Submission of revised version of the manuscript:

CPD 11, 5605-5649, 2015:

Regional climate signal vs. local noise: a two-dimensional view of water isotopes in Antarctic firn at Kohnen station, Dronning Maud Land

Thomas Münch et al.

This document contains a list of the major changes made for the revised manuscript, a one-to-one reply to the reviewer comments including all respective, detailed changes made in the manuscript, and a marked-up manuscript version created with `latexdiff`.

We briefly summarise first the major changes that we made in the revised manuscript version:

- The entire manuscript was shortened and simplified. Special attention was here given to:
    - overall improvement of the text flow and the readability, reduction of technical terminology, clean-up/clarification of nomenclature
    - discussion of the seasonal layer profiles (Sect. 3.1)
    - application of the statistical noise model (Sect.s 3.5 and 4.2/4.3)
    - discussion of the climate representativity of firn cores together with the derived implications (Sect.s 4.2 and 4.3)
    - description and discussion of the Monte Carlo approach/linear trend detection experiment (Sect. 4.3)
- The sectioning was revised in order to improve the overall logical structure of the manuscript:
    - Sect.s 3.3 and 3.4 were swapped
    - Sect. 3.5 was introduced as a new section
    - the former Sect.s 4.1 and 4.2 were merged into a single Sect. 4.1

- Fig. 3 (horizontal variance of T1 as a function of depth) and its discussion was removed from the manuscript.
- Appendix A – the derivation of the statistical noise model – was completely rewritten and restructured to improve the comprehensibility, and complemented by two supporting figures and a table. Further, we introduce an additional part which discusses explicitly the estimation of the model parameters based on our data set.
- The discussion (Sect. 4) was largely rewritten to clearly state the limitations of our approach in order to present a realistic but not overly pessimistic view. This includes as a major point the change of the best-case scenario for the post-depositional annual noise variance which we now model based on white noise. All relevant results (Fig.s 8+9 as well as in the text) were updated accordingly.
- Appendix B with Fig. (B1) was removed; instead, a new appendix is introduced discussing the effect diffusion has on the reduction of the post-depositional noise variance.
- The layout of the figures was improved

Response to the reviewers

CPD 11, 5605-5649, 2015: Regional climate signal vs. local noise: a two-dimensional view of water isotopes in Antarctic firn at Kohnen station, Dronning Maud Land

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3rd April 2016

This is the original review reply already published after the discussion phase with the proposed changes that we have now introduced to the revised manuscript. Parts where additional or other changes than originally stated were made are now marked by respective additional answers typeset in blue.

We thank both reviewers for their constructive comments. Based on these, the major points that we suggest for the manuscript revision are a shortening of the entire manuscript, a clarification of the used nomenclature and of the mathematical derivation of the noise model, as well as the rewriting of certain paragraphs. We would like to point out that part of the review comments are based on misunderstandings. We are sorry that our style in the manuscript was not concise enough at some points and will make efforts to improve this. Please find below our detailed answers. We will first reply to the general comments of both reviewers and afterwards answer the specific comments. The reviewer comments are typeset in *italics*, our author comments in normal font.

General comments

Anonymous referee #1:

First and most important I think that the manuscript does not read well. The writing feels *overly complicated while the mathematical treatment, the description of the statistical noise model as well as the way the latter is used with the real data sets are not presented clearly. The manuscript will benefit from a clean-up and a clarification of the mathematical symbols as well as the terminology that seem to be used carelessly to some extent. After I read the Appendix 1 and all sections relevant to the derivation and use of the noise model, it is still very unclear to me what exactly have the authors done. I can't claim that my math/statistics level is very high but can certainly relate to the average reader of CP and my problem in understanding the methods lies mostly in the rather confusing use of symbols and often in the absent explanations of how the noise model was applied.*

AC:

We would like to express our apologies that the manuscript was hard to read and to follow. We will make an effort to improve its readability. This will include a shortening as well as a simplification of the manuscript. We plan to accomplish the shortening by removing the diffusion model and its discussion, by merging sections 4.1 and 4.2 and by condensing individual paragraphs. Simplification of the manuscript will be reached by reducing technical terminology and a clean-up of the nomenclature. For this, we will extend the Data and Methods section by an additional paragraph that introduces the coordinate system that is used throughout the manuscript (including a schematic figure) as well as relevant nomenclature. We will make sure that the nomenclature introduced there will be used throughout the rest of the text. We will give more space to the statistical noise model in order to clarify both its derivation in the appendix as well as its application in the main text. To improve the comprehensibility of the derivation, we will introduce a table of symbols including their definitions in the appendix.

In addition, we shortened section 3.2 by removing Fig. (2) (horizontal T1 d18O variance as a function of depth) together with its discussion. We swapped sections 3.3 and 3.4 to improve the logical structure of the manuscript. We introduced a new section 3.5 where we introduce the statistical noise model and its validation by using parameters estimated from the trench data. This serves to give the model and its application a more central location. Section 3.4 was renamed to "Spatial correlation structure" and now also includes the discussion of the inter-trench correlations (former Fig. (8)) that was originally in the Discussion section. The cited literature was revised and less relevant literature was removed.

I believe that the manuscript falsely presents an overly pessimistic view on the use of the water isotopic ratios obtained from single firn/ice cores. The reason for this is that the signal to noise ratios and variance estimations of the 1 m deep firn cores array are in a way "extrapolated" and used for evaluating the representativity of deeper cores thus falsely giving the impression that a minimum of N cores is needed for a robust isotopic signal to be estimated. Even though a study of the top 1 m of firn is very valuable one should expect isotopic diffusion and firn densification to heavily attenuate a lot of the variance caused by post-depositional (mostly surface topography) effects. This is of course not to say that the interprofile correlation is expected to approach 1 but certainly the low covariances the authors observe for the top 1 meter are not representative of the deeper parts of a firn core. I also fear that the results the authors present regarding the last 6000 years of isotopic data from the EDML core overestimate the importance of post depositional noise and neglect the recorded climate variability. This does not necessarily mean that water isotopic records are accurate proxies of polar temperature over the Holocene; the problem of the low responsivity of the d18O signal to temperature still remains.

AC:

The reviewer states his concerns about the fact that we use noise levels inferred from the first metre of firn also to assess the representativity of much deeper firn cores, and mentions that both densification and diffusion likely affect the noise level in the deeper parts. We are certainly aware of the fact that our approach of analysing the first metre is only a limitation, and we will ensure that this is also marked as such clearly in the manuscript.

However, regarding the influence of densification and diffusion we do not fully agree. In the first metre of firn densification does not occur at our study site which is shown by the density data obtained from the trenches. It is therefore not relevant for our data. Below the first 1-2 metres where densification starts, its effect on the lateral isotopic variability is probably dependent on the sampling resolution. However, the exact effect is yet unclear. We will add a respective remark at the end of section 4.1. In the case of diffusion and densification we also have to bear in mind that it acts equally on both signal and post-depositional noise. If the variance of the climate signal in the isotopic time series does not change on the time-scale considered (e.g. inter-annual), which is a reasonable assumption, the variance ratio of signal to noise will not be affected by diffusion nor densification, and our results of the representativity will not change for the deeper parts of a firn core.

However, we also expect that the climate signal has more variance associated with longer time scales, e.g., as seen on glacial-interglacial time scales. Therefore, the signal to noise ratio will improve considerably when analysing longer time scales (e.g. centennial or millenial variations). We will add these points to the discussion in sections 4.3 and 4.4.

Regarding the interpretation of the decadal variance seen in the EDML deep ice core over the last 6000 years, we admit that so far we have neglected diffusion at this point. However, even after a full forward diffusion of our trench noise level estimates with a (pessimistic) diffusion length of 8 cm water equivalent, the effect on decadal and longer variations is small. Our inferred noise levels for the decadal time scale are consequently not strongly affected (the inter-annual noise levels estimated from the trenches are reduced by a factor of ~ 0.095 in the diffusion case instead of a factor of 1/10 in case of undiffusing white noise; see also our more detailed answer to the the respective specific comment). Thus, our statement that the EDML core decadal isotope variations might to a considerable part be noise is still valid after accounting for diffusion. We will add this discussion to the manuscript.

We account for these points by clearly stating the limitations of our approach: We have insufficient knowledge on the noise covariance for time scales above annual and have to rely on assumptions (Sect. 4.3). We base the best-case scenario of the annual noise variance now on the assumption of white noise since the reliability of the vertical autocorrelation of the noise that is suggested by our data is limited by the short data set (Sect. 4.2 + Appendix A). We discuss the influence of diffusion and densification on the signal-to-noise variance ratio (Sect. 4.2) and on the post-depositional noise level (Sect. 4.3). Specifically, we account for diffusion when calculating the decadal level of the post-depositional noise variance (Sect. 4.3 + Appendix B). Finally, we add a clear statement that we expect the climate signal and therefore the signal-to-noise ratio and representativity of firn cores to increase on longer time scales (Sect. 4.3).

I have the impression that the authors tend to statistically treat the pre-deposition isotopic signal as a stationary stochastic process when in reality it is to a large extent a deterministic signal. Additionally, water isotope time series from ice cores are found to present a red + white noise behavior in the frequency domain, likely reflecting processes in the climate system that introduce a long-term memory. As a result the approach the authors use for example *in section 4.4 when attempting to detect a warming trend is far from realistic. A warming signal in water isotopes can't possibly be just the sum of a linear trend and white noise.*

AC:

While we do not agree that large parts of the pre-depositional signal are deterministic, we are also aware that it is a mixture of many processes. On the one hand, its temperature signal consists of deterministic components (the seasonal cycle, solar and volcanic forcing, anthropogenic trends) and of a stochastic component as result of the internal variability in the climate system (red climate noise). On the other hand, it exhibits a non-temperature part including meteorologic/atmospheric effects of stochastic nature that influence the isotope content of precipitation, noise due to a varying isotope-temperature relationship, post-depositional noise, etc. In our paper we examine therefore the most simple and also most optimistic case: an anthropogenic trend + white post-depositional noise. Our Monte Carlo simulation is hence valid as an upper bound of the detection probability since all other mentioned components of a real isotope time series will complicate the detectability of an anthropogenic trend. In our oppinion it is thus only necessary to formulate the underlying assumptions in the Monte Carlo simulation much more clearly and to mention the additional complicating issues, but not to refine the approach itself.

To acount for these points, the relevant part of the manuscript was entirely rewritten. We name our Monte Carlo approach now a toy model experiment to stress that it is not a realistic scenario for the East Antarctic climate evolution of the last 50 years, but a simple model to estimate an upper bound for the trend detectability. In line with that, we formulate the assumptions and limitations of the model clearly.

Based on their results regarding the minimum number of cores required for a satisfactory representativity, the authors suggest that it is preferable to sacrifice measurement precision (wrongly referred to as accuracy in the manuscript) to higher throughput in order for more cores to be analyzed using Cavity Ring Down Spectroscopy. This recommendation sounds tentative for two reasons. Firstly with the current Cavity Ring Down instrumentation one injection is very unlikely to provide results free of memory effects regardless of the correction scheme used. I am personally not aware of a correction scheme that "behaves" when such a small number of data points are available per sample. The problem this generates is *that intra-sample memory effects are notorious for modifying the color of the noise in high resolution water isotope records. This impacts any work utilizing spectral methods as power spectral densities become biased in the low frequency part of the spectrum. Secondly a higher analytical noise level results in inferior Deuterium excess records and impacts the accuracy of temperature reconstructions based on water isotope diffusion – the latter seeing a great benefit from measurements of as high precision as possible. I would argue that the authors should reconsider this message and at least stress out that there will be a cost in following a one-injection measurement approach.*

AC:

We agree with the reviewer that reducing the number of injections on Cavity Ring Down Spectrosopy instruments down to one per sample might affect the usability of the data for diffusion-based methods as well as for the interpretation of deuterium excess. On the other hand, it would improve single-proxy reconstructions if it allowed more replicate core measurements. In the revised version, we will better stress the limitations of our suggestion.

In the revised version we removed the one-injection suggestion entirely. Instead, we only state that fast measurements can be a benefit in order to analyse many cores (Sect. 4.3), and that these can be achieved by using three injections and a memory correction as used for our data set. We clearly state that this might affect the data usability for diffusion- or d-excess-based inferences. In the Conclusions section we now state that monitoring the measurement error could allow faster measurements and that alternative, indirect methods might circumvent the problems for stratigraphic noise discussed here.

Last, though not as important, it would be nice presenting some of the d18O profiles from T1 so the reader has a feeling of how the time series look.

AC:

We do not think that this is an improvement of the manuscript since single T1 d18O profiles will not offer any new insights compared to the T2 profiles already shown. All data presented in the paper will be made public via the data base PANGAEA (http://www.pangaea.de/) so that anyone will be able to investigate it.

Anonymous referee #2:

The paper overall is very difficult to read. The writing is too complicated, often mixing nomenclature, or not defining it properly. The statistical model, especially, deserves more attention in the text, as well as more description in the Appendix. A major simplification of the story is needed. As it stands, the reader is lost in technical and often unnecessary writing. The paper could be as much as 25% *shorter just in this regard.*

AC:

Similar issues have been mentioned by the first reviewer. We therefore cite here our answer from above:

We would like to express our apologies that the manuscript was hard to read and to follow. We will make an effort to improve its readability. This will include a shortening as well as a simplification of the manuscript. We plan to accomplish the shortening by removing the diffusion model and its discussion, by merging sections 4.1 and 4.2 and by condensing individual paragraphs. Simplification of the manuscript will be reached by reducing technical terminology and a clean-up of the nomenclature. For this, we will extend the Data and Methods section by an additional paragraph that introduces the coordinate system that is used throughout the manuscript (including a schematic figure) as well as relevant nomenclature. We will make sure that the nomenclature introduced there will be used throughout the rest of the text. We will give more space to the statistical noise model in order to clarify both its derivation in the appendix as well as its application in the main text. To improve the comprehensibility of the derivation, we will introduce a table of symbols including their definitions in the appendix.

In section 4.4, the authors attempt to reconstruct a 0.5degC temperature trend using a Monte Carlo approach consisting of a signal (linear temperature trend) and random noise. Although the time period is short (50 years), this is far too simplistic a model for estimating isotopic variability. The approach must also include the atmospheric component of variability, because storm tracks and moisture sources can change over decadal time periods. At the very least, this should be clearly documented as a simplifying assumption. Water isotope signals do not only depend on noise and temperature!

AC:

We agree with the reviewer that our model neglects many contributions to the signal and noise as well as the processes causing these variations. Please see also our response to the similar issue raised by reviewer 1. However, our model, by purpose, examines a simple and also most optimistic case: an anthropogenic trend + white post-depositional noise. Our Monte Carlo simulation is hence valid as an upper bound of the detection probability since all other mentioned components of a real isotope time series will complicate the detectability of an anthropogenic trend. We will formulate the underlying assumptions in the Monte Carlo simulation more clearly, mention the limitations, and make clear that this is a thought experiment to estimate a lower limit of the number of required cores and not a realistic simulation.

The results presented largely focus on isotopic analysis in the depth/time domain, but I think it would be worth pointing out that analysis in the frequency domain of isotopic profiles would be informative, and an area of much needed research. It makes sense that post-depositional stratigraphic variations alter the isotopic signal, but is the frequency component of the data preserved? That is, do the spectra of nearby isotopic profiles in the vertical direction have the same power density values? In my opinion, this would be the major test of water isotope literature. At the end of the paper, this should be suggested (note: an analysis like this would require perhaps 100 years of data from multiple cores). Table 1 would suggest there may be large discrepancies in the frequency domain, but I also think the vertical scale of the study ($\sim 1\,m$) *prevents any useful conclusions.*

AC:

We agree with the reviewer that a spectral analysis of nearby firn cores is a very interesting approach. It is expected that temperature spectra (from climate models, for instance) will show deviations from d18O spectra of ice/firn cores due to post-depositional noise and diffusion. In fact, this is part of our ongoing research to obtain a better understanding of signal and noise in Antarctic cores. However, with respect to our manuscript we do not regard a spectral approach as meaningful due to the limited vertical extent of our data. In addition, for the rather nearby trenches we expect their spectra to be similar within uncertainty of the spectral estimate. In our data, we observe a quite considerable difference between variance levels of the mean trench profiles. For example, the estimated signal variance of the mean T1 profile on the inter-annual time scale of 1.15 (per mil)^2 is in contrast to the value of T2 of only 0.21 (per mil)^2 (see Tab. 1 in the manuscript). This discrepancy can be attributed to the fact that information is lost due to the stacking of the single profiles. We will add a sentence to the conclusions section that spectral analyses of firn cores would complement trench-like studies in order to understand the spectral shape of the noise.

Throughout the paper, an accumulation value for low-accumulation sites is poorly defined. The results of the paper are only valid for low accumulation sites, which I guess might mean something like less than 15 cm ice eq/year. It should be made clear at the beginning of the paper, and throughout.

AC:

As a reference throughout the paper, we will define a low accumulation rate to mean a value of $\leq$ 10 cm water eq./year. The East Antarctic plateau typically shows accumulation rates below this threshold.

This value for low-accumulation regions was now defined in the 2nd paragraph of the introduction.

Suggesting that only one injection on Cavity Ring Down Spectroscopy instruments be used for future multi-ice core studies, in my opinion, should not be included as a suggestion in the paper. Although throughput would increase, current CRMS instruments cannot give reliable results with a single injection - precision is lost - and this can alter the frequency component of the signal. Plus, the deuterium excess parameter requires good precision in both d18O and dD for useable results.

AC:

Also the first reviewer has critisised our recommendation in the paper to reduce the number of injections on Cavity Ring Down Spectrosopy instruments down to one per sample in order to be able to measure more cores instead. We will better state the limitations of our suggestion in the revised manuscript.

In Figure 4, seeing that the mean isotope profiles of T1 and T2 are correlated at 0.82 leads me to believe that clarification is needed in the text. Using a low accumulation site to extract temperature is problematic in many ways, and using up to 50 cores might be necessary to get some sort of temperature signal, but simply averaging a few isotopic profiles over some *depth/time is still useful to pull out a common climate signal. This must be clarified to the reader.*

AC:

The significant observed seasonal correlation of 0.81 is expected from our noise model for the seasonal time scale: The model shows that a number of five profiles at a spacing of 10 m is sufficient to obtain a representative (R>0.9) isotope signal. In T1, 38 profiles are averaged in the mean profile, thus a large number; in T2, four profiles at optimal spacings of at least 10 m are averaged. The recommendation of drilling 10–40 cores for a representative signal refers to the inter-annual case for which the signal-to-noise ratio is much smaller. Despite that, we observe a correlation between T1 and T2 for the inter-annual mean time series of 0.87. However, this value should be taken with care since its significance is doubtable as the value is only based on five observations. Both aspects will be clarified in the manuscript in section 4.3.

We discuss the correlation of 0.81 of the mean trench profiles now more clearly in section 4.1: We infer that the mean profiles show a regionally coherent isotope signal, consistent with the mean inter-profile and inter-trench correlations of single profiles. We also discuss the effect of the autocorrelation of the stratigraphic noise on the noise levels of trench profile stacks depending on the number and the spacing of the profiles.

Answers to specific comments, anonymous referee #1:

RC 1, P5610–L15:

Based on the scheme you present the results of your measurements are not calibrated on the SMOW/SLAP scale. This is unfortunately a point misunderstood by many laboratories performing water isotope analysis. Technically a calibration of your samples on the SMOW/SLAP scale requires a two fixed-point calibration. This originates from the SMOW/SLAP scale definition itself where zero is defined by SMOW and the linear scale is defined by SLAP at -55.5 per mile (precisely). The problem with a three points linear fit is that despite the fact that often the R2 value of the linear fit looks excellent the actual offsets of the points from the calibration line are large enough to cause accuracy issues that are not easy to identify.

I think your measurements will strongly benefit from fixing the two extreme water standard points, calculating a calibration line based on those two and using the 3rd mid point as an accuracy check. This in the end is a measure of your "combined uncertainty" and often it can be slightly higher than a precision estimate that is based on the of series of injections of a standard water. With this in mind the 0.09 per mile precision given in the manuscript is absolutely the upper limit of precision and very likely the combined uncertainty of the measurements is somewhat worse. Having said this, I do not think your actual results will vary significantly by choosing a 2-point calibration and thus if you make a proper comment on the calibration scheme it will be fine not readdressing all your measurement runs. It would however be very nice to apply it to one run in order to get a feel of how high your combined uncertainty is, as estimated by checking the offset of the middle standard from the calibration line.

AC:

Please excuse that, for the sake of brevity, we have apparently not adequately described our measurement and correction scheme. In fact, each measurement run includes three blocks of standard measurements, one at the beginning, one at the end and one in the middle of the run. The three-point calibration as well as the memory correction is performed with, or respectively based on, standards from the first block, the drift correction by additionally using standards from the last block. To check the precision of the entire calibration and correction scheme, an independent standard in the middle block is measured that is neither used for calibration nor memory/drift correction. Our given measurement precision is based on the deviation of this standard from its known value. It thus yields a measure of the combined uncertainty of the calibration and the measurement itself. In the revised version we will add that the given precision is based on the evaluation of an independent standard not used for calibration or correction and thus represents an combined uncertainty.

Regarding SMOW/SLAP scale we agree that, strictly speaking, the calibration is not performed onto the SMOW/SLAP scale. We will change the respective sentence to: "The isotopic ratios are calibrated by means of a linear three-point regression analysis with different in-house standards where each standard has been calibrated to the international V-SMOW/SLAP scale."

RC 2, P5611–L8:

"Signifficantly higher density" Maybe an estimate?

AC:

According to the reference given, the dunes typically exhibit snow densities about 15–50 % higher than the mean value of the surrounding firn. We will add this information to the manuscript.

