Supplementary Material:

Technical Note: Correcting for signal attenuation from noisy proxy data in climate reconstructions

Caspar M. Ammann[1], Marc G. Genton[2] & Bo Li[3]

[1]National Center for Atmospheric Research, 1850 Table Mesa Drive, Boulder, CO 80307-3000, USA. [2]Department of Statistics, Texas A&M University, College Station, TX 77843-3143, USA. [3]Department of Statistics, Purdue University, West Lafayette, IN 47907, USA.

Climate of the Past "Technical Note"

1) Justification of correcting $\tilde{\sigma}_U^2$ using $\hat{\sigma}_U^2 = \tilde{\sigma}_U^2 - k\hat{\beta}_{1*,OLS}^2$ for $k>0$

Let $Y=\beta_0+\beta_1 X+\varepsilon$ and $W=X+U$, where $\varepsilon \sim iid\ N(0, \sigma_\varepsilon^2)$, $U \sim iid\ N(0, \sigma_U^2)$, and iid stands for independent and identically distributed. From the model, we get $X= -\beta_0/\beta_1+(1/\beta_1)Y-(1/\beta_1)\varepsilon$, which, when plugged into $W=X+U$ yields:

$$W= -\beta_0/\beta_1+(1/\beta_1)Y+U-(1/\beta_1)\varepsilon.$$

Define $\beta_{1*}=1/\beta_1$ and $\varepsilon_*=U-(1/\beta_1)\varepsilon$. Then, from the equation above we get $\sigma_{\varepsilon*}^2= \sigma_U^2+ \sigma_\varepsilon^2/\beta_1^2$, hence

$$\sigma_U^2= \sigma_{\varepsilon*}^2- \sigma_\varepsilon^2/\beta_1^2= \sigma_{\varepsilon*}^2- \beta_{1*}^2\sigma_\varepsilon^2.$$

So the variance of the noise is

$$\sigma_U^2 = \sigma_{\varepsilon*}^2- k\beta_{1*}^2$$

for $k>0$. Both $\sigma_{\varepsilon*}^2$ and β_{1*}^2 can be obtained from the regression of W on Y. Then we propose to use 5-fold cross-validation to determine k. The derivation for the multiple regression case is similar.

2) Annual reconstruction and increased variance

All reconstructions shown in the main manuscript were performed at full annual resolution of the data, but Figure 2 was smoothed with a 10-year Gaussian filter for illustration of the excellent performance of the reconstruction with regard to bias. As indicated in the text, the trade-off is that the variance at interannual time scales is increased. Fig. S1 shows the raw annual reconstruction result for CPS (blue, panel A) and for the multiple regression (red, panel B) in comparison to the known truth from the NCAR CSM 1.4 simulation (Ammann et al., 2007). The increase in variance is clearly visible, albeit a generally excellent performance even at annual scales is present. In particular the amplitude of cold years caused by volcanism (see for example years 1258, 1453 and 1815/16) are well captured by the reconstruction, a skill not commonly achieved by many reconstructions (Rutherford et al., 2005; Ammann and Wahl, 2007).

3) Effect of additional noise in model grid samples

For illustrating the difference between OLS and ACOLS-based reconstructions we show in Figs. 2 and S1 how isolated samples taken from a very small number of grid points (12 points) can be used to estimate a large-scale average quantity (NH average), despite the inherent spatial variability in the full model field. We point out that the relative relationship between predictors (here grid points) and the target (NH average temperature of a model simulation) are comparable to the numbers reported in Hegerl et al. (2007). However, in reality one has to deal with a series of uncertainties. Without claiming to be comprehensive, we demonstrate here how additional noise affects the comparison of the two reconstruction methods (OLS vs. ACOLS). We consider both unstructured “white”-noise (Fig. S2) and memory-carrying “red”-noise (Fig. S3). Both examples use a signal-to-noise ratio of 1:1, and the memory in the “red-noise” case is derived from an AR(2) process (Li et al., 2007).

