact, most calibration/validation partitions are temporally well mixed and \( RE_\pi \) and \( CE_\pi \) measure the same thing (see §3).

The skill varies, but it varies on rather low levels. The 90% quantile hardly exceeds the 30% mark, and the highest median is \( RE_\pi = 26.5\% \) and \( CE_\pi = 24.6\% \) for \( \pi = 262 \). Generally, flavors of the form \( x1x \), i.e. those predicting PC1 of NHT, perform much better, with almost all medians above 20%. The other flavors are much more variable, partly caused by the degradation from rescaling mentioned above. An exception are the flavors of the form \( x61 \) which show remarkably little variance (albeit only moderate scores). This is understandable insofar as RegEM, unlike the other flavors, depends on the particular calibration set only in terms of the predictand (utilizing the full instrumental period for the predictors). This would also apply to the EM flavors (\( x51 \)), but they are probably more susceptible to overfitting. Note that the flavor \( \pi = 311 \), which has shortly been touched in §2 to exemplify shrinking, scores very little, with \( RE \) and \( CE \) values below 5%. This is about the same order of magnitude as the estimate obtained from Eq. (4).

The mindful reader has noticed that some flavors, such as \( \pi = 111 \) and \( \pi = 311 \), have identical distributions. In fact, for direct regression, with a linear dependence of the estimated model on the predictand, cf. §4.2, they are equivalent with respect to NHT and thus redundant. (Note that the RegEM flavors \( \pi = 161 \) and \( \pi = 361 \) are similar as well.)

The triangles in the figure represent the two calibrations with full temporal separation, i.e. the periods 1917–1980 (upper triangle) and 1854–1916 (lower triangle). They are more comparable to estimates of previous studies and obviously assume the role of outliers, in a positive sense for \( RE \) and in a negative one for \( CE \). While several \( RE \) values approach 50% the \( CE \) values are negative throughout. Models with trended and fully separated calibration sets are thus rewarded with high \( RE \) scores but penalized with low \( CE \) scores.

Based on such levels of performance it is difficult to declare one specific flavor as being the “winner” and being superior to others. Just from the numbers, the flavor \( \pi = 262 \) gives the best \( RE \) performance (see above). It predicts PC1 using RegEM and rescaling. But it is only marginally better than, e.g., the simpler variant 211 (simple forward regression, with median 23.4%). Note that the flavor 161 was promoted by Mann et al. (2005) and earlier to replace the original MBH98 flavor 222. From the current analysis, this cannot be justified (\( RE \) median of 21.8% compared to 25.9%).

This is somewhat in agreement with Rutherford et al. (2005) who report a millennial \( RE \) of 40% (46% for the “hybrid” case), as compared to the 51% of MBH98. Moreover, for the late calibration the 161 flavor is particularly bad (\( RE_5 = 11.9\% \)); it improves, nonetheless, when calibrating with the “classical” calibration period 1902–1980 (28%).
6 Significance

There is an ongoing confusion regarding the notion of significance of the estimated reconstruction skill. For the same model (the one used by MBH98, here the emulated flavor 222), MBH98 (resp. Huybers, 2005) and McIntyre and McKitrick (2005b) report a 99% significance level for $RE$ as different as 0% and 54%! Hence, with a reported $RE$ of 51% the model is strongly significant in the first interpretation and practically useless, i.e. indistinguishable from noise, in the latter. And what might be even more intriguing: The trivial model of §3 with an $RE$ score of 56% turns out to be significantly skillful under both interpretations. Obviously, the notion of “being significant”, or of being a “nonsense predictor”, deserves a closer look.

A major difference in the two approaches is the allowance for nonsense regressors for the significance estimation, because only that yields higher scores. Now even in the well-mixed, representative case the trivial predictor scored about 20% in both $RE$ and $CE$, which would still be significantly skillfull under a significane level of 0%. To avoid this, nonsense regressors must therefore be allowed. On the other hand we have seen how the temporal separation produces non-representative samples, and creates $RE$ “outliers” of up to 60%. The proposed significance level of $RE = 54\%$, which is based on these outliers, is thus equally inflated and must be replaced by something more representative.

A crucial question is: What kind of nonsense predictors should be allowed? – To derive a statistically sound significance level requires a null distribution of nonsense reconstructions. Now one can think of all sorts of funny predictors, things like calendar years, Indias GDP, the car sales in the U.S., or all together, etc., but that will not make up what mathematically is called a measurable set (to which probabilities can be assigned). Hence, a universal distribution of nonsense predictors does not exist. – A more managable type of nonsense predictors are stochastic processes generated from white noise, such as AR, ARMA, ARFIMA, ..., (cf. Brockwell and Davis, 1998). Once we fix the number of regressors, the type of model, say ARMA($p,q$), and the set of parameters, a unique null distribution of scores can be obtained from Monte Carlo experiments. From these, a significance level can be estimated and compared to the original score of the reconstruction. The only problem is then that each of the specified stochastic types creates its own significance level.

