

**Inferring  
accumulation records  
from ice-cores**

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# Inferring paleo-accumulation records from ice-core data by an adjoint method: application to James Ross Island's ice core

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Received: 11 August 2014 – Accepted: 14 August 2014 – Published: 24 September 2014

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Published by Copernicus Publications on behalf of the European Geosciences Union.

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## Abstract

Ice cores contain a record of snow precipitation that includes information about past atmospheric circulation and mass imbalance in the polar regions. We present a novel adjoint method to reconstruct a climatic record by both optimally dating an ice-core and deriving from it a detailed accumulation history. The motivation of our work is the recent application of phase sensitive radar which measures the vertical velocity of an ice column. The velocity is dependent on the history of subsequent snow accumulation, compaction and compression; and in our inverse formulation of this problem, measured vertical velocity profiles can be utilized directly thereby reducing the uncertainty introduced by ice flow modelling. We first apply our method to synthetic data in order to study its capability and the effect of noise and gaps in the data on retrieved accumulation history. The method is then applied to the ice core retrieved from James Ross Island, Antarctica. We show that the method is robust and that the results depend on quality of the age-depth observations and the derived flow regime around the core site. The method facilitates the incorporation of increasing detail provided by ice-core analysis together with observed full-depth velocity in order to construct a complete climatic record of the polar regions.

## 1 Introduction

A layer of snow deposited at some position and time on the surface of an ice sheet has today a burial rate and depth that is dependent on the weight and amount of snow accumulated above it, its metamorphosis to ice and the flow of the ice sheet. When the ice core is extracted, the chemical constituents trapped in the ice contain information of the climatic conditions during the time of deposition.

Dating the ice-core is paramount in order to interpret the climatic record. Different methods are used to establish the ice-core timeline, mainly: annual layer counting, localization of dated volcanic eruptions in the record, comparison of chemical species

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with other ice-cores, and ice flow modelling. Commonly, the ice-core chronology is obtained by a combination of methods (e.g., Hammer et al., 1978; Parrenin et al., 2007a; Lemieux-Dudon et al., 2010). Annual layer counting and the localization of known volcanic events offer accurate dating, but their use is restricted to the top of the ice-core or to sparse sections of the record.

Ice-core dating relies on ice-flow models in sections where direct chronological information is unavailable or incomplete. The assumption made is that the record of snow accumulation is related to past cloud temperatures that, in turn, are derived from ice core isotope analysis (e.g., Watanabe et al., 2003; Parrenin et al., 2007b). However, this assumption can not be used on coastal areas of Antarctica or Greenland (Bromwich and Robasky, 1993; Kapsner et al., 1995) where the main driver of change in accumulation is atmospheric circulation and not the thermodynamic equilibrium of the atmosphere. However, recent advances in ice core data analysis provide detailed chronologies for ice cores located in coastal areas using layer counting (e.g., Rasmussen et al., 2006; van Ommen and Morgan, 2010) or transferring timelines from reference ice-cores using stratigraphic links (e.g., Ruth et al., 2007; Lemieux-Dudon et al., 2010). The aim of this paper is to use a detailed chronology such as those, derived independently of temperature reconstructions, in order to obtain a record of paleo-accumulation.

Previous work has estimated paleo-accumulation and unknown ice-flow parameters in order to extend the age-depth record beyond the confines of layer counting or stratigraphic links. Typically, an inverse method based on direct search is applied (e.g., Dansgaard and Johnsen, 1969; Morgan et al., 1991) or, more recently, on Monte-Carlo techniques (e.g., Parrenin et al., 2001; EPICA community members, 2004; Parrenin et al., 2007b). Here, our approach is to apply a Lagrange method for constrained optimization, commonly employed in computational fluid dynamics (CFD) for inverse problems that are governed by partial differential equations (e.g., Petra and Stadler, 2011).

The motivation of our work is the increasing interest in climatic records extracted from high-resolution ice cores located in coastal areas of Antarctica and Greenland (e.g.,

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van Ommen and Morgan, 2010; Mulvaney et al., 2012). We have taken advantage of the recent development of a phase sensitivity radar, that allow the direct measurement of the present vertical velocity full-depth along the ice-core (Corr et al., 2002; Jenkins et al., 2006), this greatly reduces the uncertainty introduced by ice-flow modelling (Kingslake et al., 2014). In addition, recent advances in multi-ice-core multi-component analysis has improved the detail in ice core chronology (e.g., Rasmussen et al., 2006; Ruth et al., 2007; Lemieux-Dudon et al., 2010).