This part was now considerably shortended by removing the discussion of the details.

RC 3, P5612–L10:

The numbers you give for the RMS deviations seem very low after looking at the profiles in Figure 1b. Is there any chance you calculated mean of differences and not an RMS value?

AC:

This is a misunderstanding, please excuse that this has not become clear. For a specific layer profile, we calculate the root-mean square deviation (rmsd) for two cases: i) between the layer profile and the surface height profile, and ii) between the layer profile and the horizontal reference (a straight line). The numbers we state in the manuscript are the differene between the two rmsd values. We will rewrite the entire paragraph for clarification.

The entire paragraph was rewritten and formulated in a more concise fashion. In addition, we changed the terminology and now compare standard deviation values for the seasonal layer profiles between evaluation on absolute and surface coordinates. This reduces terminology since the two coordinate systems used are now introduced in the Data and Methods section.

RC 4, P5612–L22 and Figure 2:

The P–P values of the T2 d8O profiles ar about 10 per mile lower than of those from T1. Can you maybe comment on this?

AC:

The peak-peak value is an instable metric and depends strongly on the sample size. In T2 only four profiles were sampled which likely causes the difference between both trenches (20 per mil in T1 vs. 12 per mil in T2). More stable metrics are for example the mean and the standard deviation which indeed show much smaller differences between the trenches (mean(T1)=-44.4 per mil vs. mean(T2)=44.0 per mil; SD(T1)=3.1 per mil vs. SD(T2)=2.7 per mil). These values are also stated in the manuscript or will be added (please see answer to RC 8 of referee #2).

RC 5, P5614–L11:

For the case of an AR-1 process one would expect the correlation to continuously drop until it reaches values close to zero for high lag values. Here you observe a plateau at the value of 0.5 for spacings $\geq 10m$ *Does this imply something for the choice of the AR-1 approach for your lateral noise?*

AC:

This is a misunderstanding as our model is not an AR-1 process alone, but the sum of a noise following an AR-1 process and a coherent signal. In P5614-L13-15 we state: "We assume that each profile consists of a common signal S and a noise component ε independent of the signal. The noise component is modeled as a first-order autoregressive process (AR(1)) in the lateral direction." The inter-profile correlation then is the sum of a constant term and an AR(1) term that decorrelates with increasing distance between the profiles (see Eq. (2) in the manuscript):

$$r_{XY} = \frac{1}{1+\frac{\mathrm{var}(\varepsilon)}{\mathrm{var}(S)}} + \frac{\frac{\mathrm{var}(\varepsilon)}{\mathrm{var}(S)}}{1+\frac{\mathrm{var}(\varepsilon)}{\mathrm{var}(S)}} \times \exp\left(-\frac{|x-y|}{\lambda}\right) .$$

The constant term assumes for a variance ratio $\mathrm{var}(\epsilon)/\mathrm{var}(S) = 1.1$ as used in the manuscript a value of ~ 0.5. We will change the legend of Fig. 5 to "AR(1) noise + signal model" to make it also here immediately apparent to the reader that the model consists of a noise and a signal component.

For additional clarification, we added a sentence that the dependency on distance arises from the autocorreation of the noise.

RC 6, P5614–L18:

The term "signal to noise ratio" is normally used to describe the ratio of the powers of two signals. Is it appropriate to use this term when looking into the variance ratio?

AC:

The signal-to-noise ratio is indeed defined as the ratio of the powers of signal and noise. However, it is also routinely used in the related literature to describe the variance ratio (e.g., Persson et al., 2011, JGR; Wigley et al., 1994, Journal of Climate and Applied Meteorology).

When both signal and noise are stationary stochastic processes, their respective power is equal to their mean-squared value; which is further identical to the variance if both have zero mean. An AR(1) process is stationary stochastic; however, this is not the case for the isotopic seasonal signal since it contains a deterministic signal, the seasonal cycle. To prevent misunderstandings, for the manuscript we will name it signal-to-noise variance ratio, as, e.g., in Fisher et al., 1985.

The signal-to-noise variance ratio is now introduced in the new section 3.5 referencing Fisher (1985).

RC 7, P5617–L8:

Preferably replace "m-scale" with "meter–scale"

AC:

We will adopt this change in the manuscript.

RC 8, P5617–L11:

The relatively recent literature on vapor measurements and their interpretation has certainly showed that the isotopic composition of the upper snow is subject to change post deposition and similar changes can be observed in the vapor isotopic composition. However I do not think that the literature has showed any solid evidence that sublimation-condensation processes are the mechanism driving these changes in the upper firn (it is possible indeed). A rather simple diffusion model can show how an underlying winter layer can significantly deplete the isotopic composition of the overlying enriched summer layer in a period of hours to few days, something allowed by the extremely open porosity of the upper firn.

AC:

We agree with the reviewer but also think that our statement "Possibly, exchange of water vapour with the atmosphere by sublimation-condensation processes (Steen-Larsen et al., 2014), potentially accompanied by forced ventilation (Waddington et al., 2002; Neumann and Waddington, 2004; Town et al., 2008), acts as a further noise source." clearly reflects that this is not a solid evidence but a possibility.

This part was now entirely removed to shorten and simplify the text.

RC 9, P5618–L3:

Indeed firn diffusion plays a strong role. Do you not think that the densification process itself is also a mechanism that reduces the variance caused by surface topography noise?

AC:

In the sampling region no densification is observed within approximately the first two metres of firn (J. Freitag, personal communication), the densities measured in both trenches support this (T. Laepple et al., manuscript in preparation). Consequently, we do not consider densification to be important for our data set. Nevertheless we agree that below the first 1-2 metres, where densification starts, it may influence the noise variance given the firn is sampled in constant intervals.

The possibility that densification influences the post-depositional noise level has been added to Sect. (4.3), however, together with the statement that this is only true for undated samples and thus not relevant for the discussion in this section.

RC 10, P5618–L23:

I guess that you need a sinusoidal d18O signal in order to cancel out at a shift of $\nu/4$? Also, your observations show a plateau at a correlation of 0.5 so you do see something different in fact.

AC:

The purpose here was to assign a physical interpretation to the observed decorrelation length of the noise. However, we agree with the reviewer that the attempt to relate a sinusoidal surface variation with the exponential decorrelation of the noise is too simplistic since the autocorrelation of a periodical function is again periodical, not exponential. We will remove this part and simply state that the observed decorrelation length of $\lambda \sim 1.5$ m is of the same order of magnitude as the small-scale surface height variations, suggesting stratigraphic noise to be an important noise component in our records.

RC 11, P5619–L2:

Is the 1km value an educated guess?

AC:

The value corresponds to the rounded up distance between the trenches.

RC 12, P5619–L5:

Your comments on the validity of the isotopic thermometer and the precipitation intermittency are certainly valid but I find them irrelevant here. Your study deals with local noise and further complicating the discussion with the long standing question on the validity of the isotopic thermometer can possibly be confusing at this point in the manuscript.

AC:

We agree that the additional comments on the isotopic thermometer and precipitation intermittency might confuse some readers at this point, and we will remove this part from the manuscript.

RC 13, P5619–L15-22:

The reader here is left guessing what you have done for this section. Which model parameters from T1 do you carry over for this calculation? You mention that an averaged set of T1 profiles is used and that those profiles are chosen if they fulfill the required criteria. Can you be more specific? Inspecting Fig. 7 I see a feature of your model that is hard to understand (it also appears in Fig. 8 actually). For $N = 2$ *and* $N = 3$ *there seems to be a discontinuity in your model. A "kink" is very clearly seen. I do not see any reason why your math produces such a feature (i am referring to the* r_{xy} *definition here). Can you explain why this is the case?*

AC:

i) We are sorry that this part was apparently not clearly written. We will thoroughly rewrite it to clarify what is being done here. ii) The "kinks" seen in the model curves in Fig.s (7) and (8) are not a discontinuity of the model itself, but due to the fact that the model (and also the data) can only be evaluated for an integer number of profiles. We will add points at N=1,2,3,... to the lines in each plot to make this clear.

The discussion of the inter-trench correlations between mean profiles of T1 and T2 has now been moved to section 3.4 and formulated in a simpler fashion. Comparison to the model-based results is now carried out in the new section 3.5 together with the model-based inter-profile correlations. This highlights that for both cases the same parameter values are used which are obtained from the trench data in the text immediately above.

RC 14, P5620–L20:

Again you refer to correlation to local temperatures. This is essentially a different study and *your reference to weather station data sort of pops out of the blue here leaving the reader a bit confused.*

AC:

We think it is important to assign a physical meaning to our term of representativity. For this we stick to the classic interpretation of d18O as a proxy for local temperature, thereby assuming that the coherent isotope signal identified in the trench record is related to local temperature variations. Bearing in mind issues such as meteorology and moisture source temperatures that complicate this interpretation, our representativity can then be interpreted as an upper bound for the correlation with a nearby weather station. True correlations will certainly be lower. We want to stress again our oppinion that a physical meaning of the term representativity is a benefit for the reader and suggest to keep this, but will of course rewrite the sentence to make our reasoning more transparent.

RC 15, P5620–L25:

Can you be more specific on the time scale here. Do you simply mean "time" and not "time scale"? Also keep in mind that nowhere in the manuscript a description on how you assigned a time scale is to be found. You calculate annual means but have not described how you assign years to your data.

AC:

i) We are afraid this is a misunderstanding. In our understanding the term "time scale" is common usage in climatology to denote a typical period of time: e.g., climate variations occur on different time scales, from seasonal over inter-annual to decadal, centennial and longer variations. ii) The construction of the age-depth relationship/assignment of annual means is described in P5616 L4-8: "In order to obtain annual-mean d18O time series we define annual bins through the six local maxima determined from the averaged profile of the two mean trench profiles. The mean peak-to-peak distance of these maxima is 19.8 cm, consistent with the accumulation rate. Three alternative sets of annual bins are derived from the five local minima as well as from the midpoints of the slopes flanking these minima.", but we will try to add a more detailed description in the results section.

The part that describes the binning of the annual data (Section 3.3) was entirely rewritten together with the assignement of years to the annual data. The relevant part of section 4.2 was rewritten to clarify our usage of the term time scale in relation to climate variations.

RC 16, P5621–L10:

Would the simplest and best case scenario be assuming white noise?

AC:

Indeed, white noise would be more advantageous than autoregressive noise. However, firstly the detrended trench data are positively autocorrelated in the vertical direction, contradicting white noise. Secondly, white noise is physically quite unlikely. Since stratigraphic noise is the result of constant mixing, erosion and redistribution of the surface snow it is likely that adjacent layers show some inter-relation. We will change the wording to reflect that the first-order autoregressive noise is the best case, consistent with the available data.

Now we base our best-case annual noise level estimate on white noise to provide a true optimistic scenario. All relevant results have been updated accordingly. However, we mention that our data hint at an autocorrelation of the noise in the vertical direction but that the reliability of this finding is limited by our short data set. Therefore, the true results will likely be in between of our limiting estimates.

RC 17, P5622–L10:

I guess you would have to agree that the study from Graf et al has completely different boundary conditions than yours. Low cross correlations between the records in that case can be due to other processes that are not apparent in your case.

AC:

We are aware that the results obtained by Graf et al. also include other effects than just the stratigraphic noise. This is reflected in our manuscript (P5622-L18-21): "However, this accordance does not necessarily mean that our worst-case scenario is the more realistic one since the measured cross-correlations [in the study of Graf et al.] are also subject to potential dating uncertainties and additional variability caused by spatially varying precipitation-weighting and possibly other effects." We disagree with the reviewer that the study of Graf et al. has completely different boundary conditions: It was conducted in the same area, the firn cores are annually resolved, and they cover isotopic variations at the end of the Holocene. In summary, we would leave this part of the manuscript as it is.

Nevertheless, we have rewritten this part to make clear that the low F values found by Graf et al. are not necessarily caused by such high annual noise levels as suggested by our worst-case scenario.

RC 18, P5623–L5:

I am not sure the term "significant challenge" is appropriate here considering you only use data from the top 1 m of firn.

AC:

The corresponding part in the manuscript is: "The noise level identified in our trench data poses a significant challenge for the interpretation of firn-core-based climate reconstructions on seasonal to inter-annual time scales." Hence, we already restrict the statement to apply to seasonal to inter-annual time scales only, and not in general. We will add "in our study region" to stress that we only make a statement for the area around Kohnen station.

We have added that this applies to low-accumulation regions as defined in the manuscript.

RC 19, P5623–L21:

Replace "high-accuracy" with "high-precision". It is the precision that affects the variance of your noise in the isotopic profiles. Accuracy issues can potentially create biases but this is not exactly what you are looking at.

AC:

We will replace "high-accuracy" with "high-precision". We accidentally mixed up the two terms.

RC 20, P5624–L5-7:

I suppose you would require that the d18O signal is stationary in order to make this statement?

AC:

While we do not make any assumption about the d18O *signal* here, indeed we assume stationarity of the post-depositional noise (before densification and diffusion which does not influence the ratio of stratigraphic and measurement noise). However, we feel that this is a reasonable assumption, at least for the late-Holocene.

At the end of section 4.2 we now discuss that we do not expect first-order changes of the post-depositional noise levels over time for the Holocene given the stability of the climate in this period.

RC 21, P5624–L25:

I find it problematic that after you have used a certain color for the lateral and vertical noise in your previous calculations, now for the case of the detection of the warming trend you only assume a linear slope plus white noise for the whole signal. This is far from realistic. Take a look at high-resolution deep ice core data – there is a plethora of information in them and they certainly do not look like white noise even for the case of the relatively "boring" Holocene.

AC:

As outlined in more detail in our answer to the general comments, we do not assume at any point that the Holocene climate signal is white. The purpose of the "warming detection thought experiment" is to provide the reader with a simple demonstration what stratigraphic noise implies for the detectability of a temperature trend. Here we aim for the simplest, and also most optimistic model which is reflected in our assumption of a pure linear trend. Including any further signal components (internal climate variability, filtering and modification of the signal by meteorology etc.) would complicate the model and also the understandability for the reader, but also lead to more pessimistic results (thus requiring even more cores to detect an antropoghenic signal).

The white-noise component arises solely from modeling the post-depositional noise. It is correct that on the seasonal time scale the data suggests that the post-depositional noise is autoregressive in the vertical direction (thus in the time domain) with a decorrelation length of $\lambda \approx 6$ cm. However, on the inter-annual time scale the noise for such a λ can be well approximated by white noise as the power spectrum of an AR(1) process levels off on frequencies below the frequency associated with the decorrelation length. As an asset, white noise is more optimistic than AR(1) noise and here also simpler for the reader to understand. We will add some clarifying remarks about the relationship of the vertical noise covariance between seasonal and inter-annual time scales.

As outlined earlier, we now base the best-case scenario of the annual noise level on white noise. Regarding the covariance of the noise on longer time scales, we generally assume white-noise behaviour. These information are given now together in Appendix A as a reference for the reader.

RC 22, P5626–L16:

I assume that with the term "noise" here you refer to post depositional noise. I personally have my strong doubts that this statement is true for three reasons. Firstly a simple spectral analysis of the EDML high resolution data over the last 6000 years will reveal clear information of the diffusion process and thus past temperature. The signal to noise ratio in this case (and of course this varies through the core) is roughly 20-30 dB. Secondly as I have explained above your results are based on values that are likely an overestimate of the final contribution of post depositional noise since you are focusing only at the top 1m. Lastly (and here I have to admit I am doubting myself a bit so take this with a grain of salt..) I am not sure that the use of the statistical variance is proper for a deterministic periodic signal like this of d18O.

AC:

Regarding the reviewer's first point we have to be cautios as the reviewer contrasts two different methods. There are several things to consider:

i) The signal-to-noise ratio (SNR) the reviewer gives in the case of inferring past temperature from diffusion is in our understanding the ratio of the measurement noise (the baseline in the d18O spectra) to the measured spectral signal. This cannot be compared to our SNR contrasting isotopic signal to post-depositional noise, but rather has to be compared to the ratio of isotopic signal to our measurement precision of 0.09 per mil. In the manuscript we use as an estimate for the annual signal variance a value of 0.68 (per mil)^2. This gives a SNR of $10 \log(0.68/0.09^2) \sim 20$ dB, similar to the reviewer's lower bound. On longer time scales one should expect the signal to become stronger. However, in any case the SNR of isotopic signal to post-depositional noise is considerably smaller.

ii) We are afraid that it has not become clear that we refer all our implications for the ability of d18O firn cores to reconstruct past climate to the classical method of interpreting d18O as a proxy for (local) temperature. In this context we do not intend to say that there is no climate signal in the EDML record over the last 6000 years, but that it might be entirely masked by post-depositional noise (see below our answer to the second point). We will rephrase the respective passage to make this clear. We agree with the reviewer that the diffusion method is a powerful tool to reconstruct past temperatures. This is based on the fact that the temperature signal that is reconstructed is not inferred from the isotopic time series itself but by the diffusion acting on it. In fact, it is commonly assumed that, before diffusion, the d18O spectrum is initially white due to post-depositional noise (Gkinis et al. (2014), Johnsen et al. (2000)). We will add a clear statement to the manuscrip that all our implications refer to the classical d18O method, and mention that there are other means utilizing firn cores for climate reconstructions (such as the diffusion method or nitrogen/argon isotope ratios) to which our implications do not necessarily apply.

To the reviewer's second point: It is certainly a strong assumption to apply noise levels inferred from the first metre of firn to a time series covering 6000 years. We will carefully rephrase the respective parts to make this clear. Additionally, we admit that in the manuscript the effect diffusion has on the decadal post-depositional noise level has so far been neglected. However, even after a pessimistic estimate of the effect of diffusion, the change of our results is small: Taking the inter-annual post-depositional noise level inferred from the trenches (5.9 (per mil)^2 in the worst-case, 1.25 (per mil)^2 in the best-case scenario) and assuming the inter-annual noise to be initially white, the decadal noise level is obtained by the integral over the diffused spectrum. Accounting for full forward diffusion with a constant diffusion length of 8 cm water equivalent it turns out that the inter-annual noise level is reduced by a factor of ~ 0.095 instead of a factor of 1/10 for undiffusing white noise. This small difference is due to the fact that for the present accumulation rate at Kohnen station of 6.4 cm w.eq./yr, diffusion mainly acts on isotopic variations on sub-decadal time scales. For longer periods of time it becomes more and more negligible.
In summary, the decadal d18O variations observed in the EDML record can still not easily be interpreted as climatic variations but instead might be to a large extent post-depositional noise. For the revised manuscript, we will add our estimate of the influence of diffusion in the main text and update the noise levels given in Tab. 2 accordingly.

To the last point: We agree with the reviewer that in statistics, variance is strictly defined only in terms of random variables. However, generally climate is a mixture of stochastic and deterministic parts. This is exemplarily seen also in the EDML d18O time series over the last 6000 years which does not resemble a purely deterministic signal (see Fig. 2 of Oerter et al. (2004)). Using the variance in such cases is straightforward.
In addition, we state here that our inferences about the decadal noise level in the EDML core are only a rough estimate since our short trench data do not allow to fully assess the decadal noise covariance.

RC 23, P5626–L25:

Your phrasing on the intermittency of the accumulation may be misunderstood here. It may be a good idea to stress out that you are talking about post deposition (or redeposition) of snow causing the local variability of the accumulation.

AC:

Thanks for the comment; indeed we did not mean accumulation intermittency here but post-depositional redeposition. We will rephrase the sentence accordingly.

RC 24, Appendix A:

I would suggest that the authors spend some time to reread this section. A clean-up in the way symbols are used and what exactly do they mean (perhaps a table?) would be very helpful. In particular the use of the terms ε, $\tilde{\varepsilon}$, ε_x, ε_y, $\sigma_{\overline{x}}^2$, $\sigma_{\overline{x}}^{*2}$ *and what they represent has been very hard for me to follow when reading this section. I also think that since your data analysis is all performed in the depth domain you should substitute* t *with* z *in all the equations in Appendix A.*

Assuming one drills a vertical core and measures a signal $X(z)$ *then this signal can be seen the sum of an ideal signal* $S(z)$ *plus some noise* $w(z)$ *as:*

$$X_n(z) = S_n(z) + w_n(z) \tag{1}$$

where n *the index for core* n *drilled at lag* τ_n. *As far as I understand you consider* $w_n(z)$ *to be the sum of a white noise variance* $w_{vert}(z)$ *in the vertical direction and a variance described by an AR(1) process in the horizontal plane* $\overline{\varepsilon}_n(z)$.

So, $w_{vert}(z)$ *has a constant value and* $\overline{\varepsilon}_n(z)$ *is (simply definition of an AR(1) process):*

$$\overline{\varepsilon}_n(z) = \alpha \cdot \overline{\varepsilon}_{n-1}(z) + \overline{w}_n(z) \tag{2}$$

where $\overline{w}_n(z)$ *is white noise and for simplicity lets assume it is the same for all cores thus simply summing up eq.1 and eq.2 I combine the white noise components into one and get:*

$$X_n(z) = S_n(z) + \varepsilon_{vert}(z) + \alpha \cdot \overline{\varepsilon}_{n-1}(z) + \overline{w}_n(z) = S_n(z) + \alpha \cdot \overline{\varepsilon}_{n-1}(z) + w'(z) \tag{3}$$

Can you clarify where does the normalization parameter in your eq. A3 comes from? I can also not understand how you separate your Gaussian noise in the vertical and your AR1 lateral in the math. Can you be more specific as to what is the difference between your $\widetilde{\varepsilon_{n-1}}(t)$ *and* $\varepsilon_n(t)$. *In the text* $\tilde{\varepsilon}$ *is described as white noise but in eq. A3 it looks like AR(1). Additionally since* $S(t)$ *represents an "ideal" noise-free signal how do you practically calculate the* $var(S)$ *quantity as seen in several of the equations in the manuscript?*

In the beginning of the derivation of eq. A5 you calculate the mean value $X(t)$, *you run the indexes from 1 to* N *but for some reason the variable* n *is kept in the subscript. Is this correct?*

AC:

We are sorry that the derivation given in the appendix was not presented comprehensibly enough. For the revised manuscript, we will re-write the entire derivation in a more concise and understandable fashion, including a clean-up of the nomenclature.