It was perhaps this dilemma that originated the debate about the benchmarking of $RE$, specifically, estimating the 99% level of significance, $RE_{\text{crit}}$. In the literature, one finds the following approaches:

1. (MBH98) simple AR(1) process with specified memory: $RE_{\text{crit}} = 0\%$;
2. (MM05) inverse regression of NHT on a red noise predictor, estimated from one of the 22 proxies (the dominant PC of North American tree ring network): $RE_{\text{crit}} = 59\%$;
3. (Huybers, 2005) as 2, with rescaling: $RE_{\text{crit}} = 36\%$ using red noise version from accompanying matlab code (10 000 samples);
4. (McIntyre and McKitrick, 2005b) as 3, but with 21 additional (uncorrelated?) white noise predictors: $RE_{\text{crit}} = 54\%$.
   One might now feel inclined to provide the “correct” or “optimum” way of representing the proxies as a stochastic process. If I now add
5. as 4, but with all noise predictors (not only PC1) estimated from the original proxies,
   the series of benchmarking attempts from 1 to 5 would in fact slowly convergence to what MBH98 and similar studies should be compared to. But so much is not required. One can and must only provide a realistic lower bound on the level of significance, may it come from whatever stochastic process. With regard to 5, a benchmark has not been estimated so far, and will not be estimated here. The lesson of §3 is that all benchmarks 1–4 are inflated by the temporal separation of calibration and validation sets, and more realistic values are to be expected from multi-crossvalidation.
For each of the 36 flavors I have therefore repeated the analysis of §5, with the proxies being replaced by red noise series. Specifically, for each proxy a stochastic long-memory process is generated whose memory parameter, $d$, is estimated from the proxy using log-periodogram regression (Geweke and Porter-Hudak, 1983; Brockwell and Davis, 1996). To obtain more robust estimates of $d$ I used here, like MM05, the full proxy record from 1400 to 1980; the corresponding estimates varied between $d=-0.17$ and $d=0.85$. Note that the log-periodogram estimation is slightly different from the method suggested by Hosking (1984) and applied by MM05. Neither method is perfect, as both rest on various approximations (cf. Bhansali and Kokoszka, 2001) that provide little more than a rough guess of what the “true” long-memory parameter might be. But again, such kind of truth is not required.

The noise generation was redone in each of the 300 iterations (to remove sampling effects). The result is shown in Fig. 5. Like in Fig. 4, $RE$ and $CE$ values are similar. All scores are smaller compared to the corresponding proxy predictions, with a greater spread per flavor. They are nonetheless not negligible. Analogously to the proxies, the scores are generally better for flavors of the form 2xx, with median levels varying about 10%. For each flavor, also included are the experiments with full temporal separation. Some of the $RE$ scores exceed 50%, like the trivial predictor (54% for ϕ=311). As an example, Fig. 6 shows the distribution of the 300 predictions for the flavor ϕ=222, in terms of validation $RE$ and in comparison to the proxy predictions. We clearly see different distributions, the nonsense predictions being more spread and generally shifted to smaller $RE$ values, varying roughly about 20%. Note that this is about the score of the trivial predictor for representative calibration sets, depicted in Fig. 3. There are nonetheless outliers with very good scores (~45%). These are possible, as we saw, if the predictors are sufficiently persistent, and calibration and validation sufficiently separated in the time domain.

The degree to which the proxy predictions outperform their nonsense pendants is depicted in the last Fig. 7; it shows for each flavor the respective Mann-Whitney test statistic. Except for the flavors ϕ=13x the values are well beyond the 99% level of the standard normal null distribution of the test (obtained if both samples come from the same population). The highest values are, like in Fig. 4, attained by the 2xx flavors that are based on predictand EOF filtering. The x61 flavors, i.e. those using RegEM, are also large, which is possibly due to the overall reduction in $RE$ spread for those flavors (see above).

Now one thing is still unresolved: Why do the nonsense predictions have non-vanishing score even for the well mixed, representative samples? – A nonsense prediction has, by definition, no skill. In an ideal world, which among other things has infinite samples and truely independent validation, it would yield a cross-validity of $R_e = 0$ and thus, using Eq. (2), $CE \leq 0$ (see §2); and $RE$ would at best be artificially inflated via the $a$ bias, from Eq. (3). Positive scores of nonsense predictions are therefore an artefact of the limited sample of 127 cases/years. In fact, in the finite case, calibration and validation sets are never fully independent; they become more and more dependent if the memory of the time series gets comparable to the series length. The most plausible explanation for the spurious skill is therefore: that the validity of a regression will be partly inherited by the “independent” verification period and create skill there.