We first describe the method in Sect. 2 and its application to synthetic and existing data from James Ross Island ice core data (Mulvaney et al., 2012, 2014) in Sect. 3. At the time of writing this manuscript we do not have a suitable location where both velocity derived from phase sensitive radar and a detailed ice core timeline is available. For simplicity, we have assumed that the flow of ice follows a (Dansgaard and Johnsen, 1969) analytical approximation for the synthetic data (Sect. 3.1) and a shallow ice approximation (e.g., Hutter, 1983) combined with the measured compaction for the James Ross Island ice core case (Sect. 3.2).

## 2 Methodology

### 2.1 Description of the problem: forward model

The combined result of snow accumulation, compaction and compression is that a particle of ice deposited in the surface of a summit of ice-thickness  $H$  is transported with a velocity  $(u, v, w)$  within the ice. The age of a particle of ice  $A$ , represents the time elapsed for a particle of ice to get transported from the surface to a given position  $(x, y, z)$ , where  $z$  represents the vertical. In this paper, we consider that the distribution of the age varies in time, from a given initial state  $A_0$  to the present ( $t = T$ ), but we neglect the effect of horizontal advection and the local variation of thickness through

time ( $\frac{\partial H}{\partial t} = 0$ ). The age of ice can then be expressed as,

$$\begin{aligned}\partial_t A + w \partial_z A &= 1, & 0 \leq z \leq H, & 0 \leq t \leq T \\ A(t = 0, z) &= A_0, & 0 \leq z \leq H, \\ A(t, z = H) &= 0, & 0 \leq t \leq T.\end{aligned}\tag{1}$$

We assume that the spatial variation of the velocity only depends on rheology and not on time dependent variables such as surface accumulation or ice thickness (e.g., Parrenin et al., 2007a). This assumption is supported by the perturbation analysis of Wilchinsky and Chugunov (1997) for isothermal ice. Considering the conservation of mass, the vertical velocity is

$$w(z, t) = -m(t) + [-a(t) + m(t)] \eta(z),\tag{2}$$

where  $a$  is the surface accumulation rate,  $m$  the basal melt rate, and  $\eta$  is the shape function (sometimes referred to as thinning function). For the sake of simplicity, we refer to the surface accumulation rate and basal melt rate as accumulation and melting, as there is no possible ambiguity in this paper.

## 2.2 Inverse model

The inverse problem can be formulated as finding the records of accumulation  $a(t)$  and melting  $m(t)$ , and the initial distribution of age-depth  $A_0(z)$ , that minimize the difference

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between observations and model results:

$$\begin{aligned}
 \min_{a,m,A_0} J(a,m,A_0;A) := & \frac{1}{2} \int_0^H \frac{[A(t=T) - A_d]^2}{\sigma_d} P_d dz \\
 & + \frac{\gamma_a^R}{2} \int_0^T (\partial_t a)^2 dt + \frac{\gamma_m^R}{2} \int_0^T (\partial_t m)^2 dt \\
 & + \frac{\gamma_{A_0}^R}{2} \int_0^H (\partial_z A_0)^2 dz, \tag{3}
 \end{aligned}$$

where  $\gamma_a^R$ ,  $\gamma_m^R$  and  $\gamma_{A_0}^R$  denote the weight of the Tikhonov regularization for accumulation, melting and initial age-depth distribution.  $A_d$  is the measured age of ice,  $\sigma_d$  its estimated uncertainty and  $P_d$  is the point measurement operator for discrete data (Petra and Stadler, 2011),

$$P_d(z) = \sum_{j=1}^{N_d} \delta(z - z_j) \quad \text{for } j = 1, \dots, N_d, \tag{4}$$

where  $N_d$  is the number of point measurements and  $\delta(z - z_j)$  is the Dirac delta function.

We follow a formal Lagrange approach (e.g., Tröltzsch, 2010), that includes a Lagrangian multiplier function  $\lambda$  in order to find the solution of the minimization problem

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(Eq. 3) constrained by the forward model (Eq. 1).

$$\mathcal{L}(a, m, A_0; A, \lambda) = J(a, m, A_0; A)$$

$$+ \int_0^T \int_0^H \lambda (\partial_t A + w \partial_z A - 1) dz dt$$

$$+ \int_0^H \lambda(t=0) [A(t=0) - A_0] dz \quad (5)$$

The optimal solution is obtained by setting the variations (functional derivatives) of the Lagrangian  $\mathcal{L}$  with respect all variables to zero.