To the individual points:

We agree that it is more appropriate to use z as the vertical variable instead of t and will follow this advice. We will also add a table of symbols summarising the different definitions. The factor $\sqrt{1-a^2}$ is not a result of the derivation but was introduced as a normalization so that the variance of the AR(1) noise series is unity. However, this introduction is actually not necessary and unfortunately led to a small mistake in the manuscript regarding nomenclature of the noise variances which, however, does not affect the actual results. For the revised manuscript, we will not use the this normalization and better separate the nomenclature of the noise (see below).

The noise term $\widetilde{\varepsilon_n}$ of profile n was introduced to be following a first-order autoregressive process in the horizontal direction. Thus, according to the definition of an AR(1) process, this noise term splits into the term $a\widetilde{\varepsilon_{n-1}}$ arising from the autocorrelation of the noise with the previous profile, and a term ε_n which is noise drawn from random variables that are independent and identically distributed (white or Gaussian noise). For the revised manuscript, for the sake of clarity, we will change the notation as follows: The autocorrelated noise will be termed w_n, the independent white noise component of each noise profile ε_n. Then, w is the noise term that can be identified with the horizontal trench variance in the main text, and not ε as accidentally given.

It is unfortunately a misunderstanding that we separate the noise into a vertical and a horizontal component. The only further assumptions about the modelled post-depositional noise is that it is stationary in both the horizontal and the vertical direction, and that its variance is isotropic. Thus, the noise term of a trench profile can be described by a single term. We will state these assumptions more clearly in the revised version of the appendix. A potential depth-dependency of the noise becomes relevant for averaging the trench data from seasonal to lower (e.g. inter-annual) resolution. This depth-dependency is then represented by the covariance of the noise in vertical direction for which the two cases in the main text are discussed (autoregressive noise similar to the horizontal direction (best case), or complete inter-dependence of the noise on the sub-annual time scale (worst case)). We will also describe this discussion in greater detail in the revised manuscript.

An exact estimate of the signal variance, $\mathrm{var}(S)$, is not necessarily needed, since our model results depend only on the signal to noise variance ratio, $\mathrm{var}(S)/\mathrm{var}(\varepsilon)$. For the seasonal time scale, this ratio can be estimated from the inter-profile correlation (Fig. 5) as it is done in the manuscript, and is then used throughout the manuscript for the noise model on this time scale. However, for the inter-annual time scale, individual estimates of the annual signal and noise variance are necessary. The annual signal variance is approximated by the mean of the variances of the mean annual d18O trench time series. This assumes that the noise in the time series is sufficiently averaged out by the stacking of the profiles. We will clarify the respective parts in the manuscript to make our approach and the underlying assumptions more clear to the reader.

The reason why the variable n is kept in the subscript in the beginning of Eq. (A5) is that n denotes the horiztonal position of the profile along the trench; thus n_1 refers to the position of profile number 1, n_N to the prosition of profile number N. We will simplify the entire nomenclature in the revised version of the appendix to avoid such ambiguity.

We present an entirely new version of appendix A with an alternative and more concise derivation of our statistical noise model. In addition, we support our derivation with a table summarising the used nomenclature, a figure depicting a model firn trench, and a figure illustrating the dependency of the relative effective noise variance of a profile stack, $\sigma^{*2}_{\{i\}}$, on the number and spacing of averaged profiles. Further, we extend the appendix by a section discussing the esimation of all relevant parameters used in the noise model.

Answers to specific comments, anonymous referee #2:

RC 1, P5607-L3-4:

The stated text "the strong relationship between the isotopic ratios in precipitation and local air temperature" should be clarified. This is valid at large distances (latitude scale). Variability at a single ice core site will also depend on the trajectory of individual storm tracks, and for example, the location of low pressure zones that influence meteorology. This means that there is both a local temperature effect and an atmospheric effect. This is also mis-represented later in the paper using the Monte Carlo simulation.

AC:

Thank you for this comment. We will remove the adjective "strong" from the cited sentence as the relationship between precipitation and local temperature depends both on the spatial as well as temporal scale considered – as you mentioned and as we describe later in the introduction. In addition, we will better clarify in the manuscript here that local d18O also depends on the specific trajectory of a given precipitation event and thus on meteorology. However, still we think that our approach for the Monte Carlo simulations is valid as we aim to provide the optimistic boundary case which provides an upper bound for the reconstruction of a local temperature trend. We will describe our underlying assumptions for the Monte Carlo approach more clearly – in this context please see also our answers to the general comments.

The relevant part in the Introduction was now removed to shorten the manuscript at this point.

RC 2, P5607–L13-16:

It is mis-leading to say that outside of large-scale temperature shifts (how big? glacial-interglacial size shifts?) it is often too hard to extract climate information. There is still climate information, such as multi-year or decadal oscillations, but perhaps finding a temperature signal in a low accumulation site is too hard. Please clarify. What sort of temperature shift? What does low accumulation even mean (less than 15cm ice eq/yr perhaps)?

AC:

We are sorry that our definition in the manuscript of non-climate noise as "the part of the isotopic record that cannot be interpreted in terms of large-scale temperature variations" was ambiguous. We refer the term "large-scale" here to large spatial scales, not to the amplitude of the temperature variation. We will point this out more clearly by writing "in terms of regional or larger-scale temperature variations".

From this interpretation it follows that any local effects on the isotopic record (meteorological and post-depositional influences) are interpreted as non-climate noise in our manuscript. To our knowledge there is so far no solid evidence that decadal isotope variations observed at a single low-accumulation site, for example in the EDML deep ice-core record, can be interpreted in terms of regional temperature oscillations (as evidenced by a significant correlation to independent climate data). Thus, we think that our statement "may often be too high to accurately extract a climatic signal" is appropriate.

We will define low-accumulation here as being less than 10 cm water eq./year, please see also our answer to comment RC 4.

RC 3, P5607–L21-23:

What are non-climate influences? Do you mean noise, that must be averaged to get climate over something like 30 years or greater? This is at least partially explained in the rest of the paragraph. Perhaps state "short-term processes" or "small spatial scale processes" instead of "non-climate influences".

AC:

We do not limit our definition of "non-climate influence" to noise on small spatial or short temporal scales, but include any influence that leads to isotopic variations (or, respectively, variations of any other temperature proxy) that cannot be interpreted as a regional or larger scale temperature signal. We will rephrase our sentence here to point out that we refer again to our earlier definition of non-climate noise (see our comment on RC 2).

RC 4, P5608–L23:

Please define low-accumulation.

AC:

Albeit being a subjective choice, we will adopt as a definition of low accumulation a value of ≤ 10 cm water eq./year – all the deep ice core sites on the East-Antarcic plateau exhibit less accumulation.

For our manuscript, low-accumulation regions are now defined explicitly in the 2nd paragraph of the Introduction.

RC 5, P5609-L21:

Please state the accumulation rate in m ice eq./yr for comparison to other ice core sites.

AC:

As the unit m ice eq./year is dependent on the the value adopted for the density of ice we would prefer to change the unit to m water eq./year which is common usage in the ice-core sciences as well. The numerical value of the annual mean accumulation rate at Kohnen station would only change by order of magnitude then, being 64×10^{-3} m water eq./year.

RC 6, P5609-L27:

What is a "spirit level"?

AC:

A device with a glass tube filled with liquid and a bubble of air to test whether a surface is level by the position of the bubble.

RC 7, P5611-L5-14:

This paragraph is excellent and useful. Describing the structure of the surface of the snow, and at what locations along the horizontal trench line, allows the reader to form ideas about how this may affect the isotope profiles in the vertical direction.

AC:

Thank you.

RC 8, P5611-L15:

Please also include a standard deviation value, in addition to mean, max, and min.

AC:

The standard deviation of d18O values over the entire trench T1 is 3.1 per mil, over entire T2 2.7 per mil. We will add this information to the manuscript.

RC 9, P5611-L19:

What is a "high" d18O value? In the next line, please give standard deviation, not variance.

This sentence is important, but very confusing. Likewise in line 23, what is a lower d18O value. Please use enriched or depleted.

AC:

We meant "high" and "low" in relation to the respective mean value. However, using "enriched" and "depleted" instead is more appropriate – thanks for this suggestion.

RC 10, P5612-L2:

What is an "isoline"? Please define somewhere above this sentence for clarity. The rest of the paragraph is similarly confusing, and because of its importance, it should be carefully rewritten. Give accumulation rate in m ice eq.yr. Do "lateral layer profiles" refer to isolines? The nomenclature is difficult to follow.

AC:

An isoline is a curve along which some variable (here, d18O) has a constant value. We will add this definition to the paragraph. The lateral layer profiles are thus not identical to isolines since the former follow the seasonal maxima and not a specific constant d18O value. We will re-write the paragraph for clarification.

We have rewritten and simplified this part of the manuscript. The additional nomenclature of an isoline is not needed any longer.

RC 11, P5612-L23-24:

What are "inter-profile deviations" referring to? Deviations of isolines? Try to use one common description, rather than many types. In general, I can interpret what the author means over the preceding two paragraphs, but it should be defined more clearly.

AC:

This paragraph discusses the d18O profiles of T2 (Fig. 2) – we will add "d18O" in line 22 to clarify this. We will change "inter-profile deviations" to "differences between the profiles".

This part has been rewritten.

RC 12, P5613-L2-5:

I cannot understand what this sentence means: "On the horizontal dimension of the trenches, the observed lateral variance (Fig. 3) reflects processes that are not related to variations of atmospheric temperatures as these are coherent on this spatial scale. According to the ter-

minology adopted here, the lateral variance is non-climate noise." Do you mean that local temperature and regional atmospheric circulation should cause variations in vertical isotopes profiles, while horizontal profiles are affected by something else, such as post depositional movement superimposed on the natural climate variability? Also, please do not use "lateral", as this can mean "side-to-side" in the vertical or horizontal direction, and when used on its own, is confusing to the reader. Try to define nomenclature early in the paper, and stick to that nomenclature throughout.

AC:

Yes, you understood it correctly. However, we will re-phrase the sentence to make it easier to understand. In addition, we will add a paragraph to the "Data and Methods" section introducing the coordinate systems used in the manuscript together with a corresponding nomenclature.

The discussion of the horizontal isotopic variance observed in T1 as well as the relevant figure was now removed from the manuscript in order to shorten and simplify the manuscript. We only state the observed mean horizontal variance in the text as it is later needed for the statistical noise model.

RC 13, P5613-L17-25:

For this paragraph: 1) The first sentence repeats previous rationale. 2) In line 22, a mean of what? Units? It is unclear what is being discussed at this point. 3) Why do you call this "classical"? Can you include a reference? 4) In line 25, the author mentions vertical shifting, but it is not entirely clear why this is introduced? Is this peak matching with a max shift of 12cm? The entire paragraph needs to be clarified.

AC:

We will re-write the entire paragraph. In detail we will make the following changes: 1) We will shorten the first sentence. 2) In line 22, we discuss the correlations between single profiles of T1 and single profiles of T2. Hence we will write "mean correlation of ..." instead of just "a mean of ..." for the sake of clarity. 3) We called snow pits "classical" opposed to our more extensive two-dimensional sampling in the trenches. However, as this might be mis-leading we will remove the word "classical" and will include the reference to McMorrow et al. (2002) as an example of a snow-pit study. 4) Allowing for a vertical shift before correlating a profile of T1 with a profile of T2 is necessary as we don't have an exact height reference of T1 relative to T2. We will introduce this at the beginning of the paragraph.

RC 14, P5615-L5:

By "independent of the signal", do you mean the climate signal?

AC:

Yes. We will add the word "climate" for clarification.

This part was moved to Appendix A to simplify the manuscript at this point.

RC 15, P5615-L24:

It might be worth noting that the missing d18O winter values could have been a winter where very little precipitation fell (the seasonality effect).

AC:

This is indeed a possibility and we will add this to the manuscript.

RC 16, P5617-L14:

Spatial precipitation intermittency on scales of km's is not relevant to this study as the trenches are only spaced at 500m.

AC:

We agree to remove this part as we explicitly discuss possible causes of lateral isotopic variance only for the spatial scale of the trenches.

RC 17, P5618-L3:

*The attenuation of the signal with depth *must* be mainly explained by diffusion. Using the term 'likely' disregards physics. I think this paragraph can be shortened considerably to say: diffusion attenuates the signal with depth, and in the upper few meters, ventilation can cause even larger attenuation of the signal.*

AC:

We will shorten the paragraph considerably as you suggest (including an entire removal of the diffusion model).

RC 18, P5618-L28:

What do you mean by "the remaining correlation"?

AC:

We meant the correlation that remains after the small-scale stratigraphic noise is decorrelated. We will rephrase the sentence to make this clear.

RC 19, P5619-L22:

What "criteria"? You mean, "the following criteria"? Or something else?

AC:

We will thoroughly rewrite this part to clarify what is being done here; see also answer to RC 13 of referee #1.

RC 20, P5620-L1:

At this point, I have become somewhat lost. While the larger picture remains clear, the details are confusing. For example, "representativity" is difficult to interpret in many instances.

AC:

We will shorten and simplify the discussion of Fig. 7 to make the general picture more clear to the reader. Regarding the term of representativity that is introduced, we will emphasize the physical interpretation of the term as being an upper bound for the correlation with local temperature. We bear in mind that meteorology (storm tracks, moisture source, etc.) and possibly other effects complicate this simple interpretation. Hence, the representativity can be at most an upper bound. Please see also our answer to RC 14 of referee #1.
Discussion of Fig. 7 is now carried out in section 3.4 to improve the logical structure of the manuscript here and in general.

RC 21, P5623-L5-7:

*You must state in this sentence that the interpretation of firn-core-based climate reconstructions is challenging for *low accumulation sites* and state what accumulation value(s). For high accumulation sites, the interpretation is quite straightforward. As this important sentence is written, it is mis-leading.*

AC:

We will add the information that this is true for low-accumulation sites ($\leq$ 10 cm water eq./year).

RC 22, P5625-L22:

It should be clarified that low accumulation firn cores do not show a coherent signal at high-frequencies (i.e. probably at sub-decadal scales, depending on the accumulation rate).

AC:

We will add to our statement “single isotope profiles obtained from low-accumulation regions are poorly correlated and do not show a coherent signal” that this applies, based on our data, at least to sub-decadal time scales.

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Regional climate signal vs. local noise: a two-dimensional view of water isotopes in Antarctic firn at Kohnen station, Dronning Maud Land

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Abstract

In low-accumulation regions, the reliability of δ^{18}O-derived temperature signals from ice cores within the Holocene is unclear, primarily due to ~~small Holocene~~ the small climate changes relative to the intrinsic noise of the isotopic signal. In order to learn about the representativity of single ice cores and to optimise future ice-core-based climate reconstructions, we studied the stable-water isotope composition of firn at Kohnen station, Dronning Maud Land, Antarctica. Analysing δ^{18}O in two 50 m long snow trenches allowed us to create an unprecedented, two-dimensional image characterising the isotopic variations from the centimetre to the hundred-metre scale. Our results show ~~a clear~~ seasonal layering of the isotopic composition ~~, consistent with the accumulation rate, as well as high lateral~~ but also high horizontal isotopic variability caused by local stratigraphic noise. Based on the horizontal and vertical structure of the isotopic variations, we derive a statistical ~~model for the stratigraphic noise . Our model~~ noise model which successfully explains the trench data~~and~~. The model further allows to determine an upper bound ~~of~~ for the reliability of climate reconstructions ~~from~~ conducted in our study region on seasonal to inter-annual time scales, depending on the number and the spacing of the cores taken.~~Implications for our study region include that reliably detecting a warming trend (0.1decade^{-1}) in 50of data would require $\sim$ 10–50 replicate cores with a horizontal spacing of at least 10. More generally, our results suggest that in order to obtain high-resolution records of Holocene temperature change, fast measurements, thus allowing multiple cores, are more important than to minimise analytic uncertainty as the latter only plays a minor role in the total uncertainty.~~

1 Introduction

Ice cores obtained from continental ice sheets and glaciers are a key climate archive. They store information on past changes in temperature in the form of stable water isotopes (EPICA community members, 2006), in greenhouse gas concentrations via trapped air (Raynaud et al., 1993) and in many other parameters such as accumulation rates (e.g., Mosley-Thompson et al., 2001) or aerosols (e.g., Legrand and Mayewski, 1997).

~~The quantitative interpretation of stable water isotopes builds on the strong relationship between the isotopic ratios in precipitation and local air temperature (**??**) .~~ Analysis of the isotope ratios recorded in single deep ice cores provided milestones in the palaeoclimate research, including the investigation of glacial-interglacial climate changes (Petit et al., 1999) and the existence of rapid climate variations within glacial periods (Dansgaard et al., 1993).

In contrast to ~~this coherent view~~ the coherent view established from polar ice cores ~~on~~ for millennial and longer time scales, the reliability of single ice cores as archives of the Holocene climate evolution is less clear (Kobashi et al., 2011). The small amplitude of ~~Holocene climate~~ changes and the aim to reconstruct ~~them at a~~ climate parameters at high temporal resolution poses a challenge to the interpretation of ice-core signals. This is especially true for low-accumulation sites~~as here the~~ ; defined here for accumulation rates below 100 mm w.eq. yr^{-1} which holds for large parts of the East Antarctic Plateau. There, the non-climate noise – ~~to which we refer in this manuscript as the~~ that part of the isotopic record ~~that~~ which cannot be interpreted in terms of ~~large-scale temperature variations~~ temperature variations on regional or larger scales; hence including any meteorological, pre- and post-depositional effects that additionally influence the isotopic composition – may often be too high to accurately extract a climatic temperature signal (Fisher et al., 1985). Despite the challenges, quantifying the Holocene polar climate variability is the key foundation to determine the range of possible future climate changes (e.g., Huntingford et al., 2013, and references therein) as well as to test the ability of climate models in simulating natural climate variability (Laepple and Huybers, 2014).

The quantitative estimation of climate variability from proxy data therefore requires an understanding of the non-climate ~~influences~~ noise in order to separate them from the climate signal (e.g., Laepple and Huybers, 2013). Several mechanisms influence the isotopic composition of snow prior to and after its deposition onto the ice sheet~~and thus cause~~ . On larger spatial scales, non-climate ~~noise in~~

~~ice-core signals~~variability may be introduced by different moisture pathways and source regions (e.g., Jones et al., 2014) as well as spatial and temporal precipitation intermittency (Persson et al., 2011; Sime et al., 2009, 2011; Laepple et al., 2011) . Irregular deposition caused by wind and surface roughness along with spatial redistribution and erosion of snow is a major contribution to non-climate variance on smaller spatial scales (“stratigraphic noise”~~) (Fisher et al., 1985)~~ , Fisher et al., 1985). Wind scouring can additionally remove entire seasons from the isotopic record (Fisher et al., 1983). ~~Non-climate variability may further be introduced by spatial as well as temporal precipitation intermittency (Persson et al., 2011; Sime et al., 2009, 2011) . After deposition, vapour~~ Vapour exchange with the atmosphere by sublimation-condensation processes (Steen-Larsen et al., 2014) can influence the isotopic composition of the surface layers; diffusion of vapour into or out of the firn driven by forced ventilation (Waddington et al., 2002; Neumann and Waddington, 2004; Town et al., 2008) may represent an additional component of post-depositional change. Finally, diffusion of water vapour through the porous firn smoothes isotopic variations from seasonal to inter-annual ~~and possibly~~ or longer time scales, depending on the accumulation rate (Johnsen, 1977; Whillans and Grootes, 1985; Cuffey and Steig, 1998; Johnsen et al., 2000).

In the last two decades, a ~~growing~~ number of studies analysed ~~to which extent~~ the representativity of single ice cores ~~record a representative climate signal on~~ in recording sub-millennial ~~time scales~~climate changes. One well-studied region is Dronning Maud Land (DML) on the East Antarctic Plateau. ~~Comparing 16 annually resolved isotope records from DML spanning the last 200~~Here, Graf et al. (2002) found low signal-to-noise variance ratios ~~of 0.14 for oxygen isotope ratios and 0.04 for accumulation rates. Karlöf et al. (2006) analysed~~ (F) in 200 year ~~-long records of oxygen isotopes and electrical properties in five cores with inter-site spacings of 3.5–7 km and~~ long firn-core records for oxygen isotopes ($F = 0.14$) and accumulation rates ($F = 0.04$). On a similar time scale, Karlöf et al. (2006) detected no relationship ~~between the cores except for volcanic imprints . This result is consistent with Sommer et al. (2000a, b) who studied~~ in electrical properties apart from volcanic imprints between firn cores. Similarly, high-resolution records of chemical trace species from three ~~DML~~ shallow ice cores ~~(inter-site distances of $\sim$ 100–200 km) and discovered~~ (Sommer et al., 2000a, b) showed a lack of inter-site correlation on decadal time scales. ~~Reconstructed accumulation rates showed a weak but significant correlation between two cores only on time scales larger than 30(Sommer et al., 2000a) . The low representativity of single low-accumulation records was also~~ These results were supported by process studies comparing observed and simulated ~~snow pits, the latter modelled by combining backward trajectories with a Rayleigh-type distillation model (Helsen et al., 2006) . While~~ snow-pit isotope data (Helsen et al., 2006) . Whereas the model-data comparison ~~exercise was reasonably~~ was successful for coastal high-accumulation regions of DML, it largely failed on the dryer East Antarctic ~~plateau. Such a relationship~~ Plateau. This dependency between accumulation rate and ~~the~~ signal-to-noise ratio ~~of ice cores~~ was further demonstrated in ~~different~~ studies across the Antarctic continent (Hoshina et al., 2014; Jones et al., 2014; McMorrow et al., 2002).