This argument applies equally to the proxy predictions. Therefore, about 10% of the 25% skill are likely caused by spurious skill due to memory effects.

7 Conclusions

The analysis poses three questions:

1. How do we interpret the estimated levels of reconstruction skill?

2. How do we interpret the resulting spread in that skill?

3. How are possible answers to 1. and 2. affected by the significance analysis?

ad 1: It was found that realistic estimates of skill vary about 25%, equally for $RE$ and $CE$. The results were obtained using a well confined testbed of proxy and tem-
temperature information through 127 instrumental years, with almost no gaps. The proxies represent a standard set of what is available back to AD 1400. The set of temperature grid points does not cover the entire globe, and its areal averages serve only as approximations to the full NHT average; but it is about the largest subset that is rigorously verifiable. On this background, previous estimates of NHT reconstruction skill in the range of $RE=50\%$ appear much too large. They are inflated by the use of a non-representative calibration/validation setting in the presence of trended data.

ad 2: Crossvalidation of any type (single, double, multi) is a means to estimate the distribution of unknowns (here: the reconstruction skill). As there is no a priori criterion to prefer a specific calibration set, all such sets receive equal weights before and after the analysis (this is somewhat in conflict with Rutherford et al. (2003) who seem to prefer one set because of its validation skill). The estimated distributions were quite similar for $RE$ and $CE$, indicating that both scores actually measure the same thing. The considerable spread of most distributions simply reflects our limited ability to estimate skill any better, based on a sample size of 127 cases/years, and on an effective sample size that is even less, due to persistence.

ad 3: Reconstructions based on real proxies significantly outperform the chosen class of nonsense predictions (based on stochastic long-memory processes). It is unknown whether they outperform any such class, as strong persistence, like in the trivial predictor, enhances scores. Most flavors, moreover, reveal a non-vanishing score for the nonsense predictors, varying about 10% for many flavors. This was attributed to the degraded independence of the finite validation period by memory effects, allowing portions of the calibration information to drop into the validation. As this is equally true for the proxy predictions, a significant amount of the estimated verification skill is likely to be spurious, and further adjustments are necessary.

It is unknown how such an adjustment should be done numerically, producing a final overall verification skill that for the best flavors is somewhere between 15% and 25%, with large uncertainties. With respect to applications, that is, reconstructions, the main question is, however: Are these levels sufficient to decide the millennial NHT controversy? – 25% $RE$ translates to an amplitude error of $100-RE \sim 85\%$. If one were to focus the controversy into the single question: Was there a hemispheric Medieval Warm Period and was it possibly warmer than recent decades? – that question cannot be decided based on current reconstructions alone, at least not in a verifiable sense.

Appendix A

$RE$, $CE$, and $R_c$

Suppose true (verification) and predicted values are given by $x$ and $\hat{x}$, respectively. Without loss of generality let us assume $\langle x \rangle=0$. There are now three forms of relative bias, the calibration mean bias, $\alpha = \frac{\bar{x}}{\sqrt{\langle x^2 \rangle}}$, and the two biases in mean, $\beta = \frac{\langle \hat{x} \rangle}{\sqrt{\langle x^2 \rangle}}$, and amplitude, $\gamma = \sqrt{\frac{\langle (\hat{x}-\langle \hat{x} \rangle)^2 \rangle}{\langle x^2 \rangle}}$. Using these, we have

$$CE = 1 - \frac{\langle (\hat{x} - x)^2 \rangle}{\langle x^2 \rangle}$$

$$= \frac{\langle x^2 \rangle - \langle \hat{x}^2 \rangle + 2\langle \hat{x}x \rangle - \langle x^2 \rangle}{\langle x^2 \rangle}$$

$$= \frac{2\langle \hat{x}x \rangle - \langle \hat{x}^2 \rangle}{\langle x^2 \rangle}$$

$$= \frac{2\langle (\hat{x} - \langle \hat{x} \rangle)x \rangle - \langle (\hat{x} - \langle \hat{x} \rangle)^2 \rangle + 2\langle x \rangle\langle \hat{x} \rangle - 2\langle \hat{x} \rangle^2 + \langle \hat{x} \rangle^2}{\langle x^2 \rangle}$$

$$= \frac{2\langle (\hat{x} - \langle \hat{x} \rangle)x \rangle - \langle (\hat{x} - \langle \hat{x} \rangle)^2 \rangle}{\langle x^2 \rangle} - \frac{\langle \hat{x} \rangle^2}{\langle x^2 \rangle}$$
\[
\begin{align*}
E &= \frac{2 \langle \langle \hat{x} - \langle \hat{x} \rangle x \rangle \rangle}{\sqrt{\langle \langle \hat{x} - \langle \hat{x} \rangle \rangle^2 \rangle}} = \sqrt{\frac{\langle \langle \hat{x} - \langle \hat{x} \rangle \rangle^2 \rangle \langle x^2 \rangle}{\langle \langle \hat{x} - \langle \hat{x} \rangle \rangle^2 \rangle}}
\end{align*}
\]