Setting the variations of the Lagrangian with respect to the Lagrangian multiplier to zero ( $\delta_\lambda \mathcal{L} = 0$ ) is equivalent to the forward problem, also known as state equation (Eq. 1).

Setting the variations of the  $\mathcal{L}$  with respect to the age ( $\delta_A \mathcal{L} = 0$ ) leads to the adjoint problem for  $\lambda$ , that in the strong form can be written as,

$$\partial_t \lambda + \partial_z (w \lambda) = 0, \quad 0 \leq z \leq H, \quad 0 \leq t \leq T$$

$$\lambda = \frac{A_d - A}{\sigma_d} P_d, \quad t = T, \quad (6)$$

$$\lambda = 0, \quad z = 0.$$

Finally, the so-called control equations, are obtained by setting to zero the variations of  $\mathcal{L}$  with respect to accumulation, melting and initial distributions of age ( $\delta_a \mathcal{L} = 0$ ,

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$\delta_m \mathcal{L} = 0$  and  $\delta_{A_0} \mathcal{L} = 0$ ):

$$-\int_0^H \eta (\lambda \partial_z A) dz - \gamma_a^R \partial_t^2 a = 0,$$

$$\int_0^H (\eta - 1) (\lambda \partial_z A) dz - \gamma_m^R \partial_t^2 m = 0,$$

$$-\lambda(t=0) - \gamma_A^R \partial_z^2 A_0 = 0.$$

(7)

### 2.3 Numerical Solution

The system of equations composed by the state equation (Eq. 1), the adjoint equation (Eq. 6) and the control equations (Eq. 7) represent a complex non-linear system of equations. In order to solve the system, we use a steepest decent method (e.g., Petra and Stadler, 2011).

First we assume initial solutions for accumulation, melting and initial distribution of age ( $a^k$ ,  $m^k$  and  $A_0^k$ ). We then solve the state equation (Eq. 1), and using the inferred solution of age  $A^k$ , we solve the adjoint equation backward in time to obtain the solution of the Lagrange multiplier  $\lambda^k$ .

In order to update the solution for accumulation, melting and initial distributions of age and temperature, we increase them in the direction of steepest descent of the Lagrangian.

$$a^{k+1} = a^k + \beta_a g_a,$$

$$m^{k+1} = m^k + \beta_m g_m,$$

$$A_0^{k+1} = A_0^k + \beta_A g_A,$$

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where  $\beta_a$ ,  $\beta_m$  are  $\beta_A$  are parameters, and the gradients of the Lagrangian are the left-hand side of Eq. (7).

$$g_a = - \int_0^H \eta (\lambda \partial_z A) dz + \gamma_a^R \partial_z^2 a,$$

$$g_m = \int_0^H (\eta - 1) (\lambda \partial_z A) dz + \gamma_m^R \partial_z^2 m,$$

$$5 \quad g_A = -\lambda(t=0) - \gamma_A^R \partial_z^2 A_0. \quad (9)$$

The values of  $\beta_a$ ,  $\beta_m$  and  $\beta_A$  are calculated each iteration using a backtracking line search algorithm (e.g., Dennis and Schnabel, 1996, §6.3.2). The method assumes sufficiently large initial values for  $\beta_a$ ,  $\beta_m$  and  $\beta_A$ , that are successively halved until  
 10 the corresponding decrease in the objective function  $J$  (Eq. 3) fulfill the Armijo (1966) condition,

$$J(x^k) - J(x^{k+1}) \geq -c \beta_x g_x^2, \quad (10)$$

where  $x$  represents the state variables  $a$ ,  $m$  or  $A_0$ ;  $g_x$  the respective gradients of the objective function  $g_a$ ,  $g_m$  or  $g_A$ ;  $\beta_x$  the parameters  $\beta_a$ ,  $\beta_m$  or  $\beta_A$ ; and the constant  $c$  is fixed to  $c = 10^{-2}$ .  
 15