~~A similar question of representativity also arises for Artic and Greenlandic records, although the higher accumulation rates generally lead to a higher signal content (**??**Gfeller et al., 2014) .~~

Despite this large body of literature, quantitative information about the signal-to-noise ratios and the noise itself is mainly limited to correlation statistics of nearby cores. While a relatively good understanding of stratigraphic noise exists in Arctic records (Fisher et al., 1985), this does not apply to ~~large parts~~ low-accumulation regions of Antarctica where the ~~environment is markedly different with the accumulation being~~ accumulated snow is considerably reworked in and between storms (Fisher et al., 1985).

Here we provide a direct visualisation and analysis of the signal and noise in an East Antarctic low-accumulation region by an extensive two-dimensional sampling of the firn column in two 50 m long snow trenches. Our approach, for the first time, offers a detailed quantitative analysis of the spatial structure of isotope variability on a centimetre to hundred-metre scale. This is a first step towards a signal and noise model to enable quantitative ~~reconstruction~~ reconstructions of the climate signal and ~~its~~ their uncertainties from ice cores.

2 Data and methods

Near Kohnen station on Dronning Maud Land, close to the EPICA deep ice core drilling site ~~on Dronning Maud Land~~ (EDML, -75.0° S, 0.1° E, altitude 2892 m a.s.l., mean annual temperature $-44.5\,^\circ$C, mean annual accumulation rate ~~$64\,\mathrm{kg\,m^{-2}\,yr^{-1}}$~~64 mm w.eq. yr^{-1}, EPICA community members, 2006), two 1.2 m deep ~~, 1.2m wide~~ and approximately 45 m long trenches in the firn, named T1 and T2, were excavated during the austral-summer field season 2012/2013 using a snow blower. Each trench was aligned perpendicularly to the local snow-dune direction. The horizontal distance between the starting points of T1 and T2 was 415 m.

~~To provide an~~ An absolute height reference ~~, vertically aligned bamboo poles were stuck into the snow every 60 cm applying~~ was established using bamboo poles by adjusting their heights above ground with a spirit level. ~~Additionally,~~ A control measurement with a laser level ~~device was used to check the bamboo pole heights, yielding~~ yielded in each snow trench a vertical accuracy better than 2 cm. No absolute height reference between the two trenches could be established, but, based on a stacked laser level measurement, the vertical difference between the trenches was estimated to be less than 20 cm.

Both trenches were sampled for stable-water-isotope analysis with a vertical resolution of 3 cm. In T1, 38 profiles were taken at variable horizontal spacings between 0.1 and ~ 2.5 m. In T2, due to time constraints during the field campaign, only four profiles at positions of $0.3, 10, 30$ and 40 m from the trench starting point were sampled. All firn samples (~~a total number of~~ $N = 1507$) were stored in plastic bags and transported to Germany in frozen state. Stable isotope ratios were analysed using Cavity Ring-Down Spectrometers (L2120i and L2130i, Picarro Inc.) in the isotope laboratories of the Alfred Wegener Institute (AWI) in Potsdam and Bremerhaven. The isotope ratios are reported in the usual delta notation in per mil (‰) as

$$\delta = \left(\frac{R_{\mathrm{sample}}}{R_{\mathrm{reference}}} - 1 \right) \times 10^3 \,, \tag{1}$$

where R_{sample} is the isotopic ratio of the sample ($^{18}\mathrm{O}/^{16}\mathrm{O}$) and $R_{\text{reference}}$ that of a reference. The isotopic ratios are calibrated ~~to the international V-SMOW/SLAP scale by means of a~~ with a linear three-point regression analysis ~~with different~~ using in-house standards at the beginning of each measurement sequence, where each standard has been calibrated to the international V-SMOW/SLAP scale. Additionally, a linear drift-correction scheme and a memory-correction scheme (adapted from van Geldern and Barth, 2012) is applied~~.~~ ~~The memory correction allows the reduction of repeated measurements per sample; here, we have used~~ , using three repeated measurements ~~instead of six suggested for Picarro instruments when no memory correction is applied, thereby approximately halving the measurement time~~per sample. The analytical precision of the calibrated and corrected δ^{18}O measurements ~~of all trench samples is on average~~ is assessed by evaluating standards in the middle of each measurement sequence. This yields a mean combined measurement uncertainty of 0.09‰ ~~.~~(RMSD).

For the analysis of the measurements, we set up two coordinate systems for each trench (Fig. 1). Surface coordinates refer to a local, curvilinear system with horizontal axis tangential to the surface height profile and vertical axis denoting the firn depth below the local surface. Absolute coordinates adopt the mean surface height as a reference for a straight horizontal axis, completed by an absolute depth scale.

3 Results

3.1 Trench isotope records

The firn samples obtained from trench T1 provide a two-dimensional image of the δ^{18}O structure of the upper ~1 m of firn on a horizontal scale of ~50 m (Fig. 2a).

The surface height profile of the trench ~~exhibits a snow topography which is typical for~~ reflects the typical snow topography of the sampling region ~~. It is~~ characterised by small-scale dunes with their main ridges elongated parallel to the mean wind direction ~~, significantly higher density than the surrounding firn, and typical spatial dimensions of~~

~~4 m width, 8 m length and 20 cm maximum height~~ (Birnbaum et al., 2010). Trench T1 features one prominent dune located between 25–40 m, accompanied by a dune valley between $\sim$ 8–18 m, and some smaller-scale height variations. The peak-to-peak amplitude of the large dune undulation is $\sim$ 10 cm, the entire height variations exhibit a standard deviation (SD) of 2.9 cm. ~~The trench surface height profile is adopted as a local coordinate system (surface coordinates), the mean surface height serves as reference for absolute coordinates.~~

Overall, the trench δ^{18}O record shows a diverse picture. The delta values ~~observed~~ in T1 (Fig. 2a) span a range from −54 to −34 ‰ with a mean of −44.4 ‰ (SD 3.1 ‰). A similar range of −50 to −38 ‰ is observed in T2 (Fig. 3) with a mean of −44.0 ‰ (SD 2.7 ‰). We can ~~clearly identify~~ identify eight to ten alternating layers of ~~high and low~~ enriched and depleted isotopic composition in the T1 record. ~~Following the surface undulations, the~~ The uppermost layer (first 6 cm relative to the surface) essentially shows enriched (mean of −42.7 ‰) ~~shows high~~ but also strongly variable δ^{18}O values between −54 and −34 ‰ ~~with a variance of 19 (‰)^2, thereby already spanning~~ (SD 4.4 ‰), thus already covering the range of the entire trench record. Less ~~negative~~ enriched values tend to be located in the valleys, however, the limited data do not allow to conclude whether this is a general feature. ~~Located below this first layer~~In an absolute depth of 5–20 cm, a band of generally ~~lower~~ more depleted δ^{18}O values is found ~~in an absolute depth of 5–20 cm, likewise following the snow surface but~~ exhibiting less horizontal variability ~~with a range of −53 to −46 ‰.~~

~~To further analyse the layering, we track the pronounced maxima and minima of δ^{18}O values along the trench by automatically determining the local extrema of each isotope profile and visually assigning summer and winter to these extrema, resulting in lateral seasonal layer profiles as a function of depth (Fig. 2b). Implicitly this assumes that the past isotopic isolines are also temporal isolines which is a rough approximation considering the highly variable isotopic composition of the surface layer (Fig. 2a). Our results of the seasonal layer profiles will thus likely overestimate the respective surface height profile of each past season. To analyse the similarity between the past seasonal layer profiles and the present surface, we calculate the root-mean-square deviation (rmsd) between the vertical~~

~~anomalies, i.e., the mean-subtracted seasonal layer profiles, and the horizontal reference as well as the present surface height profile. We find that the first summer layer follows the present surface undulations (rmsd difference of 1.8 cm between the comparison to the horizontal and the surface profile reference). The next three layers show on average a much weaker link with the present surface (rmsd difference of 0.5 cm), and the layers below 40 cm are on average horizontally aligned (difference of -0.8 cm). Comparably to the present surface undulations, the vertical layer anomalies feature peak-to-peak amplitudes of 6–24 cm (average SD of 3.7 cm). Supporting our above assumptions, the vertical separation of the observed lateral layer profiles is approximately 20 cm, in accord with the local mean annual accumulation rate of snow (64 kg m^{-2} yr^{-1}) and the mean firn density of $\rho_{\text{firn}} = 340$ kg m^{-3} measured in trench T1. The layering is strongly perturbed primarily~~ compared to the first layer with a range of -54 to -45 ‰ (mean -48.5 ‰, SD 1.9 ‰). The layering appears strongly perturbed in the depth of $\sim$ 60–100 cm for profile positions $<$ 30 m. Here, a broad and diffuse region of rather constant δ^{18}O values around -40 ‰ is ~~present~~ observed, together with a ~~very~~ prominent, 20 cm-thick feature of high delta values between 18 and 28 m.

~~The four~~ The four δ^{18}O profiles obtained from trench T2 (Fig. 3) show similar ~~results~~ features as trench T1. ~~Roughly five seasonal cycles can be identified, however, with remarkable inter-profile deviations especially~~ We can identify roughly five cycles in each profile. However, the profiles diverge considerably at depths of 50–90 cm ~~. This~~ which coincides with the region of strong perturbations identified in T1. ~~As in trench~~

To further analyse the isotopic layering, we determine the pronounced local maxima and minima of each T1 ~~, the T2 profiles suggest a direct relation between the isotopic layeringand the local snow height profile for the surface snow, i.e., till a depth of 10–20 cm. Below that, the profiles diverge considerably (not shown) but show a better alignment on absolute coordinates (~~δ^{18}O profile and visually assign summer and winter to the depths of these extrema. This results in consecutive horizontal curves tracing the vertical positions of seasonal extrema along the trench (seasonal layer profiles, Fig. ~~3).~~

3.2 ~~Single-profile representativity~~

~~On the horizontal dimension of the trenches, the observed lateral variance~~ 2b). Assuming that respective isotopic extrema occur at the same point in time (summer/winter), the seasonal layer profiles reflect the surface height profile for a given season. However, considering the highly variable isotopic composition observed at the current trench surface (Fig. ~~**??**)reflects processes that are not related to variations of atmospheric temperatures as these are coherent on this spatial scale. According to the terminology adopted here, the lateral variance is non-climate noise. The link between the lateral~~ 2a), this is a rough approximation and the seasonal layer profiles will likely overestimate the past surface height profiles. Nevertheless, the vertical undulations of the layer profiles show peak-to-peak amplitudes of 6–24 cm (average SD 3.7 cm), comparable to the present surface undulations, and the layers are vertically separated by approximately 20 cm, in accord with the local mean annual accumulation rate of snow (64 mm w.eq. yr^{-1}) and the mean firn density of $\rho_{\mathrm{firn}} = 340\,\mathrm{kg\,m^{-3}}$ measured in trench T1. To study the similarity between the seasonal layer profiles ~~of the isotopic composition~~ and the present ~~snow surface decreases with depth (Fig. 2b~~surface height profile, we calculate the standard deviation of their height differences ($\mathrm{SD_{surface}}$; hence, the SD of each layer profile on surface coordinates). This ~~has direct consequences for the analysis of the lateral variability of δ^{18}O values (Fig. **??**). For the first 20 cm, the lateral variance is significantly higher when evaluated on absolute coordinates than on surface coordinates (mean of 16.0 (‰)^2 vs. 7.8 (‰)^2, $p = 0.1$~~is compared to the standard deviation of the layer profiles on absolute coordinates ($\mathrm{SD_{horiz}}$). We find that the first layer profile closely follows the present surface ($\mathrm{SD_{horiz}} - \mathrm{SD_{surface}} = 1.8\,\mathrm{cm}$). For ~~older firn layers ($z < 20\,\mathrm{cm}$) the situation seems to reverse with a mean of 3.6 (‰)^2 in the former and 4.5 (‰)^2 in the latter case. In both cases, the lateral variance shows a pronounced drop from high values in the surface layer to rather constant values deeper in the firn. The overall mean lateral variability of~~ the second layer profile, the link with the surface is weaker ($\mathrm{SD_{horiz}} - \mathrm{SD_{surface}} = 1.5\,\mathrm{cm}$), and the layer profiles below 20 cm are on average horizontally aligned ($\mathrm{SD_{horiz}} - \mathrm{SD_{surface}} = -0.6\,\mathrm{cm}$). This can be explained by an annual reorganisation of the stratigraphy so that aligning the isotopic variations on absolute coordinates is on average more appropriate than the alignement according to a specific surface height profile. The positive autocorrelation with a decorrelation length of ∼6 cm that is found from the vertical T1 ~~record is $\sigma^2_{\mathrm{l,T1}} \simeq 5.9\,(‰)^2$.~~ δ^{18}O variations after subtraction of the mean trench profile is consistent with this hypothesis.

Due to the ~~rather~~ on average horizontal stratigraphy of the isotopic composition in the ~~deeper trench parts~~ larger part of the trench record all further plots and calculations will ~~refer to the horizontal reference and not to the actual snow surface.~~ be evaluated on absolute coordinates.

~~The observation of such a considerable lateral variance or noise level poses the questions on how representative single firn profilesfrom low-accumulation sites are, and how much they reflect the original climate signal that is sought to be reconstructed. One indicator for the similarity of profiles is~~

3.2 Single-profile representativity

The isotope record of trench T1 (Fig. 2a) allows to quantify the horizontal isotopic variability of the snow and firn column in our study region. We observe considerable horizontal variability with a mean variance of $\sigma^2_{\mathrm{h,T1}} \simeq 5.9\,(‰)^2$, directly affecting the representativity of single trench profiles. To mimic the potential result obtained from correlating two snow pits taken at a distance of 500 m, similarly done in many firn-core studies (e.g., McMorrow et al., 2002) , we calculate the pairwise Pearson correlation coefficient ~~. The possible correlations ($N = 152$)~~ between single profiles of T1 and single profiles of T2. We account for potential surface undulations between the trenches by allowing bin-wise vertical shifts of ±12 cm between the T1 and T2 profiles to maximise their correlation. The estimated correlations (Fig. 4) are substantially scattered around a mean correlation of ∼0.50 (~~1 SD = 0.13). Each single correlation mimics the potential result obtained from correlating two “classical” snow pits taken at a distance of 500 m. Due to the lack of an absolute height reference between the trenches, vertical shifting of the T2 profiles of up to ±12 cm is allowed to maximise the correlations.~~ SD = 0.13). The relative majority ($\sim 43\,\%$) of ~~the profile pairs shows an optimal~~ all possible profile pairs ($N = 152$) shows a maximum correlation at a shift of $+3\,$cm which is well below the estimated upper vertical height difference of the trenches.~~Our results indicate that only by chance the classical snow-pit method can yield two profiles that share significant common features (half of the profile pairs show a correlation ≤ 0.49, only two pairs ($\sim 1.3\,\%$) exhibit a correlation above 0.8). In general, due to the inherent noise, single firn profiles cannot be regarded as representative recorders of isotopic proxy signals on the vertical scales analysed here.~~

3.3 Mean trench profiles

3.4 ~~Spatial noise structure~~

~~To quantify the spatial noise structure in the trench isotope record, we investigate the inter-profile correlation as a function of profile spacing (Fig. 6). To this end, all possible profile pairs for a given spacing are selected, allowing a tolerance in the lateral position of 5 %, and the mean inter-profile correlation of the pairs is calculated. The correlation approaches one for nearest neighbours and rapidly drops with increasing inter-profile distance before it stabilises around a value of~~ ~~~ 0.5 for spacings $\gtrsim 10\,$m~~Spatial averaging is expected to improve the correlation between the trenches compared to the single profiles.

~~This spatial correlation structure can be described using a simple statistical model: We assume that each profile consists of a common signal S and a noise component ε independent of the signal. The noise component is modeled as a first-order autoregressive process (AR(1)) in the lateral direction. The inter-profile correlation coefficient can then be expressed (see Appendix A) as~~

$$r_{XY} = \frac{1}{1 + \frac{\mathrm{var}(\varepsilon)}{\mathrm{var}(S)}} \left\{ 1 + \frac{\mathrm{var}(\varepsilon)}{\mathrm{var}(S)} \exp\left(- \frac{|x-y|}{\lambda} \right) \right\} .$$

~~Here, $\mathrm{var}(\varepsilon)/\mathrm{var}(S) =: F^{-1}$ is the inverse of the signal-to-noise variance ratio of the profiles, $|x-y|$ is the inter-profile spacing, and λ denotes the decorrelation length of the~~

~~autocorrelation. The variance ratio determines the limit of Eq. (3) for $|x-y|\to\infty$. It is estimated from the data using the mean inter-profile correlation for the profile spacings between 10–35 m, giving a value of $F^{-1}=1.1\pm0.1$. An estimate of the decorrelation length is obtained from the lateral δ^{18}O variations of~~ We therefore correlate the mean trench profiles of T1 ~~by calculating the autocorrelation at a lag of $\Delta\ell=1$ m. To account for the irregular lateral sampling, we apply the Gaussian kernel correlation discussed in Rehfeld et al. (2011) and find that the noise correlation has decreased to $1/e$ at a distance of $\lambda\simeq1.5$ m.~~

~~The signal-to-noise variance ratio can also be directly estimated from the data if we identify the noise variance with the mean lateral trench variance, var(ε) $=\sigma_{\mathrm{l}}^2$, and assume that the noise is isotropic and independent of the signal. Then, the signal variance var(S) can be estimated with the mean down-core variance σ_{v}^2 (T1: $\sigma^2_{\mathrm{v,T1}}\simeq9.5\,(‰)^2$,~~ and T2, allowing again for bin-wise vertical shifts of the T2 ~~: $\sigma^2_{\mathrm{v,T2}}\simeq7.3\,(‰)^2$) reduced by the noise variance. For T1 we obtain var(S) $\simeq\sigma^2_{\mathrm{v,T1}}-\sigma^2_{\mathrm{l,T1}}=3.6\,(‰)^2$. This gives a variance ratio of ~1.6 which is of the same order of magnitude as the estimate from the inter-profile correlationbut slightly underestimates the signal strength.~~

3.4 ~~Trench mean profiles~~

~~The spatial mean of all T1~~ profile to maximise the correlation.

The mean trench profiles (Fig. 5) ~~is highly correlated with the spatial mean of the T2 profiles~~ are highly correlated ($r_{\mathrm{T1,T2}}=0.81$ for an optimal shift of $+3$ cm; $p=0.01$, accounting for the full autocorrelation structure and allowing for vertical shifting), indicating a common ~~seasonal~~ isotopic signal reproducible over a spatial scale of at least 500 m. It is interesting to note that this value is above most of the single ~~inter-profile~~ inter-trench correlations (Fig. 4). ~~Due to the surface undulations, the number of existing observations evaluated on absolute coordinates varies for the first three depth bins. To obtain non-biased mean profiles, only the depth range covered by all profiles is used in the averaging process. A vertical shift of the mean T2 profile of ±12 cm was allowed to maximise the correlation~~

~~and, consistent to the results obtained for single profiles, an optimal shift of +3 cm was obtained. In both~~

In both mean profiles, we observe five seasonal cycles spanning a range of ∼ 6–7 ‰ at the surface, but being attenuated further down and exhibiting no clear sinusoidal shape in the ~~"fourth" year (65–90 cm depth)~~depth range of 65–90 cm. Interestingly, this obscured part without ~~any clear signal of~~ clearly depleted δ^{18}O "winter" values is found in both trenches, indicating that this feature persists over ~~several hundred of metres~~ at least 500 m and is thus likely of climatic origin~~.~~, e.g., a winter with unusually low precipitation. Despite the statistically significant correlation~~($p = 0.01$, accounting for the full autocorrelation structure and allowing for vertical shifting of ±12 cm),~~ , pronounced differences between the mean profiles are present, such as a significantly ~~lower~~ more depleted, respectively more enriched, isotopic composition of the T2 mean between 50–80 cm~~and a considerably higher one within depths up to ∼ 40 cm as well as for the lowermost region of the trenches~~.

~~In order to obtain~~ To analyse annual-mean δ^{18}O time series we ~~define annual bins through~~ use different binning methods to average the seasonal trench data with bins defined by (1) the six local maxima determined from the ~~averaged profile~~ average of the two mean trench profiles~~. The mean peak-to-peak distance of these maxima is 19.8 cm, consistent with the accumulation rate. Three alternative sets of annual bins are derived from the~~ , (2) the five local minima~~as well as from~~ , (3) the midpoints of the ~~slopes flanking these minima.~~ ascending slopes flanking the maxima and (4) the midpoints of the descending slopes. To display the data on an absolute time axis we assign the year 2012 to the first annual bin. The annual-mean time series derived from ~~these four~~ the four possible binning sets are averaged to obtain a single time series for each trench (Fig. 5). The correlation of the average annual-mean δ^{18}O time series of ~~$0.87^{+0.07}_{-0.20}$~~ $r_{T1,T2} = 0.87^{+0.07}_{-0.20}$ (range represents the four binning methods) is comparable to that of the mean seasonal profiles (0.81). However, five observations of annual means are too short to reliably estimate the correlation and its significance.