4) Reconstruction Validation

Our examples were discussed with focus on effects on amplitude of reconstructed series due to the attenuation arising from the noise in the predictors. Another visual example can be achieved even within the calibration period. In Figure S4 we provide the results of a simple way to show the necessity and appropriateness for attenuation correction. We explain this idea by conducting a small experiment based on the calibration period data. In this experiment, we first sort the data by the temperature and divide the data into two groups based on the value of the temperatures, i.e., one with larger temperatures and the other one with smaller temperatures. Panels (a) and (b) in Fig. S4 illustrate that if we simply fit the OLS model using one group (thick grey line indicates the fitted model) but predict for the other group using the fitted model (the red line indicates the prediction line), the prediction will be clearly biased. Panel (c) shows that if we fit the linear model using the whole data, the line seems to be above the center of the group with smaller temperatures while below the center of the group with larger temperatures, a clear indication for attenuation. However, panel (d) shows that the ACOLS fitted model goes through the center of both groups indicating a largely unbiased slope.

While the above example allows identification of amplitude problems already in the calibration period, in reality there exist some additional options of testing for the occurrence of attenuation. Often data from a particular time period is withheld from the calibration process to be subsequently used in a verification process (Fritts et al., 1990; Mann et al., 1998; Ammann and Wahl, 2007; Wahl and Ammann, 2007). In our examples such verification opportunities could also be included for diagnostic purposes. For example, some observational data might exist for the second half of the 19th century. A difference in mean of reconstructed compared to observed mean NH temperature for these years would be an indicator for potential attenuation. Inspecting Figs. 2, S1, S2 and S3 shows that for most OLS solutions a difference in mean over a restricted late 19th century period would likely be recognized.

References

Ammann, C. M., Joos, F., Schimel, D. S., Otto-Bliesner, B. L., and Tomas, R. A.: Solar influence on climate during the past millennium: Results from transient simulations with the ncar climate system model, P Natl Acad Sci USA, 104, 3713-3718, 2007.

Ammann, C. M., and Wahl, E. R.: The importance of the geophysical context in statistical evaluations of climate reconstruction procedures, Climatic Change, 85, 71-88, 2007.

Hegerl, G. C., Crowley, T. J., Hyde, W. T., and Frame, D. J.: Climate sensitivity constrained by temperature reconstructions over the past seven centuries, Nature, 440, 1029-1032, 2006.

Hegerl, G. C., Crowley, T. J., Allen, M., Hyde, W. T., Pollack, H. N., Smerdon, J., and Zorita, E.: Detection of human influence on a new, validated 1500-year temperature reconstruction, Journal of Climate, 20, 650-666, 2007.

Li, B., Nychka, D. W., and Ammann, C. M.: The 'hockey stick' and the 1990s: A statistical perspective on reconstructing hemispheric temperatures, Tellus Series a-Dynamic Meteorology and Oceanography, 59, 591-598, 2007.

Mann, M. E., Bradley, R. S., and Hughes, M. K.: Global-scale temperature patterns and climate forcing over the past six centuries, Nature, 392, 779-787, 1998.

Rutherford, S., Mann, M. E., Osborn, T. J., Bradley, R. S., Briffa, K. R., Hughes, M. K., and Jones, P. D.: Proxy-based northern hemisphere surface temperature reconstructions: Sensitivity to method, predictor network, target season, and target domain, Journal of Climate, 18, 2308-2329, 2005.

Wahl, E. R., and Ammann, C. M.: Robustness of the mann, bradley, hughes reconstruction of northern hemisphere surface temperatures: Examination of criticisms based on the nature and processing of proxy climate evidence, Climatic Change, 85, 33-69, 2007.