with \( R_c \) denoting the correlation between predicted and true values. Now we have

\[
RE = 1 - \frac{\langle \langle \hat{x} - x \rangle^2 \rangle}{\langle (x - x_c)^2 \rangle}
\]

\[
= \frac{\langle x^2 \rangle - 2 \langle x x_c \rangle + \langle x^2_c \rangle - \langle \hat{x}^2 \rangle + 2 \langle \hat{x} x \rangle - \langle x^2 \rangle}{\langle x^2 \rangle - 2 \langle x x_c \rangle + \langle x^2_c \rangle}
\]

\[
= \frac{2 \langle \hat{x} x \rangle - \langle \hat{x}^2 \rangle + \langle x^2_c \rangle}{\langle x^2 \rangle + \langle x^2_c \rangle}
\]

\[
= \frac{CE(x^2) + x^2_c}{\langle x^2 \rangle + \langle x^2_c \rangle}
\]

\[
= CE + \alpha^2
\]

The 4th line of Eq. (A2) is an immediate consequence of the 3rd line of Eq. (A1).

**Appendix B**

**RegEM configuration**

To control the iteration, RegEM has a number of configuration switches that can be adjusted. The following settings gave satisfactory convergence results for most of the experiments. I used: multiple ridge regression as a regression procedure; regularization parameter determined from general cross validation (GCV); minimum relative variance of residuals: 5e-2; stagnation tolerance: 3e-5; maximum number of iterations: 50; inflation factor: 1.0; minimum fraction of retained variance: 0.95. This latter setting is borrowed from Rutherford et al. (2003) who argue that the GCV regularization estimate is too crude in the presence of too many unknowns. This was true here as well. In fact, using the GCV estimate for the flavors \( \varphi = 1xx \) resulted in RegEM reconstructions that were hardly distinguishable from the calibration mean.

**Acknowledgements.** I enjoyed lively discussions with U. Cubasch, F. Nehröster and F. Kaspar. This work was partly supported by the EU project SOAP.

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Table 1. Table of the $3\times6\times2=36$ reconstruction flavors.

<table>
<thead>
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<th>PRE</th>
<th>METH</th>
<th>POST</th>
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<td>219 grid points</td>
<td>forward regression</td>
<td>no rescaling</td>
</tr>
<tr>
<td>1 EOF</td>
<td>backward regression</td>
<td>rescaling</td>
</tr>
<tr>
<td>1 global average</td>
<td>TTLS</td>
<td></td>
</tr>
<tr>
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<tr>
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</tr>
<tr>
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<td>RegEM</td>
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Fig. 1. Dependence of cross validity $\hat{R}_c^2$ on adjusted $R^2$ and corresponding shrinkage.
Fig. 2. NHT observed (thin black line) and predicted from the series of calendar years (blue line). The model is calibrated in the late portion (1917–1980) and validated in the early portion (1854–1916), yielding a $RE$ score of 56%. Also depicted is the climatology forecast of the calibration period which, by definition, scores $RE=0$ (heavy black line).

Fig. 3. The dependence of the validation scores $RE$ and $CE$ on the degree of temporal separation for the simple NHT predictor (see text). For the full separation with a late (1917–1980) calibration and early (1854–1916) validation $RE$ (solid black) approaches 60%, while the fully mixed case attains only about 30% $RE$; towards early calibration $RE$ rises again to 40% but then sharply drops to negative values. $CE$ (dashed black) shows somewhat opposite behavior, with strongly negative values for the full separation and values similar to $RE$ in the mixed case. Also shown is an index (see text) of the representativeness the corresponding calibration sets (red).
Fig. 4. Boxplot of the distribution of \( RE \) and \( CE \) for each of the 36 flavors, based on 300 resamplings of the calibration/verification period. Each box indicates the 10\%, 50\%, and 90\% quantile, and the whiskers the minimum and maximum, of the distribution. Also shown are the scores obtained from the full separation into early (upward triangle) and late (downward triangle) calibration. For readability, some flavors/experiments are not shown (too negative).

Fig. 5. As Fig. 4, using nonsense predictors.
**Fig. 6.** Histogram of $RE$ from proxy and nonsense prediction using flavor $\varphi = 222$. Proxy predictions show less spread and generally greater skill. Note, however, that high scores are also obtained from nonsense predictions.

**Fig. 7.** Testing $RE$ scores of proxy vs. nonsense predictions, using Mann-Whitney test, for all flavors. The null distribution is N(0,1), so that for almost all flavors the real predictions are significantly better than the nonsense predictors.