4 ~~Discussion~~

3.4 Spatial correlation structure

~~Climate reconstructions based on proxy data rely on the assumption that at least part of the measured signal is related to a climate parameter, such as temperature in case of ice/firn-core derived~~ We have shown that spatial averaging significantly increases the correlation between the trenches. To learn more about the spatial correlation structure of the trench isotope record, we investigate (1) the inter-profile correlation as a function of profile spacing for T1 and (2) the inter-trench correlation between different sets of mean profiles from T1 and the mean T2 profile.

The inter-profile correlation is estimated as the mean of the correlations obtained from all possible T1 profile pairs separated by a given spacing, allowing a tolerance in the horizontal position of 5 %. For the inter-trench correlation, we define a T1 profile stack as the spatial average across a certain number of T1 profiles separated by a given distance, and determine all possible equivalent stacks. The inter-trench correlation with the mean T2 profile is then recorded as the mean across the correlations between the mean T2 profile and all possible T1 stacks.

The inter-profile correlation approaches one for nearest neighbours and rapidly drops with increasing inter-profile spacing before it stabilises around a value of ~ 0.5 for spacings $\gtrsim 10\,\mathrm{m}$ (Fig. 6). For the inter-trench correlation, we find a steady increase in the correlation with the T2 reference with increasing number of profiles used in the T1 stacks (Fig. 7). Additionally, the correlation increases with a wider spacing between the individual profiles of a stack.

The observed decrease of the inter-profile correlation with distance suggests a horizontal autocorrelation of the isotopic composition. Indeed, a positive autocorrelation of the horizontal δ^{18}O ~~(**?**) .However, proxy signals are inherently noisy with uncertainties arising prior to deposition of the proxy into the archive, post-depositionally during archive storage, as well as later in the human sampling and measurement process (**?**Laepple and Huybers, 2013; **?**) .~~ variations of T1 with a decorrelation length of $\lambda \simeq 1.5\,\mathrm{m}$

is found by applying a Gaussian kernel correlation (Rehfeld et al., 2011) which accounts for the irregular horizontal sampling. As we do not expect any climate-related part of the isotopic record to vary on such small spatial scales, we attribute the observed autocorrelation to noise features.

3.5 Statistical noise model

The inter-profile correlation provides an estimate of the signal-to-noise variance ratio F of single profiles (Fisher et al., 1985) ,

$$F = r_{XY}/(1 - r_{XY}). \tag{2}$$

Neglecting the small-scale correlation, we estimate F from the data using the mean inter-profile correlation for the profile spacings between 10–35 m and find $F = 0.9 \pm 0.1$.

Based on our findings, we develop a simple statistical model: We assume that each trench profile consists of the sum of a common climate signal S and a noise component w independent of the signal. The noise component is modelled as a first-order autoregressive process (AR(1)) in the horizontal direction. Then, the inter-profile correlation coefficient between profiles X and Y becomes a function of their spacing d (see Appendix A),

$$r_{XY} = \frac{1}{1 + F^{-1}} \left\{ 1 + F^{-1} \exp\left(-\frac{d}{\lambda} \right) \right\}. \tag{3}$$

Here, $F^{-1} = \mathrm{var}(w)/\mathrm{var}(S)$ is the inverse of the signal-to-noise variance ratio. Using our estimate for F and the value for λ obtained in the previous section, the model reproduces the observed inter-profile correlations (Fig. 6). Applying the same parameter values, the theoretical inter-trench correlation (Eq. A15) is also in good agreement with the empirical results (Fig. 7). This validates the model and the parameter values (F, λ) from the intra-trench ($\sim$ 10 m) to the inter-trench spatial scale ($\sim$ 500 m).

4 Discussion

Our trench data confirm earlier results that individual ~~firn records of~~ δ^{18}O firn-core records from low-accumulation regions are strongly influenced by local noise (Fisher et al., 1985; Karlöf et al., 2006). ~~However, going~~ Going beyond this finding, our two-dimensional δ^{18}O ~~dataset~~ data set also allows to determine the spatial structure and to learn about the causes of the noise. In this section, we discuss our findings in the context of the possible noise sources and derive implications for seasonal to inter-annual climate reconstructions based on firn cores.

4.1 ~~Trench δ^{18}O variance levels~~Local stratigraphic noise and regional climate signal

A ~~hypothetical,~~ horizontally stratified trench ~~with zero isotopic variance in lateral direction~~ without horizontal isotopic variations would yield perfectly correlated single profiles. ~~However, in the actual trenches we observe a high lateral variance (see Fig. **??** for T1) with a mean variance that is comparable to the mean~~ Opposed to that, our records show a significant variability in horizontal direction with mean variances $(\sigma^2_{\mathrm{h,T1}} \simeq 5.9\,(‰)^2$, $\sigma^2_{\mathrm{h,T2}} \simeq 5.3\,(‰)^2)$ that are smaller but of the same order of magnitude as the mean downcore ~~variance (Table 1).~~variances $(\sigma^2_{\mathrm{v,T1}} \simeq 9.5\,(‰)^2$, $\sigma^2_{\mathrm{v,T2}} \simeq 7.3\,(‰)^2)$. In consequence, coherent isotopic features between single profiles separated by the trench distance are only found by chance (Fig. 4: the median correlation is 0.49, only for two pairs ($\sim 1.3\,\%$) the correlation is > 0.8). Thus, single firn profiles from our study region are no representative recorders of climatic isotope signals on the vertical scales analysed here.

~~Several pre- and post-depositional effects induce lateral variance of the firn layer, the relative importance of each varies on the spatial scales considered. Starting on the m-scale, the principal contribution is induced by the surface roughness, closely related to snow drift events including spatial redistribution, erosion, reworking and dune formation ("stratigraphic noise", Fisher et al., 1985). Possibly, exchange of water vapour with the atmosphere by sublimation-condensation~~

~~processes (Steen-Larsen et al., 2014) , potentially accompanied by forced ventilation (Waddington et al., 2002; Neumann and Waddington, 2004; Town et al., 2008) , acts as a further noise source. Going to larger spatial scales ($\gtrsim 1\,\mathrm{km}$), spatial precipitation intermittency (e.g., **?**Persson et al., 2011; Sime et al., 2009, 2011) presents an additional component, influencing a certain snow layer via spatially varying precipitation weighting.~~

~~The down-core variance includes the isotopic signal from seasonal and longer climate variations. In addition, the vertical isotope record is also subject to modifications arising prior to and after the deposition of snow. Temporal precipitation intermittency can bias the δ^{18}O record (Laepple et al., 2011) but also induces vertical variability caused by inter-annual variations of the timing of precipitation events (Persson et al., 2011; Sime et al., 2009, 2011) . Diffusion of water vapour through the porous firn along seasonal isotopic gradients (Johnsen, 1977; Johnsen et al., 2000; Whillans and Grootes, 1985; Cuffey and Steig, 1998) obsc seasonal and longer isotopic cycles, depending on the accumulation rate. Forced ventilation acts perpendicular to the pressure isolines in the firn, generated by the steady wind flow across the undulating surface (Waddington et al., 2002) . Depending on the dune undulations, this may enhance the vertical diffusion in the first tens of centimetres of firn and shorten the time for the snowpack to reach isotopic equilibration.~~

~~The pronounced drop in the lateral variance with depth (Fig. **??**) can likely be explained by isotopic diffusion. This is suggested by a simple numerical estimate diffusing an artificial trench record that initially exhibits a rectangular isotope variation (25 % summer precipitation)as well as a sinusoidal surface topography with a wavelength of 10 m and a peak-to-peak amplitude of 10 cm (Fig. **??**, see Appendix B for details). While these are promising results, the theoretical estimate of Waddington et al. (2002) as well as a numerical diffusion model including forced ventilation by Neumann and Waddington (2004) showed that the true rate of diffusion in the first metre might be higher. Furthermore, Town et al. (2008) demonstrated that forced ventilation also attenuates the seasonal cycle. In total, at the current stage of investigation we are not able~~

~~to clarify the importance of water vapour exchange and forced ventilation. For this, more field measurements and a thorough numerical treatment are necessary.~~

4.2 ~~Spatial structure of lateral variance~~

~~In Sect. **??** we showed that~~ On the horizontal scale of the ~~inter-profile correlation as a function of profile spacing (Fig. 6) can be described by a common signal overlayed by lateral noise following an AR(1) model.~~

~~This demonstrates firstly that each single trench profile features a local isotopic signal common only over a few metres which is induced by small-scale covarying noise. The decorrelation length of $\sim 1.5\,\mathrm{m}$ of this noise is related to the intermittent deposition of snow and, in particular, to the dune scale: A sinusoidal surface height variation with a wavelength ν of $\lesssim 10\,\mathrm{m}$ would lead to zero autocorrelation for a shift of $\nu/4$, similar to our observations. While the real surface topography is more complicated, it suggests~~ trenches ($\sim$ 10–500 m), we expect that stratigraphic noise dominates the isotopic variations (Fisher et al., 1985) . The observed length scale of the horizontal decorrelation of the noise ($\lambda \sim 1.5\,\mathrm{m}$) is similar in magnitude as that on which the local small-scale surface height variations occur, indicating that stratigraphic noise is ~~an important~~ in fact the prominent noise component in our ~~δ^{18}O records. In addition, vapour exchange with the atmosphere driven by forced ventilation might contribute to the overall noise level since it is likewise related to the surface roughness. Secondly, the remaining~~ data.

Despite the low single-profile representativity, the trench record contains a climate signal becoming apparent through the inter-profile correlation of ~ 0.5 ~~for inter-profile spacings of~~ remaining on scales on which the stratigraphic noise is decorrelated ($\gtrsim 10\,\mathrm{m}$~~, implying roughly the same amount of signal and noise variance in single profiles, is due to a regionally coherent~~). It appears to be regionally ($\lesssim 1\,\mathrm{km}$) ~~isotope signal, supported by the fact that it is comparable to the~~ coherent as suggested firstly by the comparable values of the inter-profile correlation for spacings $\gtrsim 10\,\mathrm{m}$ and the mean correlation between ~~individual~~ single T1–T2 records (Fig. 4)~~. However, this regional isotope signal does not directly translate into a regional climatic signal of local surface air temperature as various effects~~

~~can influence the isotopic composition of precipitation (?) . Further, there is the possibility of an additional noise component with a spatial decorrelation length larger than the distance between both trenches, for example caused by spatial precipitation intermittency.~~

~~The spatial autocorrelation structure and~~ , and secondly by the common seasonal signal observed in the ~~inter-profile correlation provide estimates of an optimal sampling strategy for firn-coring efforts in the study region. To ensure that the local noise is uncorrelated, single profiles should be spaced at distances several times the decorrelation length. Visually, we find a minimum spacing of ~ 10 m to be optimal (Fig. 6).~~

4.2 ~~Representativity of isotope signals on seasonal to inter-annual time scales~~

~~Our statistical model of covarying stratigraphic noise allows to determine the seasonal signal content depending on the number of profiles and the profile spacing. As the model is entirely based on parameters estimated from the T1 data, we can use the T2 data to validate the model. Therefore, we determine and predict the correlation of an averaged set of T1 profiles with the T2 trench mean, the latter thus serving as a reference isotopic signal.~~

~~To determine the correlation from the data for a given number of profiles and a profile spacing, all possible unique sets of T1 profiles are selected that fulfill the given criteria. Due to the uneven spacing of the T1 profiles, we allow an absolute uncertainty of the spacing between the profiles in a set of 0.5 m. The correlation is given as the mean correlation over all sets. Empirically, we find a steady increase in the correlation with the T2 reference for increasing number of profiles used in the T1 set~~ mean trench profiles (Fig. ~~7). The observed increase in correlation is expected since also for autocorrelated noise the noise variance of the set decreases with the number of profiles. Additionally, as a direct consequence of the autocorrelation structure, the correlation increases with a wider spacing between the individual profiles of the T1 set (Fig. 7). A given number of profiles at a spacing of 2.4 m~~ 5).

Noise is always reduced by averaging profiles; here, the autocorrelation causes nearby profiles to share more common noise variance than ~~the same number of~~ profiles at a larger spacing. ~~Thus, when the two profile sets are averaged, the latter set will show a higher~~

~~correlation with the reference signal. This finding also explains the comparable reduction of the noise levels of the trench mean profiles (for T1 the levels drop by 46 % compared to the mean of the individual down-core variances, for T2 by 55 % (Table 1)): The 38 T1 profiles have varying inter-profile distances from 0.1–2.5 m, whereas the four T2 profiles are already spaced at large, more optimal distances of 10–20 m.~~

~~Our noise model allows to calculate the theoretical inter-trench correlation coefficient (Eq. A16). Using the variance ratio of $F^{-1} = 1.1$ obtained in Sect. **??**,~~ Therefore, albeit the same number of profiles is averaged, stacks using a larger profile spacing will exhibit less common noise variance and hence a larger proportion of the ~~model prediction is in good agreement with the empirical data (Fig. 7). We can conclude that the first-order autoregressive noise modelcaptures the major noise component for isotopic records on spatial scales of at least 500 m as well as on temporal scales of a few years~~underlying signal (Fig. 7). Our results show a minimum profile spacing of ~ 10 m to be optimal.

4.2 Representativity of isotope signals on seasonal to inter-annual time scales

For quantitative climate reconstructions from proxy data, a robust estimate of the climate signal is necessary. Based on our statistical noise model, we can estimate the isotopic climate signal content of a profile stack for our study region depending on the number of averaged profiles and their spacing.

~~With the noise model validated between the trenches, implications for climate reconstructions using firn-core isotope records can be deduced. We define the representativity~~ To this end, we define the climate representativity of a ~~set of trench profiles~~ trench profile stack as the correlation ~~of this set with a hypothetical,~~ between the stack and a common climate signal (Eq. ~~A16~~A14). This ~~representativity can be interpreted as the upper limit~~ signal is identified with the coherent isotope signal of the trench records. A physical interpretation of the climate representativity is then the upper bound of the correlation ~~to a temperature times series obtained from a weather stationlocated in the study region~~with a local temperature record, for example from a weather station. However, bearing in mind other influences such as meteorology (variable storm tracks, changing moisture source regions, precipitation-weighting), the true correlation will be lower. In the limit of independent noise ~~terms (vanishing autocorrelation),~~ our definition of ~~representativity yields the same expression as~~ climate representativity is equivalent to the expression derived by Wigley et al. (1984).

~~The representativity is time-scale-dependent since signal and noise variance are both a function of the time scale~~In general, climate signals are time-scale dependent. For example, the seasonal ~~variability~~ amplitude of the isotopic signal is much larger than any ~~year-to-year variations of the istopic signal. Analysing seasonal variability, the representativity can be readily calculated with the variance ratio F^{-1} given above. For reconstructions on inter-annual~~ variations between the years. On the other hand, one expects larger changes of the climate signal on longer time scales, ~~the isotope records are additionally averaged in the vertical direction and thus, the results depend on the vertical noise covariance. Snow-pit studies around Vostok station have shown significant temporal non-climatic~~ such as glacial-interglacial cycles. Moreover, not only the climate signal but also the noise can be a function of the time scale. One extreme example for this are the non-climate oscillations of the isotopic composition ~~(Ekaykin et al., 2002) , indicating a vertical spatial noise structure. The observed time scales of the oscillations range from 10^0–10^1, possibly up to 10^2, and are~~ on up to centennial time scales which have been indicated by snow-pit studies around Vostok station and linked to the movement of accumulation waves ~~of various scales . Here, due to the limited data coverage in vertical direction, we are only able to investigate two limiting cases. As the simplest best-case scenario (case I), the vertical noise covariance is given by an AR(1) process as in the lateral direction.In the worst case (case II) , averaging one annual firn layer does not reduce the noise level at all, assuming a complete interdependence of the noise on the sub-annual~~ on various scales (Ekaykin et al., 2002) . Since the climate representativity (Eq. A14) depends on the ratio F of signal and noise variance, it is in consequence also a function of the time scale.

~~The variance ratio of noise over signal is for the inter-annual time scale thus given by~~

$$F^{-1}_{\text{annual}} = \frac{\text{var}(\varepsilon)_{\text{annual}}}{\text{var}(S)_{\text{annual}}} = \begin{cases} \frac{\text{var}(\varepsilon)\sigma^{*2}_{\text{annual}}}{\text{var}(S)_{\text{annual}}} & \text{case I} \\ \frac{\text{var}(\varepsilon)}{\text{var}(S)_{\text{annual}}} & \text{case II.} \end{cases}$$

~~The effective annual noise variance var(ε)$\sigma^{*2}_{\text{annual}}$ (Eq. A16)for case I depends on the autocorrelation parameter a_{annual} which is estimated from the mean autocorrelation function of the vertical δ^{18}O data of T1 after subtracting the mean seasonal profile.We obtain a value of $a_{\text{annual}} \approx 0.61$ for a lag of $\Delta\ell = 3$ cm, equivalent to a decorrelation length of $\lambda_{\text{annual}} \approx 6$ cm. As the best-possible estimate, an annual signal variance of var$(S)_{\text{annual}} \simeq 0.68\,(‰)^2$ is obtained from the mean of the variances of the annual δ^{18}O time series (Fig. 5)of the two trenches (Table 1) . The seasonal noise variance var(ε) is set to the observed mean lateral T1 variance (Table 1). Altogether, we obtain an annual variance ratio of $F^{-1}_{\text{annual}} \simeq 1.8$~~

Here, we assess the climate representativity of firn isotope profiles from our study region for two specific time scales, (1) the original (seasonal) resolution of the trench data and (2) an inter-annual time scale based on binning the trench data to annual resolution.

Analysing seasonal variability, the climate representativity can be readily calculated with the model parameters obtained in Sect. 3.5. For the inter-annual time scale, estimates of both annual signal and noise variance are necessary to assess the variance ratio F. However, the shortness of our trench data on this time scale only allows heuristic estimates (see Appendix A for details). Specifically, for the annual noise variance we discuss two limiting cases: For case I) we assume that the vertical noise is white (best-case scenario), for case II) that the vertical noise shows complete inter-dependence on the sub-annual time scale (worst case). The inverse of the annual signal-to-noise variance ratio, $F^{-1}_{\text{annual}} = \text{var}(w)_{\text{annual}}/\text{var}(S)_{\text{annual}}$, used in the model is then ~ 1.2 for case I ~~, and of $F^{-1}_{\text{annual}} \simeq 8.7$~~ and ~ 8.7 for case II. ~~Note that using the seasonal noise variance as calculated from the entire trench data might represent a slight overestimation given the exceptionally high variability observed in the surface layer (Fig. **??**)~~A summary of the noise levels is given in Table 2.

For single profiles, the estimated climate representativity on the seasonal time scale is ~~around~~ 0.69 (Fig. 8). On the inter-annual time scale, single profiles ~~have~~ show a representativity of ~~0.59~~ 0.67 in the best-case scenario (Fig. ~~8a) and a much lower one~~ 8a) and of 0.32 in the worst-case scenario (~~0.32,~~ Fig. ~~8b~~8b).

~~In general~~Similar to the correlation between the trenches (Fig. 7), the representativity increases with the number of profiles averaged ~~, and the increase is stronger~~ with a stronger increase for larger inter-profile spacings. However, spacings above 10 m do not ~~increase the representavities any further~~ yield a further increase as the stratigraphic noise is ~~practically decorrelated(Fig. 6).~~ largely decorrelated. To obtain a climate representativity of 0.8 for inter-annual signals with profiles separated by 10 m, ~~one needs to take~~ a minimum of ~~4–16 cores~~ 3–16 cores is needed (from best to worst case). Demanding a representativity of 0.9, the number of cores required increases to ~~8–37.~~ 6–37.

~~The low~~ The modelled single-profile ~~representativity on~~ climate representativity for the inter-annual time scale ~~is~~ appears consistent with previous findings from Dronning Maud Land. ~~The 16 annually resolved δ^{18}O records of the study of Graf et al. (2002) , taken in an area extending 500 km from east to west and 200 km from north to south, showed~~ Graf et al. (2002) estimated a low signal-to-noise variance ratio ~~in the individual records~~ of $F = 0.14$ obtained from the cross-correlations of 16 annually resolved δ^{18}O records from an area of 500 km × 200 km. Due to the large ~~inter-profile spacing~~inter-core spacings, the stratigraphic noise ~~covariance~~ in the records is decorrelated ~~. Then,~~ and the variance ratio F ~~from the cross-correlations directly translates into the representativity of a single profile as~~ can be translated into a single-profile representativity of $r_{SX} = 1/\sqrt{1+F^{-1}} \simeq 0.35$, consistent with our findings for the worst-case scenario~~(case II)~~. However, ~~this accordance does not necessarily mean that our worst-case scenario is the more realistic one since the measured cross-correlations~~ the records analysed in Graf et al. (2002) are also subject to ~~potential dating uncertaintiesand~~ dating uncertainties, additional variability caused by spatially varying precipitation-weighting and ~~possibly~~ other effects. Therefore, the similar representativities are not necessarily caused by the high stratigraphic noise level assumed in the worst-case scenario. In addition, our trench data indicate vertical autocorrelation of the noise (Fig. 2b and Sect. 3.1). Thus, the true climate representativity for our study region will likely be in between of our limiting estimates.

Stratigraphic noise does not only affect ~~isotopic records~~ isotopes but also other ~~proxies derived from~~ parameters measured in ice cores, such as aerosol-derived chemical constituents. Gfeller et al. (2014) investigated the seasonal to inter-annual representativity of ion records from five Greenland firn cores ~~on seasonal and inter-annual time scales,~~ taken at varying distances from 7–10 m in the vicinity of the NEEM drilling site. ~~With~~ Using the definition of representativity based on ~~the theoretical work of Wigley et al. (1984) , for inter-annual time scales Gfeller et al. (2014) found representativities of $\sim$ 0.55–0.84 for single cores, and of $\sim$ 0.84–0.95 for the average of all five cores , depending on the ions~~ Wigley et al. (1984) , they found inter-annual representativities of $\sim$ 0.55–0.95, depending on the number of averaged cores and the ion species considered. These numbers are slightly higher than our best-case-scenario results for δ^{18}O, ~~a fact~~ which is expected ~~as~~ since the accumulation rate at the NEEM site is about three times higher than at Kohnen station (NEEM community members, 2013).

