



Supplement of

Influence of the choice of insolation forcing on the results of a conceptual glacial cycle model

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This supplementary material contains three additional experiments that were not included in the main manuscript for clarity.

<u>Section S1</u> : Our model's results are shown in the case where the summer solstice insolation at 50° N is used, instead of 65° N. These results are compared to the four summer insolation forcings used in the manuscript (summer solstice insolation, caloric season, ISI above 300 W m⁻² and above 400 W m⁻², all at 65° N).

<u>Section S2</u> : Instead of using our accuracy criteria as fitting criteria as defined in the manuscript, we use the correlation coefficient and show the results obtained in that case.

<u>Section S3</u>: We investigate the model's response to an input forcing containing only precession and only obliquity. To do so, we compare the model results with 3 different forcings :

- the summer solstice insolation at 65°N as in the manuscript
- the "fixed obliquity / precession only" forcing. This forcing was obtained by computing the summer solstice insolation at 65° N but with a constant obliquity value (equal to the current value)
- the "fixed precession / obliquity only" forcing. This forcing was obtained by computing the summer solstice insolation at 65° N but with a constant value of the precession parameter (equal to the current value)

<u>Section S1 : Model results in the case of the summer solstice at 50° N input</u> <u>forcing and comparison with the four other forcings of the manuscript.</u>



<u>Figure S1</u>: (a) The four different summer insolation types at 65° N and the summer solstice insolation at 50° N, displayed over the Quaternary period. (b) Corresponding spectral analysis, normalized by the standard deviation.



<u>Figure S2</u>: Optimal deglaciation threshold V_0 over the five different periods for the four different summer insolation forcings at 65° N and the summer solstice insolation at 50° N. When several values of the deglaciation threshold V_0 maximize the accuracy criteria, the mean value is plotted and the other possible values are represented with errorbars.



<u>Figure S3</u>: Accuracy over the five different periods for the four different summer insolation forcings at 65° N and the summer solstice insolation at 50° N.



<u>Figure S4</u>: Best model fit over the whole Quaternary and corresponding spectral analysis. The middle panel represents the best fit of the model for the different summer insolation used as input, compared to the data. The data (normalized LR04 curve) are in black. The blue shading represents deglaciation periods in the data and the yellow shading deglaciation periods in the model. This results in a green shading when deglaciations are seen in the model and data at the same time. The left panel represents the spectral analysis of the best fit solution over the last million year. The right panel represents the spectral analysis over the more ancient part of the Quaternary (before 1 Ma BP)



<u>Figure S5</u>: Normalized model results over the last million year, with the four different summer insolation forcings at 65° N and the summer solstice insolation at 50° N. The full line is the best fit computed over the 100 - 1000 ka BP period. The dashed line represents the same solution,

but with an increased V_0 threshold for the last deglaciation. The data (LR04 stack curve normalized) are in black. The yellow shading represents deglaciation periods in the model (case of the increased V_0 threshold) and the blue shading represents deglaciation periods in the data. This results in a green shading when deglaciations are seen in the model and data at the same time

The summer solstice insolation at 50° N has a lower obliquity component than the summer solstice insolation at 65° N (shown in Figure S1).

With the summer solstice insolation at 50° N, we obtain the same conclusion as in the case of forcings at 65° N. The optimal deglaciation V_0 (obtained while keeping all other model parameter constants), increases over the Quaternary (Figure S2), with lower values around 3 at the start of the Quaternary and values around 5 after the MPT.

As in the case of the summer solstice insolation at 65° N, the accuracy values of the results with the summer solstice insolation at 50° N are higher on the later part of the Quaternary (Figure S3). Compared to the solstice insolation at 65° N, the solstice insolation at 50° N always leads to a lower accuracy on the earliest part of the record. This seems logical as the insolation at 50° N has a weaker obliquity component, and the pre MPT period is dominated by obliquity (41 kyr cycles). Following the accuracy criteria defined in the manuscript, their performances on the post MPT part are comparable.

On Figure S4, the best guess over the Quaternary was obtained and the spectral analysis over the pre MPT ([1000 - 2500] kyr BP) and post MPT ([0 - 1000] kyr BP) was carried out.

The spectral analysis over the pre MPT period shows that model results with the summer solstice insolation at 50° N as forcing fail to reproduce the dominance of the obliquity cycle that is visible on the data on that part of the record. As for the accuracy value on this period, this is not surprising, as the summer solstice insolation at 50° N has a low obliquity component. On the post MPT part, larger cycles are produced, but their frequency do not match the data.