The values of  $\gamma_a^R$ ,  $\gamma_m^R$  and  $\gamma_{A_0}^R$ , are chosen by inspection of the  $L$  curve (e.g., Hansen, 2000). The  $L$  curve is a plot of the norm of the regularization term ( $\int_0^T (\partial_t a)^2 dt$ ,  $\int_0^T (\partial_t m)^2 dt$  or  $\int_0^H (\partial_z A_0)^2 dz$ ) vs. the difference in age-depth between ob-  
 20 servation and model results ( $\int_0^H P_d [A(t=T) - A_d]^2 / \sigma_d dz$ ) for different values of the regularization parameter. The  $L$  curve has two distinctive sections: for sufficiently small values of the regularization parameter, the differences between data and model dominate the objective function, and a gradual increase in the smoothing of the solution

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implies a gradual increase in the differences between data and model; conversely, for sufficiently large values of the regularization parameter, the regularization term dominates the objective function, and a gradual increase in the smoothing of the solution translates into sharper differences between data and model. The curve typically is  $L$  shaped, and hence its name. The regularization parameters are chosen so that their corresponding terms in the objective function sit on the  $L$  curve between these two sections, that is, the corner of the  $L$ .

Regarding the numerical techniques, we solve the state equation (Eq. 1) and the adjoint equation (Eq. 6), by using a operator combined Semi-Lagrangian Crank-Nicholson algorithm (SLCN) (Spiegelman and Katz, 2006) in a equispaced grid of elevation  $\{z_i\}$ .

The second order two-level Semi-Lagrangian scheme (e.g., Staniforth and Côté, 1991) solves the advection terms as an ordinary differential equation along the trajectory that connects from some take-off elevation  $z_*$  at time  $t = (n)\Delta t$  to a regular grid point  $z_i$  at time  $t = (n + 1)\Delta t$  in the state equation, that is solved forward in time, and at time  $t = (n - 1)\Delta t$  in the adjoint equation, that is solved backward in time ( $\Delta t \rightarrow -\Delta t$ ,  $w \rightarrow -w$ ). The Crank-Nicolson algorithm is used to discretize the diffusive terms with a central difference.

The discretized state equation (Eq. 1) and the adjoint equation (Eq. 6), using the SLCN method are respectively,

$$A_i^{n+1} = 1 + A_*, \quad (11)$$

$$\left(1 - \frac{\Delta t}{2} \delta w_i\right) \lambda_i^{n-1} = \left(1 + \frac{\Delta t}{2} \delta w_i\right) \lambda_*; \quad (12)$$

where  $\delta$  denotes the gradient discrete operator

$$(\nabla f)|_{z_i} \approx \delta f_i = \frac{f_{i+1} - f_{i-1}}{2\Delta z},$$

and  $(*)_*$  denotes the value at the take-off position  $z_*$  and time  $t = (n)\Delta t$ .

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### 3 Results

#### 3.1 Recovering accumulation from synthetic age-depth observations

In order to study the capability of our method to recover paleo-accumulation from age-depth observations, we have generated synthetic observations by adding random noise to a reference solution.

The reference accumulation solution varies over 10 kyr as,

$$a(t) = [0.2 + 0.1 \sin(2\pi t/\tau_1) + 0.05 \sin(2\pi t/\tau_2)] \text{ m yr}^{-1},$$

where  $\tau_1 = 10$  kyr, and  $\tau_2 = 1$  kyr. Figure 1 shows the reference accumulation and age-depth.

In this section, we assume that there is no melting, and that the shape function (Eq. 2) follows Dansgaard and Johnsen (1969),

$$\eta = \begin{cases} \frac{z-z_k/2}{H-z_k/2} & z_k \leq z \leq H \\ \frac{z^2/2}{z_k(H-z_k/2)} & 0 \leq z \leq z_k \end{cases}, \quad (13)$$

where  $z_k$  is a parameter that we fix to  $z_k = 0.3H$ . In addition, we assume that the initial age distribution ( $A(t=0)$ ) is exact and that it is a steady-state solution of the velocity distribution at time  $t=0$ .

We first check our model by applying it to the age-depth derived from the reference accumulation. The algorithm (Sect. 2.3) converges in 10–20 iterations, depending on the initial guess and numerical details. The results are not reported here as the recovered accumulation and age-depth converges to the exact reference accumulation.

Then, we add a 1% random noise to the reference age-depth solution and apply our method to recover accumulation and age-depth. The results are in Fig. 1. The recovered age-depth is nearly identical to the reference age-depth (under 0.1% difference),

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even in the deepest area of the record. On the other hand, the random noise introduce differences between recovered and reference accumulation: the difference increase with time from the present up to a value of about 10 %.