Our estimates for the climate representativity of firn cores hold as long as the signal-to-noise variance ratio F does not change. Variance-affecting processes such as diffusion and densification have equal influence on signal and noise and thus do not alter the ratio F. On the other hand, only one component might change over time; e.g., the noise variance might vary due to changing environmental conditions, or the variability of the climate could have been different in the past for certain time periods. Nevertheless, given the stability of the Holocene climate, we do not expect first-order changes of the signal and noise properties over time. However, we do expect a time-scale dependency of the climate signal with more variance associated with longer time scales (e.g., Pelletier, 1998) . The signal-to-noise variance ratio and the climate representativity of firn cores will improve considerably on these scales.

4.3 Implications

~~The noise level identified in our trench data poses a significant challenge for the interpretation~~ Our noise level and implied climate representativity estimates underline the challenge of firn-core-based climate reconstructions on seasonal to inter-annual time scales ~~. In the following, we discuss examples of implications of the~~ in low-accumulation regions. For our study site, we now discuss implications of our noise model concerning (1) the required measurement precision of water isotopes in the case of classical isotope thermometry, (2) the potential noise fraction in isotope signals of the EDML ice core and (3) the detectability of ~~anthropogenic temperature trends in low-accumulation firncores.~~ an anthropogenic temperature trend.

Our estimates of the stratigraphic noise level are based on the upper one metre of firn. Due to the shortness of the data our results are limited by our insufficient knowledge of the vertical noise covariance structure for time scales above annual resolution for which we now assume white-noise behaviour. The noise of isotopic data obtained from deeper parts of the firn column is affected by diffusion and densification. The latter only is of importance for undated samples. We estimate the effect of diffusion and find that for decadal time scales even below the firn-ice transition the decadal noise level at Kohnen station is reduced by only 5 % (Appendix B and Table 2) compared to the undiffused case.

The noise of an isotopic signal ~~consists of~~ includes the stratigraphic noise ~~discussed here~~ as well as ~~the~~ noise caused by the measurement process. ~~Thus,~~ Since the stratigraphic noise is a function of the number of analysed cores, and measurement precision is often related to measurement time, obtaining the best signal is a trade-off between measurement precision and the amount of analysed samples.

For seasonal as well as on inter-annual time scales, the measurement uncertainty of the trench data of $\Delta\delta^{18}\mathrm{O} = 0.09‰$ is much lower (~~~ 4–$8\,\%$~~~ 4–$10\,\%$) than the standard deviation of the stratigraphic noise (Table 2). This ratio is independent of the temporal resolution if a lower ~~temporal~~ resolution is obtained by averaging annually resolved data as both ~~, the noise level and the measurement uncertainty,~~ contributions decrease by the same amount in the averaging process, assuming independence between the samples. In such a case, priority should be given to measuring and averaging across multiple cores in order to reduce the (stratigraphic) noise levels instead of performing ~~high-accuracy~~ high-precision measurements on single cores~~, given that we are only interested in δ^{18}O~~. As an example, ~~for~~ with the Cavity Ring-Down Spectrometers ~~as those that have been~~ used for this work ~~, much~~ faster measurements are possible by reducing the number of repeated measurements ~~down to one per sample , resulting only in a slight decrease in measurement precision when a memory correction scheme as applied to our data is used~~per sample and applying a memory correction (van Geldern and Barth, 2012) . We explicitly note that this possibility is limited to classical single-isotope (δ^{18}O) reconstructions as it can affect the data usability for diffusion- (Gkinis et al., 2014; van der Wel et al., 2015) or deuterium-excess-based (Vimeux et al., 2001) inferences.

If a lower temporal resolution is obtained by a coarser sampling of ~~firn~~ the cores, the measurement error to stratigraphic noise ratio will depend on the analysed resolution (Table 2). For a resolution corresponding to ten years, our measurement uncertainty might amount to up to ~~25 %~~ 32 % of the stratigraphic noise level, ~~assuming independence of the stratigraphic noise between the years. For our data, the~~ accounting for full diffusion. The noise level of single cores would become comparable to the measurement uncertainty for averages over ~~$\sim$ 154~~$\sim$ 104 or $\sim$ 735 years ~~(case I) or $\sim$ 728~~(~~case II~~best- or worst-case scenario of annual noise level).

The deep EPICA ~~Dronning Maud Land~~ DML ice core obtained in the vicinity of Kohnen station ~~shows~~ reflects the climate evolution in Antarctica over the last 150 000 years (EPICA community members, 2006). Oerter et al. (2004) studied a ~~section of the core~~ core section covering the last 6000 years ~~with a resolutionof ten years (their Fig. 2).~~ on decadal resolution. We find a ~~decadal~~ δ^{18}O variance for this ~~part of the core of $\sim$ 0.57 (‰)^2. If we assume that our estimates of the~~ section of $\sim$ 0.57 (‰)^2. Using our diffusion-corrected stratigraphic noise variance ~~hold over the last couple of thousand years, then $\sim$ 20–100 % of the decadal variance seen in the EDML core over this time period might be simply~~ estimates would imply that $\sim$ 15–100 % (from best to worst case) of the observed decadal variance in the core might be noise (Table 2)~~. In order to reconstruct the Holocene climate variabilityof the last millennium from low-accumulation regions, there is thus the clear need to either~~

~~average~~, masking the underlying climate variability. We note that this is only a rough estimate as the shortness of the trench data does not allow to fully assess the decadal noise covariance. In any case, averaging across multiple cores ~~based on the results of the previous section, or,~~ seems necessary in low-accumulation regions to reconstruct the climate variability of the last millennium. Alternatively, if only the magnitude of variability is of interest, ~~to correct~~ the proxy variability has to be corrected for the noise contribution (e.g., Laepple and Huybers, 2013).

As a final example of applying our noise model, we ~~estimate the ability of firn cores close to the Kohnen station to reconstruct a potential warming trend of the last decades. In the last~~ test the influence of stratigraphic noise on the detectability of a linear trend at Kohnen station. This is motivated by the finding of Steig et al. (2009) that in the last 50 years ~~,~~ the surface temperature over East Antarctica has warmed by about half a degree~~(Steig et al., 2009) . The probability to detect thistrend or to reconstruct its slope is estimated using a Monte Carlo approach creating 10^5 δ^{18}O time series consisting of a signal (the linear temperature trend) and uncorrelated Gaussian noisewith variance equal to the annual trench noise variance for the best as well as the worst case~~. While both the climate signal as well as the relationship between local temperature and isotopic signal are complex, we assess the detectability with a toy model experiment. For this, we assume the climate signal to be a purely linear trend (0.5 °C/50 yr) and a linear isotope-to-temperature relationship (1 ‰ K^{-1}), further influenced only by post-depositional noise. In a Monte Carlo approach repeated 10^5 times, we create stacks from 50 yr long δ^{18}O profiles with post-depositional noise variances based on our two limiting cases (Table 2) ~~. The trend is detected when the correlation of the time series with the signal is positive at the significance level of~~ and independent noise between the profiles (inter-profile spacings $\gtrsim$ 10 m), and vary the number of averaged profiles. A trend in the stacked profile is successfully detected for an estimated trend that is signifantly larger than zero ($p = 0.05$~~. We define the probability for determining the right slope as the fraction of cases where a linear regression yields a slope that~~); the estimated slope is defined to be correct when it lies in a range of 25 % around the true slope. ~~To simplify matters, we assume a temperature-to-isotope gradient for δ^{18}O~~

~~of 1‰ K^{-1}, given the considerable uncertainties associated with the spatial and temporal gradients discussed in the literature (e.g., **?**) . We note that in general the δ^{18}O slope very likely lies below 1‰ K^{-1} ($\sim$ 0.8‰ K^{-1} for DML, EPICA community members, 2006) which implies yet lower detectionprobabilities since the signal variance is then even smaller compared to the noise variance. Finally, in the case of multiple cores it is assumed that they are taken at distances on which the autocorrelation of the stratigraphic noise is decorrelated ($\geq$ 10 m).~~ The probability for trend detection/slope determination is then the ratio of successful reconstructions to total number of realisations.

~~The~~ Drilling a single core, the probability to detect the trend or to reconstruct its slope is ~~below~~ around 20 % ~~for single cores~~ in the best-case and below 10 % in the worst-case scenario (Fig. 9). To reliably ($>$ 80 % of the cases) detect the warming over the East Antarctic ~~plateau~~Plateau, our results suggest that averaging across at least ~~$\sim$ 10–50~~ $\sim$ 7–50 firn cores taken at spacings of 10 m (Fig. 9) is needed, depending on the scenario for the annual noise variance. Inferring the right slope would need three times that number of cores. We note that more realistic assumptions about the isotopic signal (natural climate and atmospheric variability, varying isotope-temperature relationship, etc.) further complicate the trend detectability.

5 Conclusions

We presented extensive oxygen stable water isotope data derived from two snow trenches excavated at Kohnen station in Dronning Maud Land, Antarctica. The two-dimensional approach allowed a thorough investigation of the representativity of single firn-core isotope profiles, as well as of the spatial structure of the signal and noise over spatial scales of up to 500 m and a time span of approximately five years.

The trench data confirm previous studies that single ~~isotope profiles obtained from~~ low-accumulation ~~regions are poorly correlated and do not~~ ($\lesssim$ 100 mm w.eq. yr^{-1}) isotope profiles only show a ~~coherent signal, but also demonstrated~~ weak coherent signal at least on sub-decadal time scales. We also demonstrate that the spatial average of a sufficient number of profiles provides ~~a representative isotopic signal. We further show that single profiles are strongly influenced by local , small-scale noise that exhibits a spatial covariance.~~ representative isotopic signals, consistent with our finding that the local noise has a small horizontal decorrelation length (~ 1.5 m). This also suggests stratigraphic noise to be the major contribution to the horizontal isotopic variability. A statistical ~~model describing this noise as~~ noise model based on a first-order autoregressive process successfully explains the observed covariance structure and allows to reproduce the ~~observed~~ correlation statistics between the trenches. ~~The autocorrelation of the noise occurs on spatial scales that are of the same order of magnitude as the surface height variations introduced by sastrugi and dunes and the intermittent deposition of snow, suggesting stratigraphic noiseas a major noise source. Extending the ordinary stacking of isotope records, our results are used to infer appropriate sampling strategies. We derive the~~

Based on these results we infer appropriate sampling strategies. At our low-accumulation (64 mm w.eq. yr^{-1}) site an optimal spacing of about 10 m is necessary for a sufficient decorrelation of the stratigraphic noise. For seasonal and annual resolution, we estimate the climate representativity of isotope profiles ~~for seasonal to annual resolution~~ depending on the number of averaged firn cores and the inter-core spacing. ~~For our low-accumulation (64 mm w.eq. yr^{-1}) study region, we find an optimal profile spacing of about 10 m where the noise covariance is sufficiently decorrelated. The representativity depends on the time scale: For seasonal resolution , five profiles taken with the optimal spacing are sufficient~~ Our estimates show that for seasonal resolution five cores at this spacing are necessary to obtain representative (~~$R > 0.9$~~$r > 0.9$) isotope signals; on inter-annual time scales ~~, ~ 2–8~~ up to ~ 8 times as many ~~profiles would be needed.~~ cores are needed. As climate variations are typically stronger on longer time scales than analysed here, the climate representativity of firn- and ice-core reconstructions for slower climate changes will likely be higher.

~~The low representativity of single firn profiles at our site hampers the~~ We present two explicit examples of how the stratigraphic noise might hamper the quantitative interpretation of isotope in terms of climate variations ~~. The noise level observed in the trench data suggests that large parts~~ at our study site. Our data suggest that at least 15 % of the decadal variations seen in the EPICA DML ice core over the last 6000 years might be ~~noise. In addition, we show that faithfully reconstructing the~~ post-depositional noise, but the climate signal might also be masked by a much higher decadal noise level. A toy model experiment shows that the faithful reconstruction of the recent positive temperature trend observed over the East Antarctic ~~plateau is impossible by drilling only single cores; instead, averaging at least 10–50 firn coreswould be necessary. This task is~~ Plateau likely requires averaging across at least 7–50 firn cores. For single-proxy (δ^{18}O) reconstructions this task could be rendered easier by the fact that the annual noise level is substantially larger than typical measurement uncertainties. ~~Therefore,~~ Thus, monitoring the measurement error depending on sample throughput could allow fast measurements for ~~high-resolution single-proxy reconstructions it might be more advantageous to conduct less precise measurements, e.g., by operating Cavity Ring-Down Spectrometers with only one injection per sample , for~~ the benefit of analysing many cores. Alternatively, using indirect methods based on diffusion (Gkinis et al., 2014; van der Wel et al., 2015) or gas isotope ratios (Kobashi et al., 2011) might circumvent the problem of stratigraphic noise.

Since the stratigraphic noise is related to ~~the intermittent deposition~~ irregular re-deposition and erosion of snow and the formation of surface dunes, it ~~depends primarily~~ primarily depends on the local accumulation ~~rates~~rate, besides further factors such as wind strength, temperature, seasonal timing of the precipitation and snow properties. Therefore, ~~to a first approximation~~ we expect that our representativity results improve (worsen) for regions with higher (lower) accumulation rates. In effect, ~~results similar to ours likely hold~~ our results are likely applicable for large parts of the East Antarctic ~~plateau, but trench-like approaches~~ Plateau, but similar studies in West Antarctica and Greenland – regions with considerably higher accumulation rates – are needed. In addition, studies with deeper trenches that cover ~~longer times of isotopic variations~~ a longer time period, complemented by spectral analyses of nearby firn cores, are necessary to enhance our knowledge ~~about~~ of the vertical noise covariance structure~~which~~. This is crucial to determine the climate representativity on longer time scales. Deeper trenches would also allow to link our representativity results to actual correlations with temperature time series derived from weather stations. The latter is part of ongoing work at Kohnen station.

Appendix A: Derivation of noise model

~~The Pearson pairwise correlation coefficient of two time series, or profiles , X and Y reads~~

$$r_{XY} = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y},$$

~~where σ_X and σ_Y are the standard deviations of profile$X$ and profile Y, respectively, and cov(X,Y) is the covariance of the profiles given by~~

$$\mathrm{cov}(X,Y) = \langle XY \rangle - \langle X \rangle \langle Y \rangle .$$

~~Here, $\langle \cdot \rangle$ denotes the temporal average, thus the spatial average in vertical direction for a trench profile.~~ *Definitions*

We consider isotope profiles $X_i(z)$ at equidistant spacings $\Delta\ell$ where z is depth on absolute coordinates and i refers to the profile's horizontal position along a snow trench, $\ell_i = \ell_0 + i \cdot \Delta\ell$, with some arbitrary starting position ℓ_0 (Fig. A1). This and all subsequent nomenclature is summarised in Table A1.

We ~~now assume that a trench isotope profile $X_n(t)$ consists of a signal part $S(t)$ and a noise component $\widetilde{\varepsilon}_n(t)$ that is independent from the signal and following a standard normal distribution. In addition, to account for the spatial covariance of the noise in lateral direction, we assume each noise term to be following~~ assume each $X_i(z)$ as a sum of a common signal $S(z)$ and a noise term $w_i(z)$ independent of S,

$$X_i(z) = S(z) + w_i(z). \tag{A1}$$

The noise $w_i(z)$ is modelled as an AR(1) ~~autoregressive proccess,~~

$$\begin{aligned} X_n(t) &= S(t) + \widetilde{\varepsilon_n}(t) \\ &= S(t) + a\widetilde{\varepsilon_{n-1}}(t) + \sqrt{1-a^2}\,\varepsilon_n(t)\,. \end{aligned}$$

~~Here,~~ process in the horizontal direction,

$$w_i(z) = a w_{i-1}(z) + \varepsilon_i(z)\,, \tag{A2}$$

where a is the autocorrelation parameter with $0 \leq a \leq 1$ ~~, and~~ and $\varepsilon_i(z)$ are independent random normal variables (white noise). We assume the same variance var(w) of the ~~square-root term in front of $\varepsilon_n(t)$ is a normalisation. If we consider P equidistant trench profiles numbered $1,2,\ldots,P$, the noise term of profile n can be given recursively,~~

$$X_n(t) = S(t) + a^{n-1}\varepsilon_1(t) + \sqrt{1-a^2}\sum_{i=2}^{n} a^{n-i}\varepsilon_i(t)\,.$$

~~With the help of Eq. (A3), we can calculate the spatial~~ noise in both the horizontal and the vertical direction.

The mean of a set of N trench ~~profiles,~~

$$\begin{aligned} \overline{X}(t) :=& \quad \overline{\{X_{n_1}(t), X_{n_2}(t), \ldots, X_{n_N}(t)\}} \\ =& \quad S(t) + \tfrac{1}{N}\Big\{ \left(\textstyle\sum_{i=n_1}^{n_N} a^{i-1}\right)\varepsilon_1(t) \\ & + \sqrt{1-a^2}\left(\textstyle\sum_{i=2}^{n_1} a^{n_1-i}\varepsilon_i(t) + \cdots + \sum_{j=2}^{n_N} a^{n_N-j}\varepsilon_j(t)\right)\Big\} \\ =& \quad S(t) + \tfrac{1}{N}\Big\{ \left(\textstyle\sum_{\nu} a^{\nu-1}\right)\varepsilon_1(t) + \sqrt{1-a^2}\textstyle\sum_{i=2}^{\nu^*}\left(\sum_{k\in\{\nu>1,\nu\geq i\}} a^{k-i}\right)\varepsilon_i(t)\Big\} \end{aligned}$$

~~where we have defined $\nu := \{n_1, n_2, \ldots, n_N\}$ and $\nu^* := \max(\nu)$.~~

~~From Eq. (A1) the inter-profile correlation coefficient can be calculated for general kinds of covarying~~ isotope profiles $\overline{X}_{\{i\}}$ (profile stack) is defined by the indices

$\{i\} = \{i_1, i_1 + i_2, i_1 + i_2 + i_3, \ldots, i_1 + i_2 + \cdots + i_N\}$. This nomenclature of incremental steps simplifies the expressions obtained later. $\overline{X}_{\{i\}}(z)$ is given by the signal $S(z)$ and the mean of the noise terms, ~~$\mathrm{cov}(\varepsilon_X, \varepsilon_Y) \neq 0$. With $\mathrm{cov}(X,Y) = \mathrm{var}(S) + \mathrm{cov}(\varepsilon_X, \varepsilon_Y)$, $\mathrm{var}(\varepsilon_X) = \mathrm{var}(\varepsilon_Y) \equiv \mathrm{var}(\varepsilon)$ and therefore $\mathrm{var}(X) = \mathrm{var}(S) + \mathrm{var}(\varepsilon_X) \equiv \mathrm{var}(Y)$ we obtain~~

$$r_{XY} = \frac{\mathrm{var}(S) + \mathrm{cov}(\varepsilon_X, \varepsilon_Y)}{\mathrm{var}(S) + \mathrm{var}(\varepsilon)}.$$

~~Further, the idendity $\mathrm{cov}(\varepsilon_X, \varepsilon_Y) = \langle \varepsilon_X \varepsilon_Y \rangle$, holds for noise . Thus, for~~

$$\overline{X}_{\{i\}}(z) = S(z) + \frac{1}{N}\left(w_{i_1} + w_{i_1+i_2} + \cdots + w_{i_1+i_2+\cdots+i_N}\right)(z). \tag{A3}$$

The Pearson correlation of two single profiles X_i and X_{i+j} is

$$\mathrm{corr}\left(X_i, X_{i+j}\right) = \frac{\mathrm{cov}\left(X_i, X_{i+j}\right)}{\sqrt{\mathrm{var}(X_i)\mathrm{var}(X_{i+j})}} = \frac{\mathrm{var}(S) + \mathrm{cov}\left(w_i, w_{i+j}\right)}{\mathrm{var}(S) + \mathrm{var}(w)}, \tag{A4}$$

using the independence of signal and noise and the stationarity of w.

The correlation of a profile stack $\overline{X}_{\{i\}}$ and the signal is given by

$$\mathrm{corr}\left(\overline{X}_{\{i\}}, S\right) = \frac{\mathrm{cov}(\overline{X}_{\{i\}}, S)}{\sqrt{\mathrm{var}(\overline{X}_{\{i\}})\mathrm{var}(S)}} = \frac{\mathrm{var}(S)}{\sqrt{\mathrm{var}(\overline{X}_{\{i\}})\mathrm{var}(S)}}. \tag{A5}$$

Similarly, the correlation of two profile stacks with indices $\{i\}$ and $\{j\}$, assuming independent noise between the sets, is obtained from

$$\mathrm{corr}\left(\overline{X}_{\{i\}}, \overline{X}_{\{j\}}\right) = \frac{\mathrm{cov}(\overline{X}_{\{i\}}, \overline{X}_{\{j\}})}{\sqrt{\mathrm{var}(\overline{X}_{\{i\}})\mathrm{var}(\overline{X}_{\{j\}})}} = \frac{\mathrm{var}(S)}{\sqrt{\mathrm{var}(\overline{X}_{\{i\}})\mathrm{var}(\overline{X}_{\{j\}})}}. \tag{A6}$$

Derivation of model correlations

To derive the explicit correlations (A4)–(A6) for the AR(1) ~~autocorrelated noise (Eq. A3) the covariance reads~~

$$\mathrm{cov}(\varepsilon_X,\varepsilon_Y) = a^\xi \mathrm{var}(\varepsilon)\,\mathrm{with}\,\xi := \frac{|x-y|}{\Delta\ell}.$$

~~Here, $|x-y|$ is the distance between profile X and Y, and $\Delta\ell$ ist~~ noise model, we need expressions for the noise variance, var(w), the noise covariance in horizontal direction, cov(w_i,w_{i+j}), and the variance of a profile stack, var$(\overline{X}_{\{i\}})$.