On the last million year, the model output with the summer solstice insolation at 50° N as forcing reproduces quite well the data. As in the case of the summer solstice insolation at 65° N, there is no need to increase the V₀ threshold over the last 100 kyr in order to reproduce the last cycle. This is visible in Figure S5, where the best fit over the [0 - 1000] kyr BP period with a constant V₀ threshold on this period is shown in full lines. The dashed lines represent the case where the V₀ threshold is increased over the last 100 kyr. For the summer solstice insolation at 65°N and 50°N, the full and dashed curve overlap, as increasing the V₀ threshold over the last 100 kyr does not change the results.

Section S2 : Model results in the case of the correlation coefficient as accuracy criteria



<u>Figure S6</u> :Optimal deglaciation threshold V₀ over the five different periods for the four different summer insolation forcings at 65° N. The left part displays the result when the initial accuracy criteria of the manuscript is used. The right part displays the result when the correlation coefficient is used as accuracy criteria. When several values of the deglaciation threshold V₀ maximize the accuracy criteria, the mean value is plotted and the other possible values are represented with errorbars.



<u>Figure S7</u>: Accuracy over the five different periods for the four different summer insolation forcings at 65° N. The right part displays the result when the correlation coefficient is used as accuracy criteria.

Best guess over the Quaternary with initial criteria



<u>Figure S8</u>: Best model fit over the whole Quaternary and corresponding spectral analysis when the accuracy criteria is the initial criteria



<u>Figure S9</u>: Best model fit over the whole Quaternary and corresponding spectral analysis when the accuracy criteria is the correlation coefficient.

Best guess over the last Myr with initial criteria



<u>Figure S10</u>: Normalized model results over the last million year, with the four different summer insolation forcings at 65° N when the accuracy criteria is the initial criteria of the manuscript.



<u>Figure S11</u> : Normalized model results over the last million year, with the four different summer insolation forcings at 65° N when the accuracy criteria is the correlation coefficient.

Changing the accuracy criteria from the initial criteria proposed in the manuscript to the correlation coefficient changes the value of the optimal V_0 threshold, as visible in Figure S6. However, this does not change the general tendency : lower values of the V_0 threshold allow to

better fit the earliest part of the record, while higher values lead to a better fit on the post MPT period. For all insolation forcings (except the summer solstice insolation, detailed below), there is a clear tendency of increase of the V_0 threshold over the different time periods.

The correlation coefficient is better on the latest part of the record compared to the pre MPT part for all insolation forcings, and the summer solstice insolation leads to poorer results than all the other forcings on the pre MPT period, as visible in Figure S7.

On Figure S8, the optimal fit over the Quaternary when the initial criteria is used as the fitting criterion is displayed. On Figure S9, the optimal fit over the Quaternary when the correlation criteria is used as the fitting criterion is displayed. When the caloric season and the ISI above 300 W/m^2 and 400 W/m^2 are used as input forcing, the spectral analysis over the pre MPT period does not change much. However, for the summer solstice insolation, the spectral analysis shows a less pronounced obliquity peak than in the case of the initial fitting criteria. This is due to the fact that the model output does not match well the data on the [2000 - 2500] kyr BP period, where the optimal V₀ threshold is high in comparison to all other forcings.

This shows us that the correlation coefficient might not be the best criteria to determine which model output best fits our data. In this case (for the summer solstice insolation forcing on the [2000 - 2500] kyr BP period), the highest correlation coefficient is obtained for a relatively high V_0 threshold value (V_0 = 4.6), that leads to too large ice volumes compared to the data on this period.

On Figures S10 and S11, the best fit over the last Myr period is displayed. Figure S10 corresponds to the original figure of the manuscript, where the initial accuracy criteria is used, whereas Figure S11 corresponds to the case where the correlation coefficient is used as fitting criteria.

On the [0 - 1000] kyr BP period, the optimal V_0 values obtained with the correlation coefficient as a fitting criteria lead to a greater model-data mismatch than in the case of our initial accuracy criteria. The deglaciations are not all at the right place.

To summarize, the exact values obtained with the coefficient correlation for the V₀ threshold and the corresponding model realizations are different from the values obtained when our initial accuracy criteria is used. However, this does not change the main results of the paper, as we are able to produce a shift from 41 kyr to larger cycles, by increasing the V₀ threshold, and thus for all insolation forcings.