Next, we investigate the effect of increasing noise. We compare the recovered accumulation from synthetic data obtained adding 1%, 5% and 10% random noise to the reference age-depth. The results are shown in Fig. 2. The solution deteriorates when random noise increases, but the recovered accumulation loses information for small wavelengths.

In the final numerical experiment (Fig. 3), we study the effect of not having data in an area. In the simulation we assumed that there is no data in  $400\text{m} < z < 600\text{m}$  and that there is a 1% random noise in the synthetic age-depth. We find that the effect of not having any data over a given depth range of the bore hole on the inversion, is a period of time (area highlighted in Fig. 3) over which the retrieved accumulation is mainly constrained by the initially assumed profile and the regularization applied.

### 3.2 Recovering paleo-accumulation from James Ross Island ice core

In this section, we apply the method to the ice core extracted from James Ross Island, a large island off the south-east side and near the northeastern extremity of the Antarctic Peninsula. The data used in this section and its climatological implications are discussed in Mulvaney et al. (2012) and Mulvaney et al. (2014).

Figure 4 shows the timeline for the James Ross Island ice core that is provided by fixed markers derived from local and global volcanic events with unequal estimated uncertainty (details in Mulvaney et al. (2012), Supplement Table 1).

We assume that the shape function (Eq. 2) can be expressed as,

$$\eta = \frac{\rho_1}{\rho} \eta_1,$$

where  $\rho$  is the measured density at a given depth,  $\rho_1 = 910\text{kg m}^{-3}$  is the reference density of ice, and  $\eta_1$  is the shape function for ice. We further assume that  $\eta_1$  follows

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a shallow ice approximation (e.g., Hutter, 1983),

$$\eta(s) = 1 - s \left( \frac{n+2}{n+1} - \frac{s^{n+1}}{n+1} \right),$$

where  $s$  is the normalized depth ( $s = (H - z)/H$ ). In order to measure the sensitivity to the uncertainty in ice-flow in the recovered age-depth distribution and accumulation record, we assume that the rheological index  $n$  varies in the range  $2 \leq n \leq 5$ .

The recovered age-depth (Fig. 4) follows the intervals of the observed measurements, and the uncertainty introduced by the ice-flow parameter  $n$  is smaller than a 1%, indiscernible in the figure. On the other side, the recovered paleo-accumulation (Fig. 5) follows roughly the stable water isotope ( $\delta D$ ), that is a proxy for temperature, but the result is not clear due to the small number of age markers (12 markers in 3 kyr) and the uncertainty introduced by the ice-flow parameter  $n$ . The uncertainty increases with time from the present and it becomes about 20% of the estimated value after 3 kyr.

Finally, to estimate the sensitivity of the method to age-depth uncertainty, we use a basic Monte Carlo method (e.g., Campolongo et al., 2000). The error propagation is obtained by calculating the standard deviation, at a given depth, from a set of one hundred results for both accumulation and age-depth. For each one of the one hundred simulations, we resample the measured age-depth observations assuming a normal distribution with standard deviation equal to one third of the uncertainty estimation (so that 99% of the samples lie within the range of observations).

The results are shown in Fig. 6. The estimation of uncertainty in observed age-depth is variable from 1% at certain depths to about 10% in the deepest sections of the ice core. Similarly to the uncertainty propagation introduced by the ice-flow (included in the figure), uncertainty in accumulation increases with time from the present but reaching a maximum of about 15% after 3000 yr.

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## 4 Discussion

The quality of the information recovered from a model inversion is always related to the quality of the observed measurements and to the nature of the assumptions made in the forward model. In this paper, we have analyzed the effect of random noise (Fig. 2), gaps (Fig. 3) and uncertainty (Fig. 6) in the age-depth distribution (observed measurements) on paleo-accumulation (recovered information). We also evaluate the propagation of uncertainty introduced by assumptions in the forward model, that follows the compaction and compression of ice with time (Fig. 5). That uncertainty in ice-flow is simplified in this study as the uncertainty in the rheological index  $n$ .

The experiments with synthetic data (Sect. 3.1) show that our method is robust: in perfectly sampled data (Fig. 2), it recovers the reference accumulation in the presence of random noise, and, even when the increase in noise deteriorates the accumulation, it recovers time-averaged values as the details are first lost in the high temporal frequencies. Besides, when there are gaps in the data (Fig. 3), the retrieved profile is constrained by the regularization. We find that the smoothing in the accumulation transforms, after solving the forward model, into a smooth age-depth (Fig. 3). In contrast, a typical polynomial interpolation of the age-depth leads to spurious accumulation records due to the non-linearity introduced by the Age equation (Eq. 1) (Fudge et al., 2014).