The former two are given by (Chatfield, 2004)

$$\mathrm{var}(w) = \frac{\mathrm{var}(\varepsilon)}{1-a^2}, \tag{A7}$$

$$\mathrm{cov}(w_i,w_{i+j}) = \frac{\mathrm{var}(\varepsilon)}{1-a^2}\,a^j = \mathrm{var}(w)\,a^j. \tag{A8}$$

The index j can be expressed here by the distance $d=\ell_{i+j}-\ell_i$ between the profiles X_i and X_{i+j} and the spacing of adjacent profiles ~~. This can be seen if we set, without loss of generality, $X=X_1$ and $Y=X_n$ and calculate the spatial mean $\langle\varepsilon_{X_1}\varepsilon_{X_n}\rangle$, noting that only products of identical noise terms have non-vanishing covariance. The parameter a is the value of the autocorrelation function at lag one ,~~ $\Delta\ell$ as $j=d/\Delta\ell$. Further, for an AR(1) process the lag one autocorrelation is given by $a=\exp(-\Delta\ell/\lambda)$ ~~, where λ is the typical length scale on which the autocorrelation decreases to the value of 1/e. Thus, the covariance of the noise terms~~ with the decorrelation scale λ. It follows from (A8) that the horizontal noise covariance decreases exponentially with ~~increasing inter-profile spacing $|x-y|$.~~distance d as

$$\mathrm{cov}(w_i,w_{i+j}) = \mathrm{var}(w)\exp\left(-\frac{d}{\lambda}\right). \tag{A9}$$

~~To obtain the representativity of a trench profile set, we correlate the profile set with the signal $S(t)$,~~

$$r_{S\overline{X}}=\frac{\text{cov}(S,\overline{X})}{\sigma_S\sigma_{\overline{X}}};$$

~~correlating two profile sets yields~~ The variance of a profile stack $\overline{X}_{\{i\}}$ is calculated according to

$$\begin{aligned}\text{var}(\overline{X}_{\{i\}})&=\left\langle\overline{X}^2_{\{i\}}(z)\right\rangle-\left\langle\overline{X}_{\{i\}}(z)\right\rangle^2\\&=\text{var}(S)+\frac{1}{N^2}\left\langle\left(w_{i_1}+w_{i_1+i_2}+\dots+w_{i_1+i_2+\dots+i_N}\right)^2(z)\right\rangle\end{aligned}\tag{A10}$$

where $\langle\cdot\rangle$ denotes the expected value. Using the multinomial identity $(\xi_1+\xi_2+\dots+\xi_N)^2=\sum_{i=1}^{N}\xi_i^2+2\sum_{i=1}^{N-1}\sum_{j>i}\xi_i\xi_j$ yields

$$\begin{aligned}\text{var}(\overline{X}_{\{i\}})=\ &\text{var}(S)+\frac{1}{N^2}\Bigg\{N\text{var}(w)+\\&2\Big(\langle w_{i_1}w_{i_1+i_2}\rangle+\langle w_{i_1}w_{i_1+i_2+i_3}\rangle+\dots+\\&\langle w_{i_1}w_{i_1+i_2+i_3+\dots+i_N}\rangle+\langle w_{i_2}w_{i_2+i_3}\rangle+\dots+\\&\langle w_{i_2}w_{i_2+i_3+\dots+i_N}\rangle+\dots+\langle w_{i_{N-1}}w_{i_{N-1}+i_N}\rangle\Big)\Bigg\}.\end{aligned}\tag{A11}$$

By applying (A8) for the horizontal covariance of the ~~amount of variance shared by the sets,~~

$$r_{\overline{X}\overline{Y}}=\frac{\text{cov}(\overline{X},\overline{Y})}{\sigma_{\overline{X}}\sigma_{\overline{Y}}}.$$

~~For statistically independent signal and noise terms we have cov($S,\overline{X}$) = var(S). For cov($\overline{X},\overline{Y}$) we assume that one profile set is derived from~~ noise we obtain

$$\text{var}(\overline{X}_{\{i\}}) = \text{var}(S)+\text{var}(w)\times \underbrace{\frac{1}{N^2}\left\{N+2\left(a^{i_2}+a^{i_2+i_3}+\dots+a^{i_2+\dots+i_N}+a^{i_3}+\dots+a^{i_3+\dots+i_N}+\dots+a^{i_N}\right)\right\}}_{\sigma^{*2}_{\{i\}}} \qquad \text{(A12)}$$

where we define $\sigma^{*2}_{\{i\}}$ as the relative effective noise variance of the profile stack. In the limiting case of $a=0$ (zero autocorrelation) $\sigma^{*2}_{\{i\}}=1/N$, in the limit of $a=1$ (perfect autocorrelation) $\sigma^{*2}_{\{i\}}=1$. In general, $\sigma^{*2}_{\{i\}}$ is a function of both N and the spacing of the profiles averaged (Fig. A2).

For final expressions of the correlation functions (A4)–(A6), we define the signal-to-noise variance ratio $F := \frac{\text{var}(S)}{\text{var}(w)}$ and use (A9) and (A12) to obtain

$$\textit{inter-profile corr.:}\quad \text{corr}\,(X_i,X_{i+j}) = \frac{1}{1+F^{-1}}\left\{1+F^{-1}\exp\left(-\frac{d}{\lambda}\right)\right\}, \qquad \text{(A13)}$$

$$\textit{stack-signal corr.:}\quad \text{corr}\left(\overline{X}_{\{i\}},S\right) = \frac{1}{\sqrt{1+F^{-1}\sigma^{*2}_{\{i\}}}}, \qquad \text{(A14)}$$

$$\textit{stack-stack corr.:}\quad \text{corr}\left(\overline{X}_{\{i\}},\overline{X}_{\{j\}}\right) = \frac{1}{\left\{\left(1+F^{-1}\sigma^{*2}_{\{i\}}\right)\left(1+F^{-1}\sigma^{*2}_{\{j\}}\right)\right\}^{1/2}}. \qquad \text{(A15)}$$

Estimation of parameters

To evaluate the correlation functions (A13)–(A15) we need estimates of the decorrelation length λ and of the time-scale dependent variance ratio F^{-1}.

For the trench data on the seasonal time scale, we obtain a variance ratio of $F^{-1}\simeq 1.1\pm 0.1$ from the observed inter-profile correlations of T1 ~~, the other from T2.~~

~~As the trenches are separated by $\sim 500\,\mathrm{m}$, the noise terms are to a good approximation decorrelated, and therefore $\mathrm{cov}(\overline{X},\overline{Y}) \simeq \mathrm{var}(S)$. What is left to calculate is the variance $\sigma_{\overline{X}}^2$ of a profile set. A straightforward calculation, again noting that only products of identical noise terms do not vanish in the averaging process, yields~~

$$\sigma_{\overline{X}}^2 = \langle \overline{X}^2 \rangle - \langle \overline{X} \rangle^2 = \langle \overline{X}^2 \rangle - \langle S \rangle^2$$

$$= \mathrm{var}(S) + \mathrm{var}(\varepsilon) \frac{\sigma_{\overline{X}}^{*\,2}}{N^2}\,.$$

~~Here, $\mathrm{var}(\varepsilon)\sigma_{\overline{X}}^{*\,2}$ is the effective noise variance of the profile set using the definition~~

$$\sigma_{\overline{X}}^{*\,2} := \left(\sum_{\nu} a^{\nu-1}\right)^2 + (1-a^2)\sum_{i=2}^{\nu^*}\left(\sum_{k\in\{\nu>1,\nu\geq i\}} a^{k-i}\right)^2.$$

~~By combining Eqs. (A10)~~(Fig. 6) for profile spacings $\geq 10\,\mathrm{m}$, and an estimate of the decorrelation length of $\lambda \simeq 1.5\,\mathrm{m}$ from the horizontal autocorrelation of the T1 δ^{18}O data. We validate the parameters by comparing the predicted (A15) and observed correlations between profile stacks derived from T1 and ~~(A12) with Eq. (A16) , respectively, we finally obtain expressions for the representativity of a trench profile set as well as for the shared variance of a~~ T2 (Fig. 7). This assumes independent noise between T1 and ~~a T2 profile set:~~

$$r_{S\overline{X}} = \frac{1}{\sqrt{1+\frac{\mathrm{var}(\varepsilon)}{\mathrm{var}(S)}\frac{\sigma_{\overline{X}}^{*\,2}}{N^2}}}\,;$$

$$r_{\overline{X}\overline{Y}} \simeq \frac{1}{\left\{\left(1+\frac{\mathrm{var}(\varepsilon)}{\mathrm{var}(S)}\frac{\sigma_{\overline{X}}^{*\,2}}{N_{\overline{X}}^2}\right)\left(1+\frac{\mathrm{var}(\varepsilon)}{\mathrm{var}(S)}\frac{\sigma_{\overline{Y}}^{*\,2}}{N_{\overline{Y}}^2}\right)\right\}^{1/2}}\,.$$

~~For vanishing autocorrelation, $a \to 0$, Eq. (A16)gives $\sigma^{*}_{\overline{X}}{}^{2} \to N$. Thus, the representativity of a profile set, Eq. (A16) , simplifies to the classical result~~

$$r_{S\overline{X}} \xrightarrow{a\to 0} \frac{1}{\sqrt{1+\frac{1}{N}\frac{\mathrm{var}(\varepsilon)}{\mathrm{var}(S)}}},$$

~~where the noise variance scales with the number of profiles averaged.~~T2, a valid approximation given that the trench distance (~ 500 m) is much larger than λ. Relying on the assumption of equal noise variance in the horizontal and vertical direction, a second estimate of $F^{-1} \sim 1.6$ can be obtained from the observed mean T1 down-core variance (identified with signal and noise) subtracted by the observed mean T1 horizontal variance ($=$ noise).

~~For the full trench data, Eqs. (A16) –(A16)are referred to as the representativities on the seasonal time scale with the corresponding seasonal variance ratio of $\frac{\mathrm{var}(\varepsilon)}{\mathrm{var}(S)}$. On the~~ Going from the original seasonal resolution of the trench data to an explicit inter-annual time scale, ~~this variance ratio is replaced by the corresponding annual ratio of $\frac{\mathrm{var}(\varepsilon)_{\mathrm{annual}}}{\mathrm{var}(S)_{\mathrm{annual}}}$, where for the~~ the short data sets only allow limited estimations. We thus make use of the following simple heuristic arguments. The annual signal variance is estimated from the mean annual δ^{18}O time series of each trench neglecting the residual noise contributions and averaging both variance estimates to obtain $\mathrm{var}(S)_{\mathrm{annual}} \simeq 0.68$ (‰)^2. The annual noise variance, ~~$\mathrm{var}(\varepsilon)_{\mathrm{annual}}$,~~ $\mathrm{var}(w)_{\mathrm{annual}}$, is calculated from the seasonal noise variance estimated by the mean horizontal T1 variance of $\mathrm{var}(w) \simeq 5.9$ (‰)^2. Physically, we expect a vertical autocorrelation of the noise due to the underlying processes (stratigraphic noise, Fisher et al., 1985; Ekaykin et al., 2002 ; diffusion) which is also indicated by our data (Fig. 1b). However, due to the limited vertical trench data, the vertical noise autocorrelation cannot be reliably estimated and we discuss two limiting cases: case I) the vertical noise is independent (white noise) and the seasonal noise variance therefore reduced by the number of samples included in the ~~two limiting cases discussed in the text are used.~~annual average ($N \approx 7$), case II) the vertical noise shows complete inter-dependence on the sub-annual time scale and its variance is not reduced by taking annual means. The resulting inter-annual variance ratios of noise over signal are

$$F_{\text{annual}}^{-1} = \frac{\text{var}(w)_{\text{annual}}}{\text{var}(S)_{\text{annual}}} \simeq \frac{1}{0.68} \times \begin{cases} 0.84\,, \\ 5.9 \end{cases} = \begin{cases} 1.2, \text{ for case I}\,, \\ 8.7, \text{ for case II}\,. \end{cases} \tag{A16}$$

For all longer time scales, we generally assume white-noise behaviour for the noise covariance.

Appendix B: ~~Estimate of the influence of isotopic diffusion~~

Appendix B: Reduction of noise level by diffusion

~~To estimate the effect of isotopic diffusion through the porous firn on the lateral δ^{18}O variance of the trenches, we apply a simple numerical approach. An artificial δ^{18}O trench of 45 m length and 1.2 m depth is built by creating isotope profiles with a rectangular δ^{18}O variation (expressed as relative variation between −1 and 1) adopting a summer fraction of 25 %. The lateral resolution is set to 0.6 m, resulting in 76 profiles; the vertical resolution is fixed at 0.5 cm. Each profile is vertically shifted to mimic a surface height variation d of the form~~

$$d(x) = \Delta \cdot \sin\left(\frac{2\pi}{\lambda}x\right)$$

~~with a peak-to-peak amplitude of $2\Delta = 10$ cm and a wavelength of $\lambda = 10$ m.~~ The integral over the power spectrum $P(f)$ of a time series $X(t)$, where f denotes frequency and t time, is equal to the total variance of X (Chatfield, 2004) ,

$$\text{var}(X) = 2\int_0^{f_0} P(f)\,df\,. \tag{B1}$$

Here, f_0 is the Nyquist frequency according to the sample resolution of X.

For ~~the numerical diffusion calculation,~~ a given diffusion length σ and local annual layer thickness $\dot{b}$, diffusion changes the initial power spectrum $P_0(f)$ according to (van der Wel et al., 2015)

$$P(f) = P_0(f)\exp\left(-2\pi\sigma\dot{b}^{-1}f\right)^2 \tag{B2}$$

For white noise, the initial power spectrum is a constant, $P_0(f) = P_0 = \mathrm{const}$. In this case, the integral (B1) is straightforward to solve,

$$2P_0\int\limits_0^{f_0}\exp\left(-2\pi\sigma\dot{b}^{-1}f\right)^2 df = P_0\sqrt{\pi}/(2\pi\sigma\dot{b}^{-1})\,\mathrm{erf}(2\pi\sigma\dot{b}^{-1}f_0). \tag{B3}$$

We assume a layer thickness of ice of $\dot{b} = 7\,\mathrm{cm\,yr^{-1}}$ (equivalent to the ~~diffusivity is taken approximately as a constantover the first metre of firn with a value for δ^{18}O of $D \approx 2.9\times10^{-8}\,\mathrm{cm^2\,s^{-1}}$, which has been calculated according to Johnsen et al. (2000) adopting the relevant parameters for Kohnen station . The diffusion length is modeled to vary with time as (Johnsen et al., 2000)~~

~~$\sigma_{\mathrm{diff}}(t) \sim \sqrt{2Dt}$,~~

~~assuming zero vertical strain rate. The time t of burial since deposition is expressed in terms of the depth of the respective snow parcel using the present accumulation rate $\dot{b}$ of snow, $t(z) = z/\dot{b}$ with $\dot{b} = 0.2\,\mathrm{m\,yr^{-1}} \approx 6.3\times10^{-9}\,\mathrm{m\,s^{-1}}$. In the numerical approach, for each depth $z(t)$ the trench profiles are diffused with respect to the respective diffusion length σ_{diff} by convoluting the original signal with a Gaussian with a standard deviation of $\sigma_{\mathrm{diff}}(t(z))$.~~ present accumulation rate at Kohnen station of 6.4 cm w.eq. yr^{-1}) to obtain an upper limit of the diffusion effect. Given an initial noise power P_0 for annual resolution, a constant diffusion length of $\sigma = 8\,\mathrm{cm}$ (Johnsen et al., 2000) and a Nyquist frequency of $f_0 = 0.05\,\mathrm{yr}^{-1}$ according to decadal resolution, evaluation of (B3) yields a reduction of the annual noise power of $\sim 0.095[\mathrm{yr}^{-1}]P_0$, similar to the case of undiffused white noise (reduction by a factor of 10). At our site, diffusion thus only has a minor effect on decadal and longer time scales.

~~The numerical lateral δ^{18}O trench variance after diffusion is in qualitatively good agreement with the observational data of trench T1 (Fig. **??**).~~

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Table 1. ~~The variance~~ Variance levels ~~observed for~~ of the two trenches: The ~~lateral~~ horizontal variance is the mean ~~horizontal~~ variance ~~over~~ of all depth layers on absolute coordinates, the down-core variance ~~gives~~ is the mean vertical variance ~~over~~ of all respective trench profiles. The seasonal as well as the inter-annual variance levels denote the variances of the respective mean seasonal and inter-annual δ^{18}O time series of the two trenches (Fig. 5). All numbers are in units of (‰)^2.

trench	~~lateral σ_l^2~~ horizontal σ_h^2	down-core σ_v^2	seasonal $\overline{\sigma}_v^2$	inter-annual $\overline{\sigma}_a^2$
T1	5.9	9.5	5.1	1.15
T2	5.3	7.3	3.3	0.21

Table 2. ~~The noise~~ Noise variance and standard deviation (SD) of the trench data ~~and~~ together with the ratio of ~~the~~ measurement uncertainty ($\Delta\delta^{18}\mathrm{O} = 0.09\,‰$) and ~~the~~ respective noise SD, given for different time scales and for the two ~~scenarios~~ limiting cases of the annual noise variance. The decadal noise level estimates are calculated from the annual noise variances accounting for full forward diffusion.

time scale	variance in $(‰)^2$	SD in ‰	$\Delta\delta^{18}\mathrm{O}/\mathrm{SD}$
seasonal	5.9	2.43	4 %
annual: case I	~~1.25~~ 0.84	~~1.12~~ 0.92	~~8~~10 %
annual: case II	5.9	2.43	4 %
10 yr-avg.: case I	~~0.13~~ 0.08	~~0.36~~ 0.28	~~25~~32 %
10 yr-avg.: case II	~~0.59~~ 0.56	~~0.77~~ 0.75	12 %

Table A1. Summary of the nomenclature used for the statistical noise model.

symbol	description
z	absolute depth below mean snow height
X_i	trench isotope profile at position ℓ_i
$\Delta\ell$	spacing of adjacent profiles
S	climate signal contained in X_i
w_i	noise contained in X_i
ε_i	white noise component of w_i
$\overline{X}_{\{i\}}$	profile stack
a	autocorrelation parameter; $a = \exp(-\Delta\ell/\lambda)$
λ	horizontal noise decorrelation length
d	inter-profile distance
N	number of profiles
$\sigma^{*2}_{\{i\}}$	relative effective noise variance of stack $\overline{X}_{\{i\}}$
F	signal-to-noise variance ratio

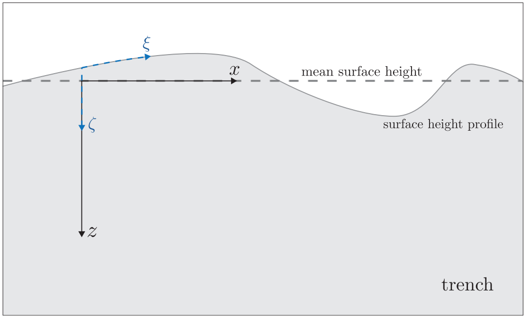

Figure 1. ~~**(a)**~~ ~~The two-dimensional δ^{18}O profile~~ Coordinate systems used for the analysis of the trench ~~T1. The depth scale is relative~~ isotope data: (1) a curvilinear coordinate system (ξ,ζ) (blue dashed lines, surface coordinates) with horizontal axis tangential to the ~~mean snow~~ surface height profile and vertical axis denoting the depth below the local surface; (~~long-dashed~~ 2) a Cartesian system (x,z) (black ~~line~~lines, absolute coordinates) ~~;~~ defined by the mean surface height.

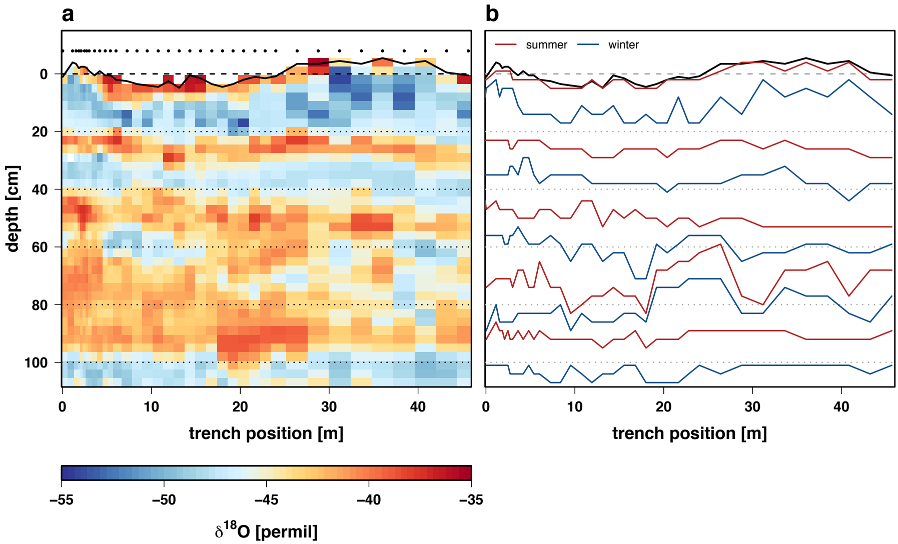

Figure 2. **a:** The two-dimensional δ^{18}O data set of trench T1 displayed on absolute coordinates. The solid black line shows the ~~local snow~~ surface height ~~at~~ profile, the ~~sampling~~ long-dashed black line the mean surface height. Sampling positions ~~which~~ are ~~indicated~~ marked by the black dots above~~the snow profile~~. White gaps indicate missing data. ~~(b)~~ **b:** The stratigraphy of trench T1 expressed as the seasonal layer profiles ~~by~~ tracking the local δ^{18}O extrema as explained in the text.