Our criteria is better suited for this model (that is threshold - based) and the choice of the solution corresponding to the highest initial criteria gives better results than when choosing the one maximizing the correlation coefficient, in terms of deglaciations placement.

<u>Section S3 : Model results in the case of forcings with precession only /</u> <u>obliquity only</u>

In this section, we compare the model results with 3 different forcings :

- the summer solstice insolation at 65°N as in the manuscript
- the "fixed obliquity / precession only" forcing. This forcing was obtained by computing the summer solstice insolation at 65° N but with a constant obliquity value (equal to the current value)
- the "fixed precession / obliquity only" forcing. This forcing was obtained by computing the summer solstice insolation at 65° N but with a constant value of the precession parameter (equal to the current value)



<u>Figure S12</u>: (a) Summer solstice at 65°N, fixed obliquity and fixed precession forcings, displayed over the Quaternary period. (b) Corresponding spectral analysis, normalized by the standard deviation.



<u>Figure S13</u>: Optimal deglaciation threshold V₀ over the five different periods for the summer solstice at 65°N, fixed obliquity and fixed precession forcings. When several values of the deglaciation threshold V₀ maximize the accuracy criteria, the mean value is plotted and the other possible values are represented with errorbars.



<u>Figure S14</u>: Accuracy over the five different periods for the summer solstice at 65°N, fixed obliquity and fixed precession forcings.



<u>Figure S15</u>: Best model fit over the whole Quaternary and corresponding spectral analysis. The middle panel represents the best fit of the model for the different insolation used as input, compared to the data. The data (normalized LR04 curve) is in black. The blue shading represents deglaciation periods in the data and the yellow shading deglaciation periods in the model. This results in a green shading when deglaciations are seen in the model and data at the same time. The left panel represents the spectral analysis of the best fit solution over the last million year. The right panel represents the spectral analysis over the more ancient part of the Quaternary (before 1 Ma BP)



<u>Figure S16</u> : Normalized model results over the last million year, with the summer solstice at 65°N, fixed obliquity and fixed precession forcings. The colored lines are the best fit computed over the 0 - 1000 ka BP period. The data (LR04 stack curve normalized) is in black. The yellow shading represents deglaciation periods in the model and the blue shading represents deglaciation periods in a green shading when deglaciations are seen in the model and data at the same time

Figure S12 represents the three forcings. As expected, the spectral analysis of the "fixed obliquity" forcing has only components from the precession, and conversely the "fixed precession" forcing has only an obliquity component.

Figure S14 shows that the summer solstice insolation allows a better accuracy in reproducing the last part of the record than the fixed obliquity and fixed precession forcings.

Figure S15 shows the best fit over the whole Quaternary. With a fixed obliquity (precession only) forcing, our model is not able to reproduce the pre-MPT 40 kyr signal, whereas this is possible with the fixed precession (obliquity only) forcing. It is not surprising that an input forcing with no obliquity component does not allow to reproduce the pre MPT, obliquity-dominated record. On the latest part of the Quaternary, the results obtained are less satisfying with the fixed precession and fixed obliquity forcings than with the summer solstice insolation. Concerning the spectral analysis, for the fixed precession (obliquity only) forcing, obliquity continues to dominate after the MPT. For the fixed obliquity (precession only) forcing, there is a 100 kyr peak after the MPT. However, concerning the terminations placement, both of these forcings fail to successfully represent the post MPT part of the record.

Figure S16 shows the best fit over the [0 - 1000] kyr BP period. When the optimization is done over the [0 - 1000] kyr period, the respective accuracy over this period is 0.92 for the summer solstice forcing and 0.67 for both the fixed obliquity and fixed precession forcings. With the fixed obliquity, and fixed precession forcing, some terminations are misplaced.

For instance, with the fixed obliquity (precession only) forcing, Termination V (around 420 kyr BP) is triggered too late. With the fixed precession (obliquity only) forcing, Termination VII and Termination IX (around 620 and 790 kyr BP) are misplaced.

These results show that when an obliquity only or precession only forcing are used, our model is not able to represent the latest part of the Quaternary record as well as when forcings with contributions from both the precession and the obliquity are used.

In the case of the ISI 300 W/m² forcing studied in the manuscript, the precession component is small, but is not absent, and allows a better fit to data. With our accuracy criteria, the accuracy of this forcing is comparable to the other forcings, but differences are indeed noticeable when looking at the record.