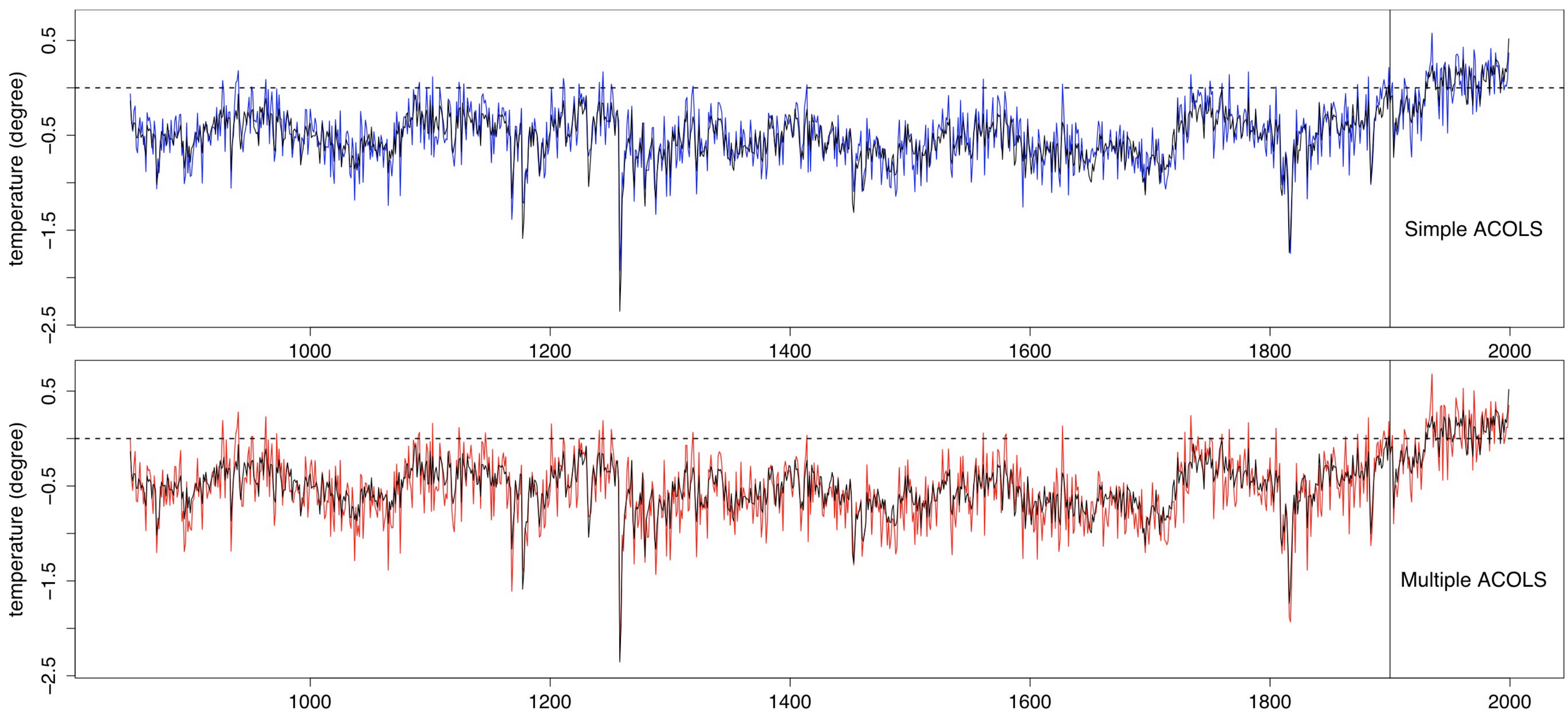

Figure S1: Same as Fig. 2, but for full annual resolution. CPS (blue) and multiple (red) regression reconstructions of NH mean temperature using ACOLS based on a network of twelve grid-points – comparable to Hegerl et al.(Hegerl et al., 2006) – here subsampled from output of a coupled GCM (Ammann et al., 2007) where the true climate is known.