The sensitivity analysis to the James Ross ice core data (Fig. 6), shows that between 1 % and 10 % of uncertainty in the observations results in similar uncertainty in paleo-accumulation. On the other hand, the propagation of uncertainty from the unknown parameter in ice flow model has a stronger effect on the accumulation (about 20 %) than in the age-depth (under 1 %).

It is not the objective of this study to evaluate the use of modelling in ice core dating, but this later result shows that, in our case study, there is a relatively small the sensitivity (under a 1 %) of age-depth to uncertainty in ice-flow modelling. Age-depth depends on accumulation; and in the ice cores where accumulation can be extracted

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from temperature reconstructions and age-depth is constrained by fixed-markers, our test case supports the use of modelling in ice core dating (e.g., Parrenin et al., 2007a; Ruth et al., 2007; Lemieux-Dudon et al., 2010).

We only consider simplified ice flow shape functions (derived from Dansgaard and Johnsen (1969) in Sect. 3.1 and Shallow Ice Approximation in Sect. 3.2) as they are easier to analyze and because our aim is to incorporate the measured full-depth vertical velocity as they become available in the future. Our main assumption is that the spatial variability of the velocity depends on compaction and rheology, and the time dependence is proportional to the combined result of accumulation and surface thinning. This assumption is partially supported by the perturbation analysis of Wilchinsky and Chugunov (1997), that assumes isothermal ice; but we do not consider the effect of time evolution on firnification (e.g., Arthern and Wingham, 1998), temperature along the core (e.g., Nereson and Waddington, 2002), ice fabric (e.g., Martín and Gudmundsson, 2012), dome migration (e.g., Martín et al., 2009), horizontal advection or thinning (e.g., Huybrechts et al., 2007). We expect that the analysis of full-depth velocity datasets (Kingslake et al., 2014) provides information on this processes on future and existing ice cores.

## 5 Conclusions

The present study describes a novel adjoint method to derive pale-accumulation records from ice core chronology. The method is designed for ice cores in coastal areas of Antarctica and Greenland because it does not rely on temperature reconstructions.

Our approach is to follow a Lagrange method for constrained optimization, commonly employed in CFD inverse problems governed by partial differential equations. We apply our method to synthetic and existing data from the James Ross Island ice core. We show that the method is robust, as it retrieves paleo-accumulation from data with random noise and it reacts well to gaps in the data. We also show that the quality of the

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pale-accumulation depends on the quality of the chronology, uncertainty and sampling, and the uncertainty in ice-flow.

Besides, in our case study on James Ross Island ice core, where full-depth velocity measurements are not available, the largest uncertainty in the inferred paleo-accumulation is a consequence of the unknown ice-flow parameters. It increases with time until a maximum of about 20 %. In addition, an uncertainty in the chronology between 1 % and 10 % propagates into a similar uncertainty in paleo-accumulation.

Our ambition is to incorporate to the method full-depth vertical velocity obtained with phase sensitive radar, in order to reduce the uncertainty introduced by ice-flow modelling, and a detailed chronology, provided by ice core analysis, in order to obtain an optimized chronology and a detailed paleo-accumulation record.

*Acknowledgements.* This study was funded by core funding from the Natural Environment Research Council to the British Antarctic Survey's IceSheets Program.

