

Validity of the linearity assumption in the decomposition of isotopic controls

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1 Reminder on the decomposition method

In the paper, we decompose the isotopic ratio of water vapor in the control simulation, $R_{v,control}$, into four terms:

$$R_{v,control} = \Delta R_{kin} + \Delta R_{RH_s} + \Delta R_{SST} + (R_{v,RH_sSSTcste} - \Delta R_{kin}) \quad (1)$$

- The first term, ΔR_{kin} , represents the effect of kinetic fractionation during ice condensation. Its is calculated as $\Delta R_{kin} = R_{v,control} - R_{v,nokin}$, i.e. the difference between a simulation with $\lambda = 0$ (λ being the supersaturation parameter) and the control simulation with $\lambda = 0.004$.
- The third term, ΔR_{RH_s} , represents the effect of surface relative humidity RH_s during ocean surface evaporation. Its is calculated as $\Delta R_{RH_s} = R_{v,control} - R_{v,RH_scste}$, i.e. the difference between a simulation with $RH_s = RH_{s,cste} = 60\%$ during ocean surface evaporation and the control simulation.
- The second term, ΔR_{SST} , represents the effect of sea surface temperature SST during ocean surface evaporation. Its is calculated as $\Delta R_{SST} = R_{v,RH_scste} - R_{v,RH_sSSTcste}$, i.e. the difference between a simulation with $SST=SST_{cste}=15^\circ\text{C}$ and $RH_s = RH_{s,cste} = 60\%$ during ocean surface evaporation and the simulation with $RH_s = RH_{s,cste} = 60\%$.
- The fourth term is calculated as a residual, so that the decomposition is exact from a mathematical point of view.

2 Linearity assumption and goal of this note

In the paper, we assume that the fourth term represents the effect of all processes other than kinetic fractionation during ice condensation and ocean surface evaporative conditions. However, this assumption is true only if all processes add up linearly. Otherwise, the fourth term could also include non-linear terms associated with kinetic effects during ice condensation and ocean surface evaporative conditions.

The goal of this note is thus to assess the validity of the assumption that all processes add up linearly.

3 Simple framework to evaluate the validity of the linearity assumption

To do so, we design a simple analytical model of $R_{v,control}$, and we compare the true value of $R_{v,control}$ to the value deduced from equation 1. The model is very simple but the goal is not to reproduce realistic $R_{v,control}$. The goal of this model is rather to have a framework to assess the validity of the linearity assumption.

The simple analytical model represents an initial water vapor parcel whose composition is given by the [Merlivat and Jouzel, 19 closure and which follows a Rayleigh distillation. Therefore:

$$R_v = \frac{R_{oce}}{\alpha_{eq} \cdot (\alpha_K + RH_s \cdot (1 - \alpha_K))} \cdot f^{\alpha_{eff}-1} \quad (2)$$

where symbols are the same as in the paper and f is the residual fraction of the vapor at the end of the distillation. α_{eq} depends on SST. α_{eff} is the effective fractionation coefficient taking into account kinetic effects following [Jouzel and Merlivat, 1984]. It depends on λ and on temperature. For simplicity we use the temperature at the end of the distillation. We assume that air parcels start their distillation at $SST=SST_{cste}=15^\circ\text{C}$ and $RH_s = RH_{s,cste}=60\%$ and cool by a temperature difference dT . f is thus a function of dT .

Therefore, $R_{v,control}$ is a function of 4 parameters: SST and RH_s during ocean surface evaporation, cooling dT and supersaturation parameter λ :

$$R_{v,control} = R_v(SST, RH_s, dT, \lambda)$$

We can apply the decomposition method to $R_{v,control}$ as:

$$\begin{aligned} R_{v,control} &= R_v(SST, RH_s, dT, \lambda) - R_v(SST, RH_s, dT, 0) \\ &+ R_v(SST, RH_s, dT, \lambda) - R_v(SST, RH_{s,cste}, dT, \lambda) \\ &+ R_v(SST, RH_{s,cste}, dT, \lambda) - R_v(SST_{cste}, RH_{s,cste}, dT, \lambda) \\ &+ R_v(SST_{cste}, RH_{s,cste}, dT, 0) + \epsilon \end{aligned}$$

The four terms have the exactly the same signification as in equation 1. The fourth term represents:

- the effect of all processes other than kinetic fractionation during ice condensation and ocean surface evaporative conditions, i.e. the effect of distillation without kinetic effects in the simple model, calculated as $R_v(SST_{cste}, RH_{s,cste}, dT, 0)$
- the effect of non linearities, ϵ , calculated as a residual.

Therefore, to assess the validity of the linearity assumption, we need to calculate ϵ .

4 Results

To calculate an upper bound for ϵ , we calculate it for extreme values of SST (from 1°C to 30°C), of RH_s (from 20% to 90%) and of dT (from 0°C to -50°C). λ is set to 0.004 as in LMDZ. The results are given in table 1.

Values for ϵ are very small: $< 0.1\text{‰}$ for $\delta^{18}\text{O}$ and of the order of numerical precision for ^{17}O -excess. These values are much lower than any variations that are considered in the paper. Very low values for ^{17}O -excess reflect the fact that ^{17}O -excess little insensitive to distillation ([Landais et al., 2008]). Values of ϵ are significant only for d-excess, but remain $< 1\text{‰}$ even for extreme cooling and extreme variations in SST and RH_s . In all cases, values of ϵ increase as the cooling increases.

5 Conclusion

The linearity assumption made in our decomposition method is valid, except for an uncertainty of up to 1‰ in the decomposition of d-excess in very cold areas.

References

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SST	1°C	30°C	1°C	30°C	1°C	30°C	1°C	30°C	1°C	30°C	1°C	30°C
RH_s	20%	20%	90%	90%	20%	20%	90%	90%	20%	20%	90%	90%
dT	0°C	0°C	0°C	0°C	-25°C	-25°C	-25°C	-25°C	-50°C	-50°C	-50°C	-50°C
ϵ for $\delta^{18}O$ (in ‰)	0	0	0	0	0.012	$4 \cdot 10^{-3}$	$1 \cdot 10^{-3}$	0.010	0.09	0.03	0.01	0.08
ϵ for d-excess (in ‰)	0	0	0	0	0.11	0.17	0.18	0.10	0.43	1.06	1.06	0.44
ϵ for ^{17}O -excess (in permeg)	0	0	0	0	0	0	0	0	0	0	0	0

Table 1: Absolute values of ϵ for $\delta^{18}O$, d-excess and ^{17}O -excess, for extreme values of SST, RH_s and dT . Values lower than 10^{-20} are written as 0.