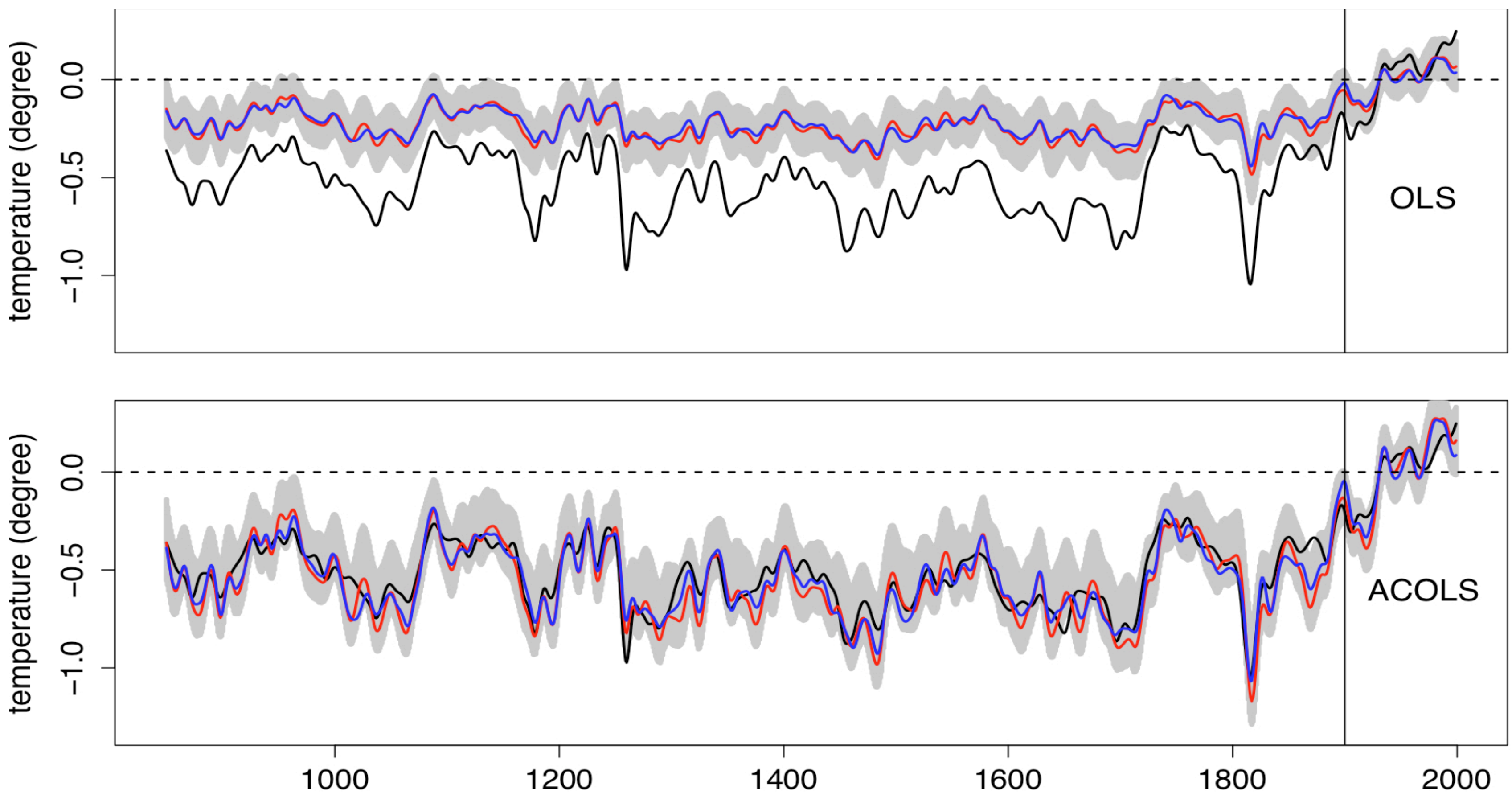

Figure S2: Same as Figure S1, but for additional “white” noise added to each grid point sample using a Signal to Noise ratio of 1:1.