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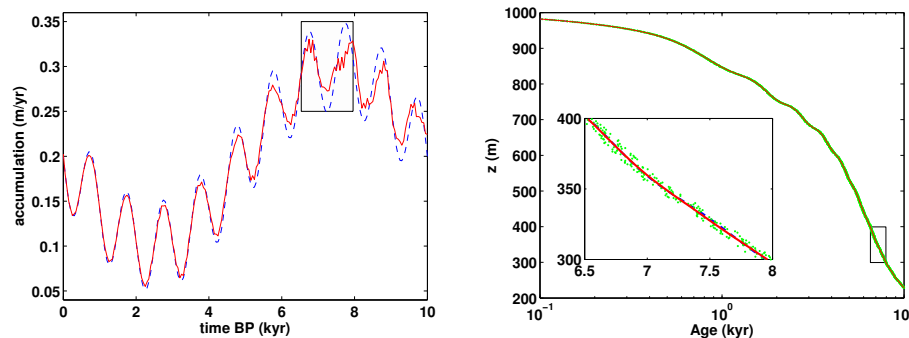
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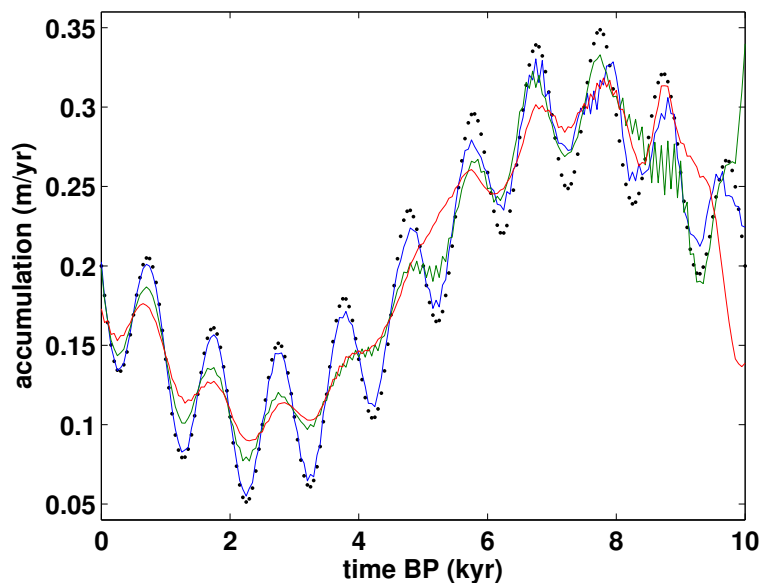


**Figure 1.** Recovered accumulation (red, left panel) and age-depth distribution (red, right panel) from age-depth synthetic data (green, right panel). The synthetic age-depth distribution has been obtained by adding 1 % uniform noise to the reference solution (blue right-panel) derived from the reference accumulation record (blue, left panel). Inset in the right panel zooms into the age-depth profile in an area that is highlighted both in the accumulation record and in the age-depth distribution.

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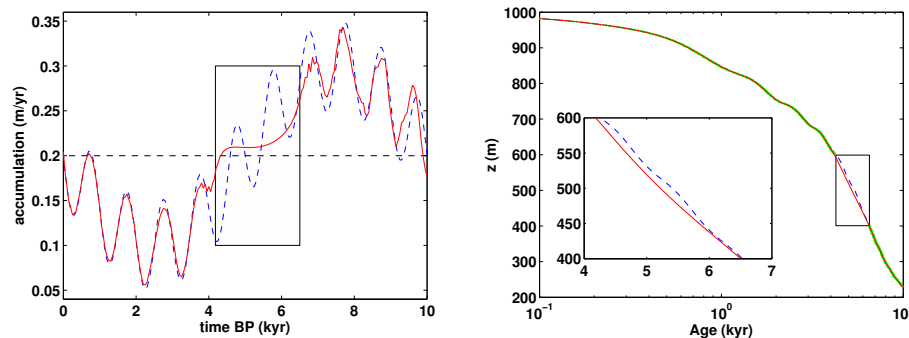


**Figure 2.** Recovered Accumulation from the synthetic age-depth distribution obtained adding 1 % (blue), 5 % (green) and 10 % (red) uniform noise to the solution derived from the reference accumulation (dotted line).

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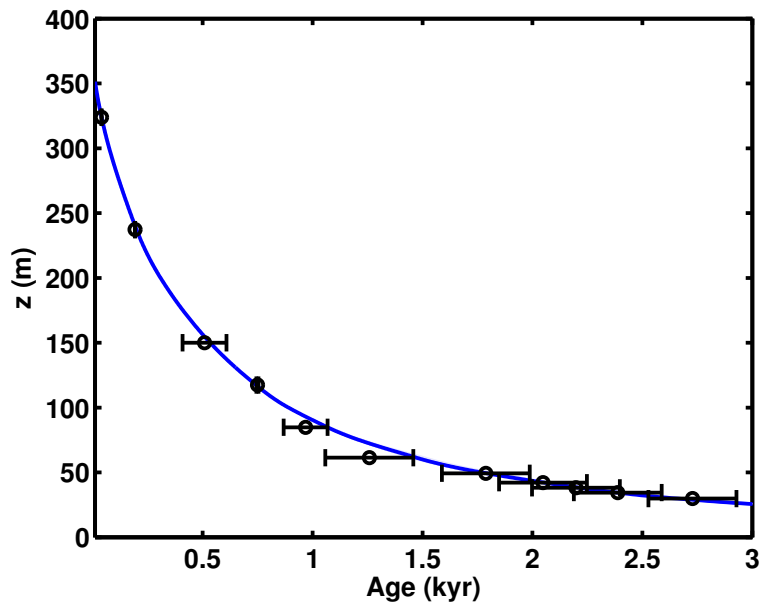


