Supplement of Clim. Past, 16, 1617–1642, 2020
https://doi.org/10.5194/cp-16-1617-2020-supplement

Supplement of

Investigating interdecadal salinity changes in the Baltic Sea in a 1850–2008 hindcast simulation

Hagen Radtke et al.

Correspondence to: Hagen Radtke (hagen.radtke@io-warnemuende.de)

1 Changes in inflow due to runoff

Inflow across Drogden Sill can be calculated from sea level at Viken and Klagshamn, η_V and η_K, by the following formula (**?**):

$$Q_{Sound} = \text{sign}(\eta_V - \eta_K) K \sqrt{|\eta_K - \eta_V|} \quad (1)$$

with the coefficient $K = 74770 \text{ m}^{\frac{5}{2}} \text{ s}^{-1}$. The sea level at Klagshamn relates to the mean sea level of the Baltic Sea,

$$\eta_K = \eta_{BS} + \eta'_K , \quad (2)$$

where the deviation η'_K is mostly meteorologically caused by wind stress. The Baltic Sea mean sea level is related to the total volume of the Baltic Sea by

$$\frac{dV_{BS}}{d\eta_{BS}} = A . \quad (3)$$

The term A_{BS} denotes the surface area of the Baltic Sea ($4.12 \cdot 10^{11}$ m^2). If we neglect density variations (barotropic view), the volume of the Baltic Sea changes by three processes: Runoff, Precipitation minus evaporation, and volume flux from the North Sea,

$$\frac{dV_{BS}}{dt} = Q_{runoff} + Q_{PME} + Q_{NorthSea} , \quad (4)$$

which means

$$\frac{d\eta_{BS}}{dt} = \frac{1}{A} \left(Q_{runoff} + Q_{PME} + Q_{NorthSea} \right) . \quad (5)$$

With a ratio of 7:3:1 for the transports between Great Belt, Sound and Little Belt (**?**), we get $Q_{NorthSea} = 11/3 \cdot Q_{Sound}$, meaning

$$\frac{d\eta_{BS}}{dt} = \frac{1}{A} \left(Q_{runoff} + Q_{PME} + \frac{11}{3} \text{sign}(\eta_V - \eta_K) K \sqrt{|\eta_K - \eta_V|} \right) . \quad (6)$$

If we neglect wind variations and assume $\eta_{BS} = \eta_K = \eta$, we get

$$\frac{d\eta}{dt} = \frac{1}{A} \left(Q_{runoff} + Q_{PME} + \frac{11}{3} \text{sign}(\eta_V - \eta) K \sqrt{|\eta - \eta_V|} \right) . \quad (7)$$

The stationary solution is given by

$$0 = Q_{runoff} + Q_{PME} - \frac{11}{3} K \sqrt{|\eta - \eta_V|} \quad (8)$$

$$\eta - \eta_V = \left(\frac{3}{11K} \left(Q_{runoff} + Q_{PME} \right) \right)^2 . \quad (9)$$

With typical values ($Q_{runoff} = 14000$ m^3 s^{-1}, $Q_{PME} = 1300$ m^3 s^{-1}) this gives a value of 3.1 mm which should be the sea level difference if the outflow does not vary. But now let us assume that the variation in η only vanishes in the temporal mean:

$$0 = < Q_{runoff} + Q_{PME} > + \frac{11}{3} K < \mathrm{sign}(\eta_V - \eta)\sqrt{|\eta - \eta_V|} > . \qquad (10)$$

Now we can differentiate with respect to η and obtain

$$\begin{aligned}
0 &= \frac{d}{d\eta} < Q_{runoff} + Q_{PME} > + \frac{11}{3} K < \frac{d}{d\eta}\left(\mathrm{sign}(\eta_V - \eta)\sqrt{|\eta - \eta_V|}\right) > \\
&= \frac{d}{d\eta} < Q_{runoff} + Q_{PME} > + \frac{11}{3} K < \mathrm{sign}(\eta_V - \eta)\frac{d}{d\eta}|\eta - \eta_V|^{0.5} > \\
&= \frac{d}{d\eta} < Q_{runoff} + Q_{PME} > + \frac{11}{3} K < \mathrm{sign}(\eta_V - \eta) 0.5 |\eta - \eta_V|^{-0.5} \mathrm{sign}(\eta - \eta_V) > \\
&= \frac{d}{d\eta} < Q_{runoff} + Q_{PME} > - \frac{11}{6} K < \sqrt{|\eta - \eta_V|}^{-1} > \qquad (11) \\
\frac{11}{6} K < \sqrt{|\eta - \eta_V|}^{-1} > &= \frac{d}{d\eta} < Q_{runoff} + Q_{PME} > \qquad (12)
\end{aligned}$$

We can easily calculate that from the observed hourly time series and obtain a value of 488353 m^2 s^{-1} for $dQ/d\eta$. This means that a 7% change in runoff will change Baltic Sea level systematically by about 2.2 mm.

These 2.2 mm mean a volume of 0.9 km^3, or 1.1% of the volume of a typical DS5 inflow, or 4.8% of the typical inflow volume of 18.75 km^3. This explains roughly half of the observed variation (around 10% at the 30-year time scale). It takes 10 days to get the additional volume out by the enhanced outflow then.

2 Budget calculations

$$
\begin{aligned}
Q_{up}^{s} &= Q_{bottom}^{s} - \frac{d}{dt} s_{bottom} && (13)\\
Q_{up}^{V} &= Q_{bottom}^{V} - \frac{d}{dt} V_{bottom} && (14)\\
Q_{surface}^{s} &= Q_{up}^{s} - \frac{d}{dt} s_{surface} && (15)\\
Q_{surface}^{V} &= Q_{up}^{V} + Q_{river}^{V} - \frac{d}{dt} V_{surface} && (16)\\
\tilde{Q}_{surface}^{s} &= \alpha Q_{surface}^{V} \frac{s_{surface}}{V_{surface}} && (17)\\
\frac{d}{dt} s_{surface} &= Q_{up}^{s} - \alpha Q_{surface}^{V} \frac{s_{surface}}{V_{surface}} && (18)\\
s_{surface} &= a(t) \left(s_{surface}(t_0) + \int_{t_0}^{t} \frac{Q_{up}^{s}(t')}{a(t')} \, dt' \right) && (19)\\
a(t) &= \exp \left(- \int_{t_0}^{t} \frac{Q_{surface}^{V}}{V_{surface}} \, dt' \right) && (20)
\end{aligned}
$$