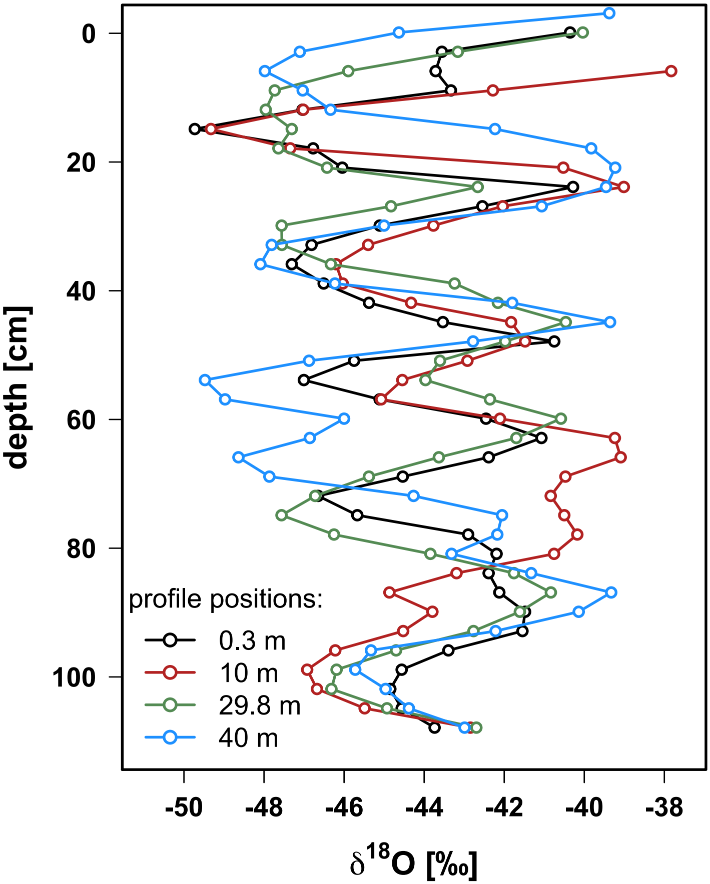
depth [cm]
0
20
40
60
80
100
-50
-48
-46
-44
-42
-40
-38
δ^{18}O [‰]
profile positions:
0.3 m
10 m
29.8 m
40 m

Figure 3. The four ~~isotope~~ δ^{18}O profiles obtained from trench T2 ~~as a function of depth below the mean snow height~~displayed on absolute coordinates.

~~The lateral variance of T1 as a function of depth below the mean snow height. Blue lines with circles give the lateral variance as calculated horizontally, red lines with circles display the variance computed for consecutive slices following the present snow surface. Dashed horizontal lines show the mean variance of each variance profile for the depth ranges of 0–20 and 20–$\sim$110 cm where the shadings represent the 90 % confidence intervals of the respective mean.~~

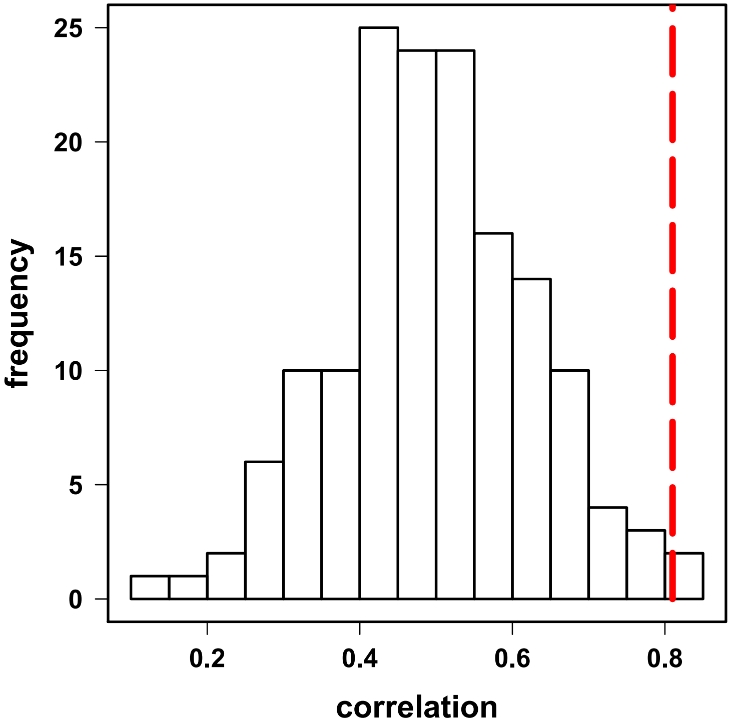

Figure 4. Histogram of all possible pairwise correlations ($N = 152$) between single profiles of trench T1 and single profiles of trench T2. Displayed are the maximum correlations allowing vertical shifts of the T2 profiles of up to ±12 cm. Shown ~~as a~~ in red ~~line~~ is the correlation between the mean ~~δ^{18}O profile~~ profiles of T1 and ~~the mean δ^{18}O profile of~~ T2 (Fig. 5).

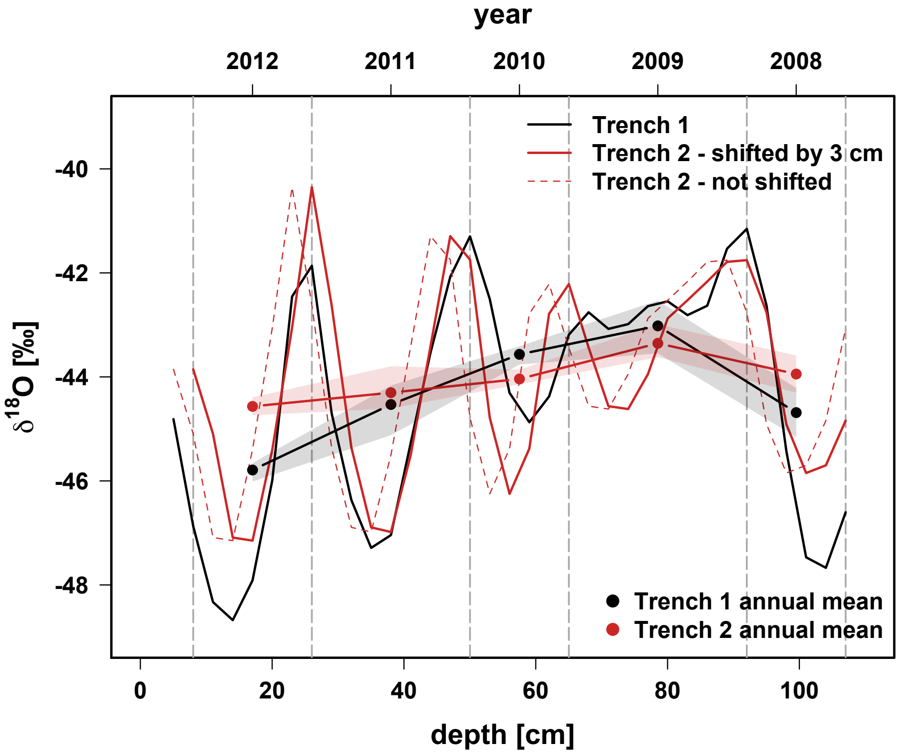
year
2012
2011
2010
2009
2008
Trench 1
Trench 2 - shifted by 3 cm
Trench 2 - not shifted
Trench 1 annual mean
Trench 2 annual mean
δ^{18}O [‰]
-40
-42
-44
-46
-48
0
20
40
60
80
100
depth [cm]

Figure 5. ~~The mean inter-profile correlation as a function of profile spacing for T1 (black line with filled circles). Shadings denote the standard error of the mean (undefined if just one profile pair is found for a given spacing), for each spacing calculated adopting an effective number of profile pairs that is set to the lower value of the actually found number of pairs and the effective degrees of freedom for the trench record in lateral direction. The dashed black line denotes the theoretical inter-profile correlation calculated for first-order autoregressive noise (AR(1)).~~

Comparison of the mean ~~seasonal~~ δ^{18}O profiles ~~as a function of depth below the~~ (lines: seasonal, points: annual mean~~snow height obtained~~) from ~~trench~~ T1 (black~~solid line~~) and T2 (red~~solid line~~). ~~Vertical shifting of ± 12 cm was allowed to~~ To maximise the seasonal correlation ($r_{\mathrm{T1,T2}} = 0.81$), ~~resulting in a shift of~~ trench T2 was shifted by $+3$ cm. For the first three depth bins, the number of existing observations varies on absolute coordinates between the ~~original T2 mean profile (black dashed line)~~trench profiles. ~~The~~ To obtain non-biased seasonal mean profiles ~~are well correlated with $r_{\mathrm{T1,T2}} = 0.81$. Additionally, red and black points with lines give~~ only the ~~approximate annual-mean δ^{18}O time series for the trenches~~depth range covered by all profiles is used. Shadings represent the range of the approximate annual-mean profiles due to different binning definitions. Note that ~~the~~ their first and last value ~~of the annual-mean time series (years 2012 and 2008)~~ are biased since the trench data are incomplete here. ~~The vertical~~ Vertical dashed grey lines ~~are~~ mark the ~~positions of the~~ six local maxima of the average ~~profile obtained from the trench~~ of both seasonal mean profiles.

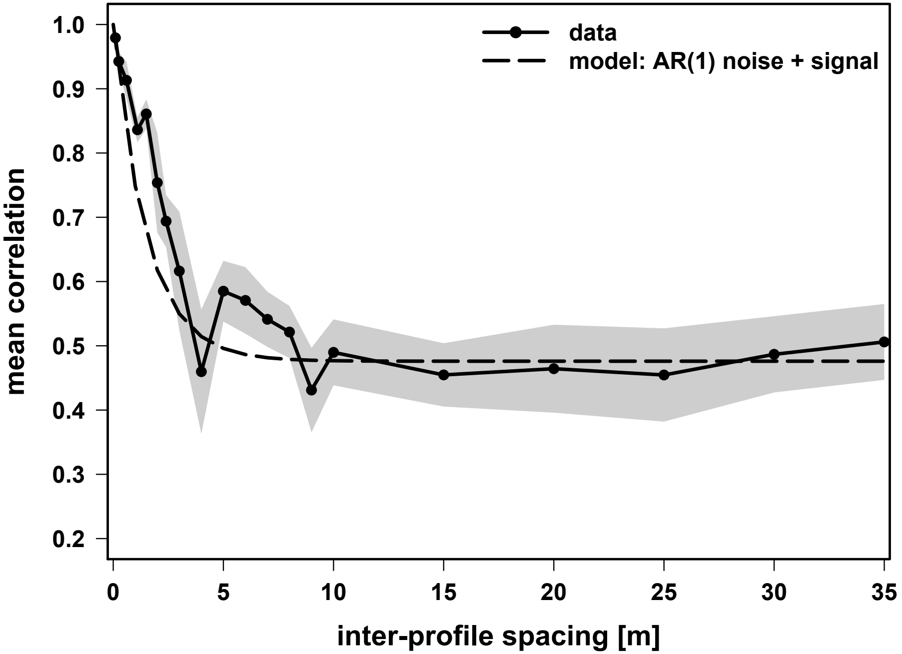

Figure 6. ~~The~~ Observed and modelled inter-profile correlation ~~between~~ as a ~~set~~ function of ~~averaged~~ profile spacing for T1~~profiles~~. Observations for a given spacing are the mean across all possible profile pairs. Shadings denote the standard error of the mean assuming maximum degrees of freedom (DOF) of $N = 12$ (estimated from the effective DOF of the horizontal trench data accounting for autocorrelation).

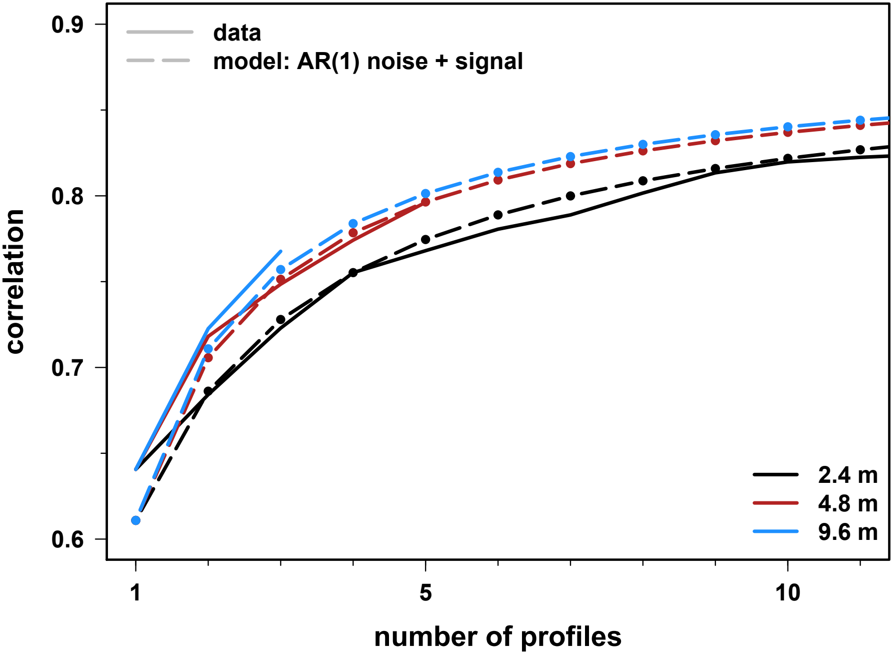

Figure 7. Observed and modelled correlations between T1 profile stacks and the mean of all T2 profiles depending on the number of profiles in the T1 ~~set and their~~ stack for three selected inter-profile ~~spacing. Three different~~ spacings~~are investigated: 2.4 m (black), 4.8 m (red) and 9.6 m (blue)~~. ~~Solid lines show the~~ Observed results for ~~the actual trench data, dashed lines display the theoretical correlations calculated for AR(1) autoregressive noise. The trench results are~~ given ~~as~~ spacing and number of profiles are the mean ~~of~~ across the correlations obtained for all possible unique ~~sets of profiles separated by the given spacing~~ stacks and ~~are~~ only calculated when at least 15 ~~sets~~ stacks are available.

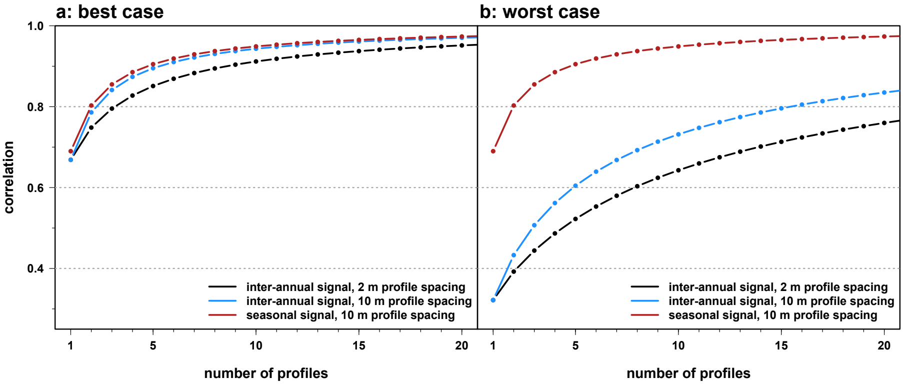

Figure 8. ~~The representativity~~ Representativity of ~~an average set of~~ a δ^{18}O firn ~~profiles~~ profile stack expressed as the correlation with a hypothetical ~~regional~~ climate signal depending on the number of profiles averaged ~~as well as~~ and their inter-profile spacing. ~~The dashed red line shows the representativity on the seasonal time scale for 10 m profile spacing.~~ For the inter-annual time scale, the two limiting cases discussed in the text are displayed (~~**a**:~~ **a:** best-case scenario~~/case I~~, ~~**b**:~~ **b:** worst-case scenario~~/case II~~), each for 2 m ~~profile spacings~~ (black) as well as 10 m ~~profile spacings~~ (blue) inter-profile spacing. As a reference, in each case the seasonal representativity is shown in red for 10 m inter-profile spacing.

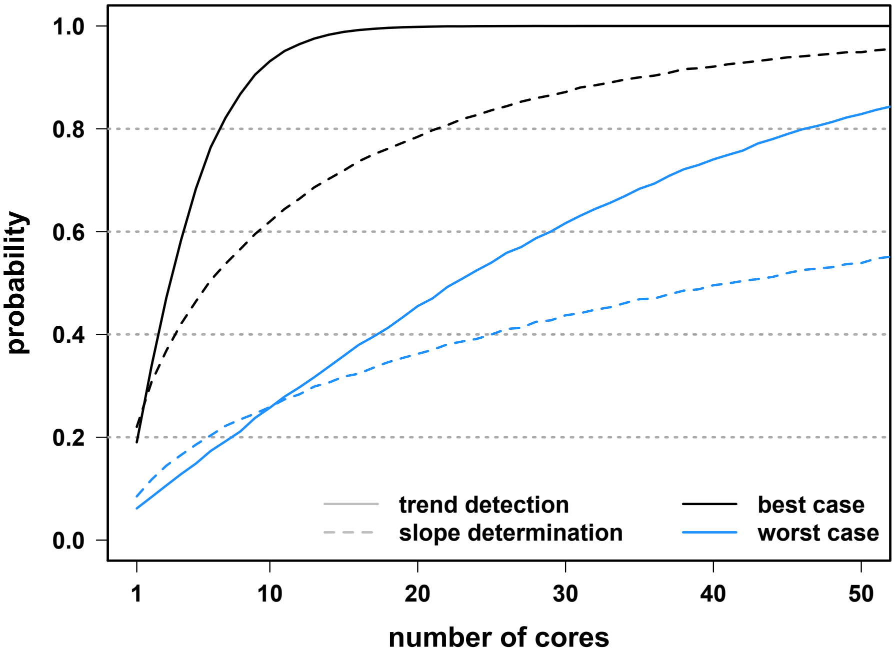

Figure 9. ~~The probability~~ Probability of detecting a linear temperature trend of $0.5\,^\circ\mathrm{C}/50\,\mathrm{yr}$ (~~correlation > 0, $p \geq 0.05$~~$p = 0.05$) (solid lines) and of determining the strength of the trend with an accuracy of 25 % (dashed lines) ~~, each~~ as a function of the number of ~~annually resolved~~ firn cores averaged and for the two scenarios of the annual noise variance discussed in the text (black~~lines~~: best case~~/case I~~, blue~~lines~~: worst case~~/case II~~).

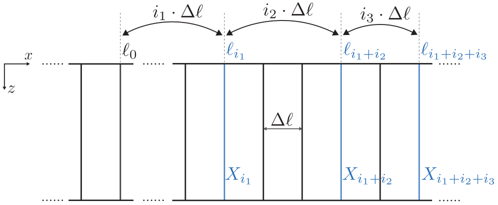

Figure A1. ~~The lateral variance~~ Sketch of ~~T1 as~~ a ~~function of depth below the mean~~ snow ~~height. Blue lines with circles give the lateral variance as calculated horizontally, red lines with circles display the variance computed~~ trench used for ~~consecutive slices following~~ the ~~present snow surface. Greyish-blue dashed lines depict the numerical estimate~~ derivation of the ~~vertical variance~~ statistical noise model. Vertical isotope profiles X_i are spaced at constant intervals of ~~a diffused artificial trench record (see text for details)~~ $\Delta\ell$ at locations $l_i = l_0 + i\Delta\ell$. The horizontal distance d between two profiles X_{i_1} and $X_{i_1+i_2}$ is defined by the incremental index i_2, $d = i_2\Delta\ell$.

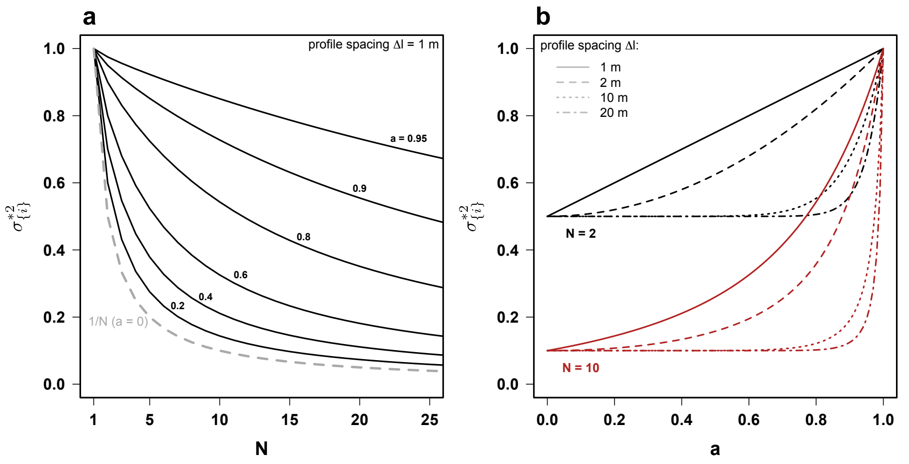

Figure A2. **a:** Relative effective noise variance $\sigma^{*2}_{\{i\}}$ of a profile stack $\overline{X}_{\{i\}}$ as a function of the number of profiles averaged for a profile spacing of $\Delta\ell = 1\,\mathrm{m}$ and for different values of the autocorrelation parameter a. The limiting case of white noise ($a = 0$) is indicated by a dashed line. **b:** $\sigma^{*2}_{\{i\}}$ as a function of the autocorrelation parameter a for different numbers of averaged profiles and profile spacings.