**Figure 3.** Recovered accumulation (red, left panel) and age-depth distribution (red, right panel) from age-depth synthetic data (green, right panel). The synthetic age-depth distribution has been obtained by adding 1 % uniform noise to the reference solution (blue right-panel) derived from the reference accumulation record (blue, left panel), and considering a gap in the data ( $400\text{ m} < z < 600\text{ m}$ ). Inset in the right panel zooms into the age-depth results where there is a gap in the age-depth data.

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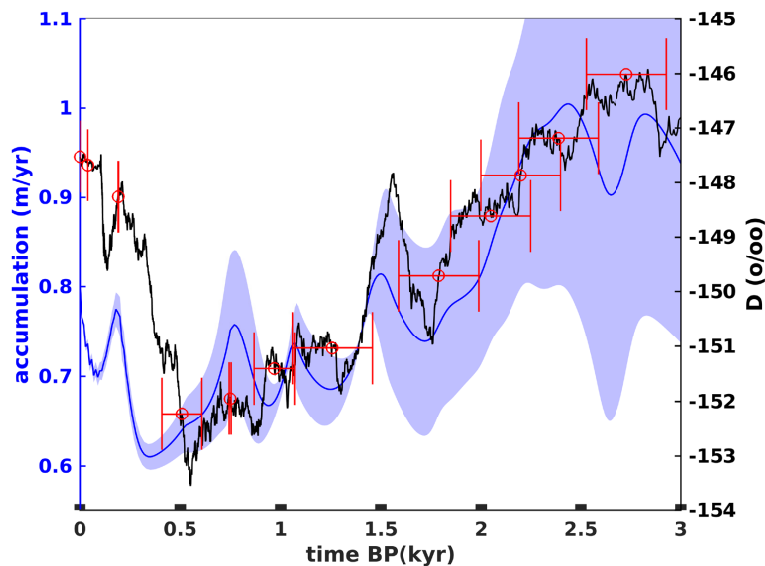


**Figure 4.** Recovered age-depth (blue) and observed age-depth (black) from (Mulvaney et al., 2012). The uncertainty in recovered accumulation due to unknown ice-flow parameter  $n$  ( $2 \leq n \leq 5$ ) is indiscernible in the plot.

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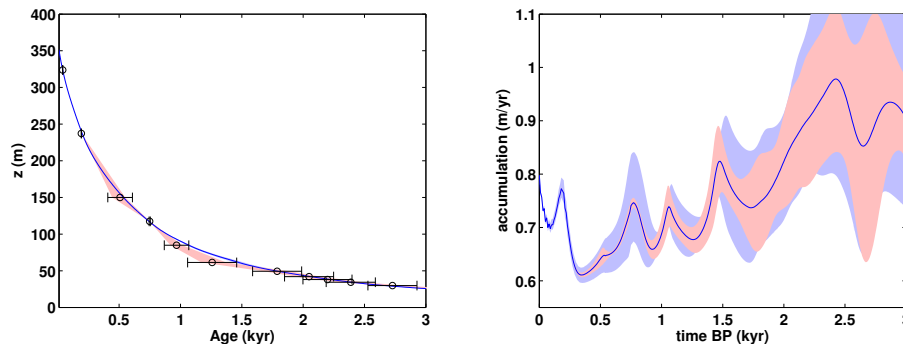
**Figure 5.** Recovered accumulation (blue) from James Ross ice core observed age-depth and stable water isotope  $\delta D$  (black). Red symbols represent the uncertainty in age-depth at the points where absolute markers were found (Mulvaney et al., 2012). The area in blue represents uncertainty in recovered accumulation due to unknown ice-flow parameter  $n$  ( $2 \leq n \leq 5$ ). The stable water isotope  $\delta D$  has not been used for the inversion and is plotted only for reference.

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**Figure 6.** Sensitivity analysis to uncertainty in chronology (red) and shape function  $\eta$  (blue). The uncertainty in chronology is assumed to follow a normal distribution. The uncertainty in the shape function  $\eta$  is represented by the range  $2 \leq n \leq 5$  for the rheological index in the Glen flow law.

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