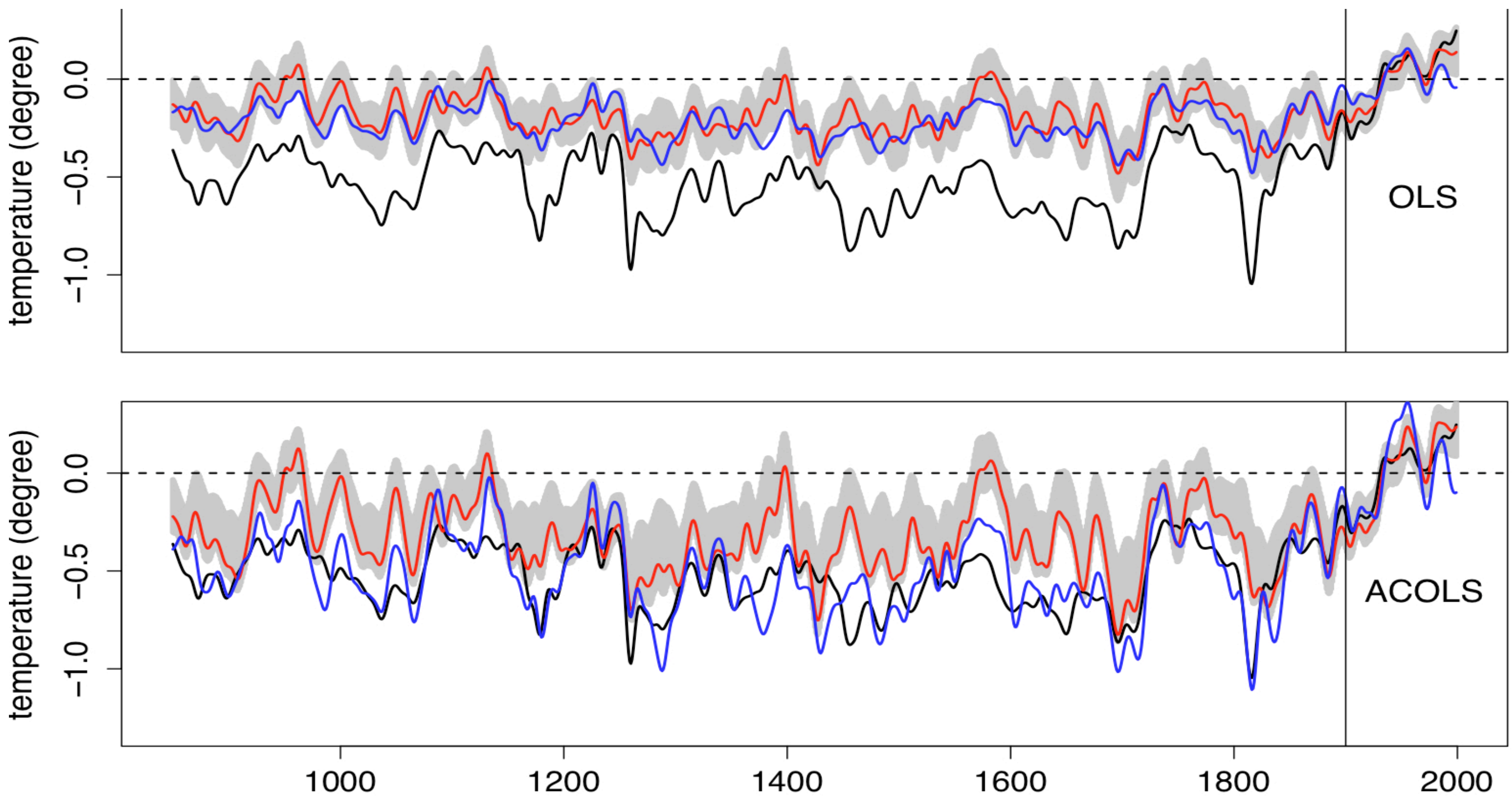

Figure S3: Same as Figure S2, but for additional "red" noise added to each grid point sample using a AR(2) process after Li et al. (2007) and a Signal to Noise ratio of 1:1.

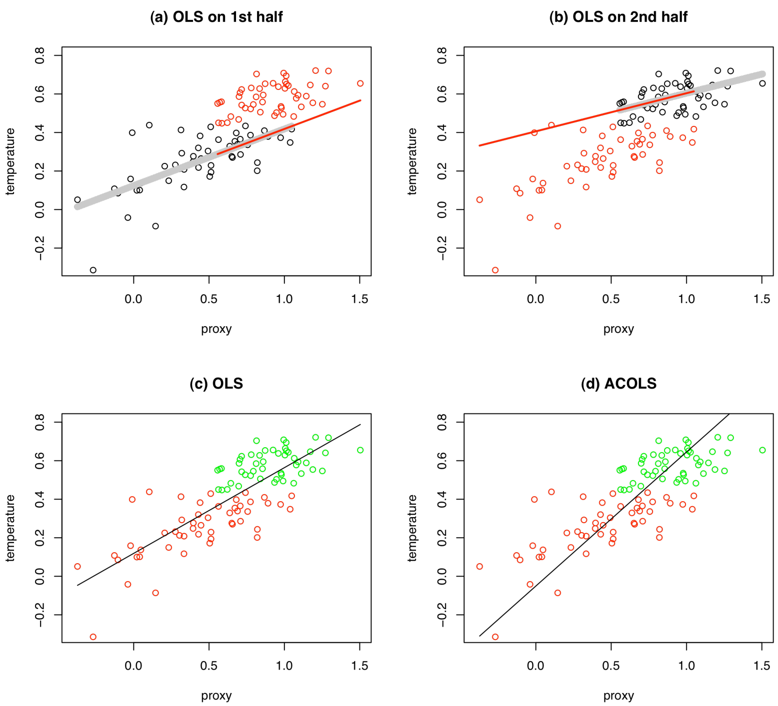

Figure S4: Calibration period exercise to identify potential for signal attenuation. Calibration data was split in half, each group either containing the warmest or the coolest observed temperatures, respectively. Panels (a) and (b) show the calibration on half the data (gray) and prediction (red) of the relationship between proxy and observational temperature. A clear bias is seen in either case. Panel (c) shows the OLS fit based on the full calibration dataset. The regression line is improved but still biased with most of the warm points (green) above the regression line, and most of the cold points below. Panel (d) shows the corresponding ACOLS solution that has both groups more faithfully represented.